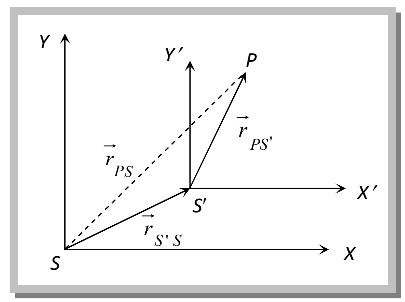
Relative velocity.

(1) **Introduction:** When we consider the motion of a particle, we assume a fixed point relative to which the given particle is in motion. For example, if we say that water is flowing or wind is blowing or a person is running with a speed *v*, we mean that these all are relative to the earth (which we have assumed to be fixed).

Now to find the velocity of a moving object relative to another moving object, consider a particle *P* whose position relative to frame *S* is $\vec{r}_{PS}$ while relative to S' is $\vec{r}_{PS'}$. If the position of frames S' relative to *S* at any time is $\vec{r}_{S'S}$ then from fig.



$$\vec{r}_{PS} = \vec{r}_{PS'} + \vec{r}_{S'S}$$

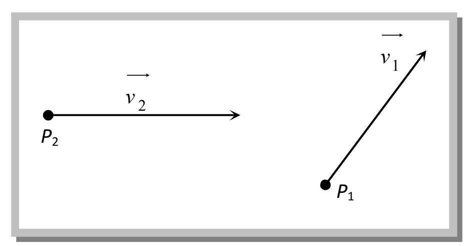
Differentiating this equation with respect to time

$$\frac{d\vec{r}_{PS}}{dt} = \frac{d\vec{r}_{PS'}}{dt} + \frac{d\vec{r}_{S'S}}{dt}$$

or $\vec{v}_{PS} = \vec{v}_{PS'} + \vec{v}_{S'S}$ [as $\vec{v} = d\vec{r}/dt$]

or $\vec{v}_{PS'} = \vec{v}_{PS} - \vec{v}_{S'S}$

(2) **General Formula :** The relative velocity of a particle P_1 moving with velocity $\vec{v}_1$ with respect to another particle P_2 moving with velocity $\vec{v}_2$ is given by,



$\vec{v}_{r_{12}} = \vec{v}_1 - \vec{v}_2$

(i) If both the particles are moving in the same direction then:

$$\upsilon_{r_{12}} = \upsilon_1 - \upsilon_2$$

(ii) If the two particles are moving in the opposite direction, then:

$$\upsilon_{r_{12}} = \upsilon_1 + \upsilon_2$$

(iii) If the two particles are moving in the mutually perpendicular directions, then:

$$\upsilon_{r_{12}} = \sqrt{\upsilon_1^2 + \upsilon_2^2}$$

(iv) If the angle between $\vec{\upsilon}_1$ and $\vec{\upsilon}_2$ be θ, then $\upsilon_{r_{12}} = \left[\upsilon_1^2 + \upsilon_2^2 - 2\upsilon_1\upsilon_2\cos\theta\right]^{1/2}$.

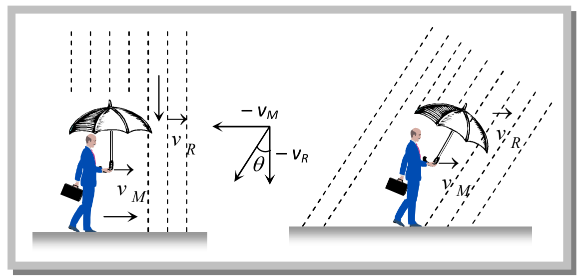
(3) **Relative velocity of satellite:** If a satellite is moving in equatorial plane with velocity $\vec{v}_s$ and a point on the surface of earth with $\vec{v}_e$ relative to the center of earth, the velocity of satellite relative to the surface of earth

$$\vec{v}_{se} = \vec{v}_s - \vec{v}_e$$

So if the satellite moves form west to east (in the direction of rotation of earth on its axis) its velocity relative to earth's surface will be $v_{se} = v_s - v_e$

And if the satellite moves from east to west, *i.e.,* opposite to the motion of earth, $v_{se} = v_s - (-v_e) = v_s + v_e$

(4) **Relative velocity of rain:** If rain is falling vertically with a velocity $\vec{v}_R$ and an observer is moving horizontally with speed $\vec{v}_M$ the velocity of rain relative to observer will be $\vec{v}_{RM} = \vec{v}_R - \vec{v}_M$



Which by law of vector addition has magnitude

$$v_{RM} = \sqrt{v_R^2 + v_M^2}$$

Direction $\theta = \tan^{-1}(v_M / v_R)$ with the vertical as shown in fig.

(5) **Relative velocity of swimmer:** If a man can swim relative to water with velocity $\vec{v}$ and water is flowing relative to ground with velocity $\vec{v}_R$ velocity of man relative to ground $\vec{v}_M$ will be given by:

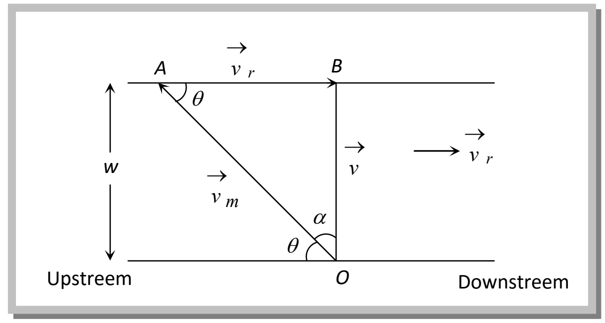
$$\vec{v} = \vec{v}_M - \vec{v}_R \text{, } i.e., \vec{v}_M = \vec{v} + \vec{v}_R$$

So if the swimming is in the direction of flow of water, $v_M = v + v_R$

And if the swimming is opposite to the flow of water, $v_M = v - v_R$

(6) **Crossing the river:** Suppose, the river is flowing with velocity υ_r. A man can swim in still water with velocity υ_m. He is standing on one bank of the river and wants to cross the river two cases arise.

(i) To cross the river over shortest distance: That is to cross the river straight, the man should swim making angle θ with the upstream as shown.



Here OAB is the triangle of vectors, in which $\vec{OA} = \vec{v}_m, \vec{AB} = \vec{\upsilon}_r$. Their resultant is given by $\vec{OB} = \vec{\upsilon}$. The direction of swimming makes angle θ with upstream. From the triangle OBA, we find,

$$\cos\theta = \frac{\upsilon_r}{\upsilon_m} \qquad \text{Also} \qquad \sin\alpha = \frac{\upsilon_r}{\upsilon_m}$$

Where α is the angle made by the direction of swimming with the shortest distance (OB) across the river.

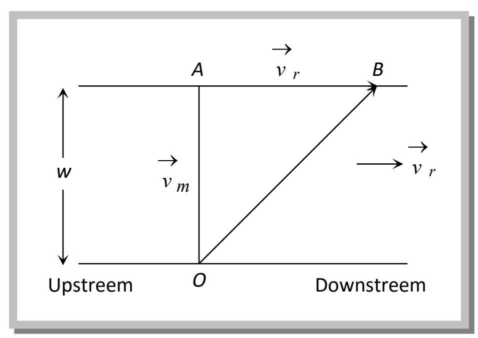
Time taken to cross the river:If w be the width of the river, then time taken to cross the river will be given by

$$t_1 = \frac{w}{\upsilon} = \frac{w}{\sqrt{\upsilon_m^2 - \upsilon_r^2}}$$

(ii) To cross the river in shortest possible time: The man should swim perpendicular to the bank.

The time taken to cross the river will be:

$$t_2 = \frac{w}{\upsilon_m}$$



In this case, the man will touch the opposite bank at a distance AB downstream. This distance will be given by:

$$AB = \upsilon_r t_2 = \upsilon_r \frac{w}{\upsilon_m} \qquad \text{Or} \qquad AB = \frac{\upsilon_r}{\upsilon_m} w$$