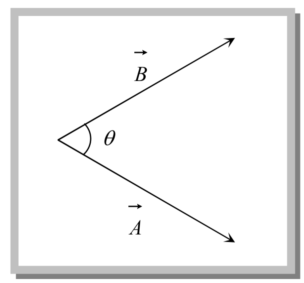
Scalar product of two vectors.

(1) **Definition:** The scalar product (or dot product) of two vectors is defined as the product of the magnitude of two vectors with cosine of angle between them.

Thus if there are two vectors $\vec{A}$ and $\vec{B}$ having angle θ between them, then their scalar product written as $\vec{A}.\vec{B}$ is defined as $\vec{A}.\vec{B} = AB\cos\theta$

(2) **Properties:** (i) It is always a scalar which is positive if angle between the vectors is acute (*i.e.*, < 90°) and negative if angle between them is obtuse (*i.e.* 90° < θ < 180°).



(ii) It is commutative, *i.e.* $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

(iii) It is distributive, *i.e.* $\vec{A}.(\vec{B}+\vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$

(iv) As by definition $\vec{A}.\vec{B} = AB\cos\theta$

The angle between the vectors $\theta = \cos^{-1}\left[\frac{\vec{A}.\vec{B}}{AB}\right]$

(v) Scalar product of two vectors will be maximum when $\cos\theta = \max = 1$, *i.e.* $\theta = 0^o$, *i.e.*, vectors are parallel $(\vec{A}.\vec{B})_{\max} = AB$

(vi) Scalar product of two vectors will be minimum when $|\cos\theta| = \min = 0$, *i.e.* $\theta = 90^o$ $(\vec{A}.\vec{B})_{\min} = 0$

i.e., if the scalar product of two nonzero vectors vanishes the vectors are orthogonal.

(vii) The scalar product of a vector by itself is termed as self dot product and is given by $(\vec{A})^2 = \vec{A}.\vec{A} = AA\cos\theta = A^2$

i.e., $A = \sqrt{\vec{A}.\vec{A}}$

(viii) In case of unit vector $\hat{n}$

$\hat{n}.\hat{n} = 1 \times 1 \times \cos 0 = 1$ so $\hat{n}.\hat{n} = \hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

(ix) In case of orthogonal unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$, $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 1 \times 1 \cos 90 = 0$

(x) In terms of components $\vec{A}.\vec{B} = (\hat{i}A_x + \hat{j}A_y + \hat{k}A_z).(\hat{i}B_x + \hat{j}B_y + \hat{k}B_z)$
$= [A_x B_x + A_y B_y + A_Z B_z]$

(3) **Example:** (i) Work W: In physics for constant force work is defined as, $W = Fs\cos\theta$ ....(i)

But by definition of scalar product of two vectors, $\vec{F}.\vec{s} = Fs\cos\theta$ ....(ii)

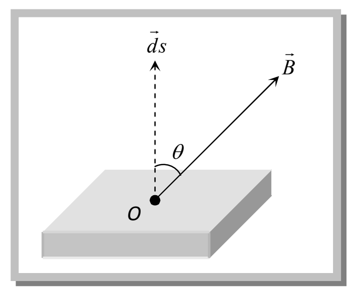
So from *eq*^{n} (i) and (ii) $W = \vec{F}.\vec{s}$ *i.e.* work is the scalar product of force with displacement.

(ii) Power P:

As $W = \vec{F}.\vec{s}$ or $\frac{dW}{dt} = \vec{F}.\frac{d\vec{s}}{dt}$ [As $\vec{F}$ is constant]

or $P = \vec{F}.\vec{v}$ *i.e.*, power is the scalar product of force with velocity. $\left[\text{As} \frac{dW}{dt} = P \text{ and } \frac{d\vec{s}}{dt} = \vec{v}\right]$

(iii) Magnetic Flux ϕ:



Magnetic flux through an area is given by $d\phi = B\,ds\cos\theta$ ......(i)

But by definition of scalar product $\vec{B}.d\vec{s} = Bds\cos\theta$ ......(ii)

So from *eq*^{n} (i) and (ii) we have

$d\phi = \vec{B}.d\vec{s}$ or $\phi = \int \vec{B}.d\vec{s}$

(iv) Potential energy of a dipole U: If an electric dipole of moment $\vec{p}$ is situated in an electric field $\vec{E}$ or a magnetic dipole of moment $\vec{M}$ in a field of induction $\vec{B}$, the potential energy of the dipole is given by :

$U_E = -\vec{p}.\vec{E}$ and $U_B = -\vec{M}.\vec{B}$