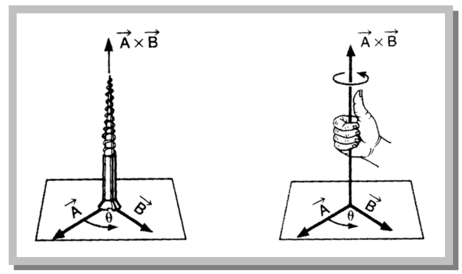
Vector product of two vector.

(1) **Definition**: The vector product or cross product of two vectors is defined as a vector having a magnitude equal to the product of the magnitudes of two vectors with the sine of angle between them, and direction perpendicular to the plane containing the two vectors in accordance with right hand screw rule.

$$\vec{C} = \vec{A} \times \vec{B}$$

Thus, if $\vec{A}$ and $\vec{B}$ are two vectors, then their vector product written as $\vec{A} \times \vec{B}$ is a vector $\vec{C}$ defined by

$$\vec{C} = \vec{A} \times \vec{B} = AB \sin\theta\, \hat{n}$$



The direction of $\vec{A} \times \vec{B}$, *i.e.* $\vec{C}$ is perpendicular to the plane containing vectors $\vec{A}$ and $\vec{B}$ and in the sense of advance of a right handed screw rotated from $\vec{A}$ (first vector) to $\vec{B}$ (second vector) through the smaller angle between them. Thus, if a right handed screw whose axis is perpendicular to the plane framed by $\vec{A}$ and $\vec{B}$ is rotated from $\vec{A}$ to $\vec{B}$ through the smaller angle between them, then the direction of advancement of the screw gives the direction of $\vec{A} \times \vec{B}$ *i.e.* $\vec{C}$

(2) **Properties:**

(i) Vector product of any two vectors is always a vector perpendicular to the plane containing these two vectors, *i.e.*, orthogonal to both the vectors $\vec{A}$ and $\vec{B}$, though the vectors $\vec{A}$ and $\vec{B}$ may or may not be orthogonal.

(ii) Vector product of two vectors is not commutative, *i.e.*, $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$ [but $= -\vec{B} \times \vec{A}$]

Here it is worthy to note that

$|\vec{A} \times \vec{B}| = |\vec{B} \times \vec{A}| = AB \sin\theta$

i.e., in case of vector $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$ magnitudes are equal but directions are opposite.

(iii) The vector product is distributive when the order of the vectors is strictly maintained, *i.e.*

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

(iv) As by definition of vector product of two vectors $\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n}$

So $|\vec{A}\times\vec{B}| = AB\sin\theta$ *i.e.*, $\theta = \sin^{-1}\left[\frac{|\vec{A}\times\vec{B}|}{|\vec{A}||\vec{B}|}\right]$

(v) The vector product of two vectors will be maximum when $\sin\theta = \max = 1$, *i.e.*, $\theta = 90^o$

$$[\vec{A}\times\vec{B}]_{\max} = AB\ \hat{n}$$

i.e., vector product is maximum if the vectors are orthogonal.

(vi) The vector product of two non- zero vectors will be minimum when $|\sin\theta| =$ minimum = 0, *i.e.*, $\theta = 0^o$ or 180^o

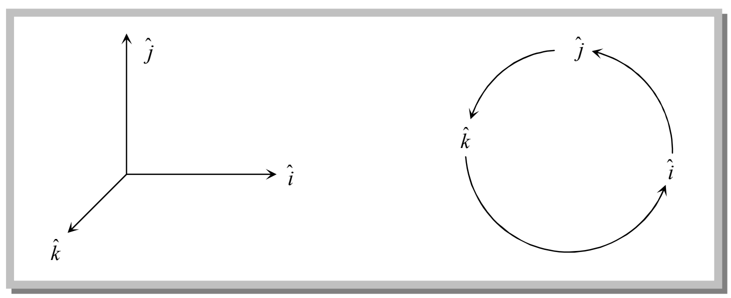
$$[\vec{A}\times\vec{B}]_{\min} = 0$$

*i.e.*if the vector product of two non-zero vectors vanishes, the vectors are collinear.

(vii) The self-cross product, *i.e.*, product of a vector by itself vanishes, *i.e.*, is null vector $\vec{A}\times\vec{A} = AA\sin 0^o\ \hat{n} = \vec{0}$

(viii) In case of unit vector $\hat{n}\times\hat{n} = \vec{0}$ so that $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \vec{0}$

(ix) In case of orthogonal unit vectors, $\hat{i},\hat{j},\hat{k}$ in accordance with right hand screw rule :



$$\hat{i}\times\hat{j} = \hat{k}, \qquad \hat{j}\times\hat{k} = \hat{i} \qquad \text{and} \qquad \hat{k}\times\hat{i} = \hat{j}$$

And as cross product is not commutative,

$$\hat{j}\times\hat{i} = -\hat{k} \qquad \hat{k}\times\hat{j} = -\hat{i} \qquad \text{and} \qquad \hat{i}\times\hat{k} = -\hat{j}$$

(x) In terms of components

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}(A_yB_z - A_zB_y) + \hat{j}(A_zB_x - A_xB_z) + \hat{k}(A_xB_y - A_yB_x)$$

(3) **Example:**Since vector product of two vectors is a vector, vector physical quantities (particularly representing rotational effects) like torque, angular momentum, velocity and force on a moving charge in a magnetic field and can be expressed as the vector product of two vectors. It is well – established in physics that:

(i) Torque $\vec{\tau} = \vec{r} \times \vec{F}$

(ii) Angular momentum $\vec{L} = \vec{r} \times \vec{p}$

(iii) Velocity $\vec{v} = \vec{\omega} \times \vec{r}$

(iv) Force on a charged particle q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by $\vec{F} = q(\vec{v} \times \vec{B})$

(v) Torque on a dipole in a field $\overrightarrow{\tau_E} = \vec{p} \times \vec{E}$ and $\overrightarrow{\tau_B} = \vec{M} \times \vec{B}$