Velocity in S.H.M.

Velocity of the particle executing S.H.M. at any instant, is defined as the time rate of change of its displacement at that instant.

In case of S.H.M. when motion is considered from the equilibrium position

$$y = a\sin\omega t$$

So $$v = \frac{dy}{dt} = a\omega\cos\omega t$$

$\therefore$ $$v = a\omega\cos\omega t \qquad \text{.....(i)}$$

or $$v = a\omega\sqrt{1-\sin^2\omega t} \qquad \text{[As } \sin\omega \text{ t = y/a]}$$

or $$v = \omega\sqrt{a^2 - y^2} \qquad \text{.....(ii)}$$

Important points

(i) In S.H.M. velocity is maximum at equilibrium position.

From equation (i) $v_{max} = a\omega$ when $|\cos\omega t| = 1$ i.e. $\theta = \omega t = 0$

From equation (ii) $v_{max} = a\omega$ when $y = 0$

(ii) In S.H.M. velocity is minimum at extreme position.

From equation (i) $v_{min} = 0$ when $|\cos\omega t| = 0$ i.e. $\theta = \omega t = \frac{\pi}{2}$

From equation (ii) $v_{min} = 0$ when y = a

(iii) Direction of velocity is either towards or away from mean position depending on the position of particle.