Time Period and Frequency of S.H.M.

For S.H.M. restoring force is proportional to the displacement

$F \propto y$ or $F = -ky$ ...(i) where k is a force constant.

For S.H.M. acceleration of the body $A = -\omega^2 y$ ...(ii)

$\therefore$ Restoring force on the body $F = mA = -m\omega^2 y$ ...(iii)

From (i) and (iii) $ky = m\omega^2 y \Rightarrow \omega = \sqrt{\frac{k}{m}}$

$\therefore$ Time period $(T) = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

or Frequency (n) $= \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

In different types of S.H.M. the quantities m and k will go on taking different forms and names.

In general m is called inertia factor and k is called spring factor.

Thus $T = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}}$

or $n = \frac{1}{2\pi}\sqrt{\frac{\text{Spring factor}}{\text{Inertia factor}}}$

In linear S.H.M. the spring factor stands for force per unit displacement and inertia factor for mass of the body executing S.H.M. and in Angular S.H.M. k stands for restoring torque per unit angular displacement and inertial factor for moment of inertia of the body executing S.H.M.

For linear S.H.M. $T = 2\pi\sqrt{\frac{m}{k}} = \sqrt{\frac{m}{\text{Force/Displacement}}} = 2\pi\sqrt{\frac{m \times \text{Displacement}}{m \times \text{Acceleration}}}$

$= 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\frac{y}{A}}$

or $n = \frac{1}{2\pi}\sqrt{\frac{\text{Acceleration}}{\text{Dispalcement}}} = \frac{1}{2\pi}\sqrt{\frac{A}{y}}$