Algebra.

(1) **Quadratic equation:** An equation of second degree is called a quadratic equation. Standard quadratic equation $ax^2 + bx + c = 0$

Here *a* is called the coefficient of x^2, *b* is called the coefficient of *x* and *c* is a constant term, *x* is the variable whose value (roots of the equation) are to be determined

Roots of the equation are: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

This formula can be written as

$$x = \frac{-\text{Coefficient of } x \pm \sqrt{(\text{Coefficient of } x)^2 - 4(\text{Coefficient of } x^2) \times (\text{Constant term})}}{2(\text{Coefficient of } x^2)}$$

Note: If α and β be the roots of the quadratic equation then

Sum of roots $\alpha + \beta = -\dfrac{b}{a}$ and product of roots = $\dfrac{c}{a}$

(2) **Binomial theorem:** If *n* is any number positive, negative or fraction and *x* is any real number, such that *x*< 1 *i.e.* *x* lies between – 1 and + 1 then according to binomial theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + ...$$

Here 2! (Factorial 2) = 2 × 1, 3! (Factorial 3) = 3 × 2 × 1 and 4! (Factorial 4) = 4 × 3 × 2 × 1

Note: If $|x| << 1$ then only the first two terms are significant. It is so because the values of second and the higher order terms being very small, can be neglected. So the expression can be written as

$(1 + x)^n = 1 + nx$

$(1 + x)^{-n} = 1 - nx$

$(1 - x)^n = 1 - nx$

$(1 - x)^{-n} = 1 + nx$

(3) **Arithmetic progression:** It is a sequence of numbers which are arranged in increasing order and having a constant difference between them.

Example: 1, 3, 5, 7, 9, 11, 13, ……or 2, 4, 6, 8, 10, 12, …..

In general arithmetic progression can be written as a_0, a_1, a_2, a_3, a_4, a_5 …….

(i) n^{th} term of arithmetic progression $a_n = a_0 + (n-1)d$

a_0 = First term, n = Number of terms, d = Common difference = $(a_1 - a_0)$ or $(a_2 - a_1)$ or $(a_3 - a_2)$

(ii) Sum of arithmetic progression $S_n = \frac{n}{2}\left[2a_0 + (n-1)d\right] = \frac{n}{2}\left[a_0 + a_n\right]$

(4) **Geometric progression:** It is a sequence of numbers in which every term is obtained by multiplying the previous term by a constant quantity. This constant quantity is called the common ratio.

Example: 4, 8, 16, 32, 64, 128 …… or 5, 10, 20, 40, 80, …….

In general geometric progression can be written as a, ar, ar^2, ar^3, ar^4, ….

Here a = first term, r = common ratio

(i) Sum of 'n' terms of G.P. $S_n = \frac{a(1-r^n)}{1-r}$ if $r < 1$

$S_n = \frac{a(r^n - 1)}{r-1}$ if $r > 1$

(ii) Sum of infinite terms of G.P. $S_\infty = \frac{a}{1-r}$ if $r < 1$

$S_\infty = \frac{a}{r-1}$ if $r > 1$

(5) **Some common formulae of algebra**

(i) $(a + b)^2 = a^2 + b^2 + 2ab$

(ii) $(a - b)^2 = a^2 + b^2 - 2ab$

(iii) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

(iv) $(a + b)(a - b) = a^2 - b^2$

(v) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

(vi) $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

(vii) $(a + b)^2 - (a - b)^2 = 4ab$

(viii) $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

(ix) $a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$

(x) $a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$

(6) **Componendo and dividendo method:** If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$