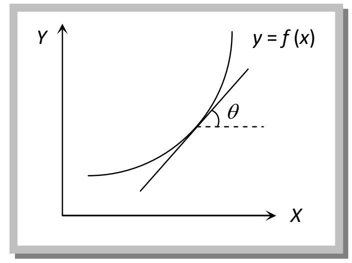
Differential calculus.

The differential coefficient or derivative of variable y with respect to variable x is defined as the instantaneous rate of change of y *w.r.t.* x. It is denoted by $\frac{dy}{dx}$



Geometrically the differential coefficient of $y = f(x)$ with respect to x at any point is equal to the slope of the tangent to the curve representing $y = f(x)$ at that point

i.e. $\frac{dy}{dx} = \tan\theta$.

Note: Actually $\frac{dy}{dx}$ is a rate measurer.

- If $\frac{dy}{dx}$ is positive, it means y is increasing with increasing of x and vice-versa.
- For small change Δx we use $\Delta y = \frac{dy}{dx}.\Delta x$

Example: (1) Instantaneous speed $v = \frac{ds}{dt}$

(2) Instantaneous acceleration $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

(3) Force $F = \frac{dp}{dt}$

(4) Angular velocity $\omega = \frac{d\theta}{dt}$

(5) Angular acceleration $\alpha = \frac{d\omega}{dt}$

(6) Power $P = \frac{dW}{dt}$

(7) Torque $\tau = \frac{dL}{dt}$

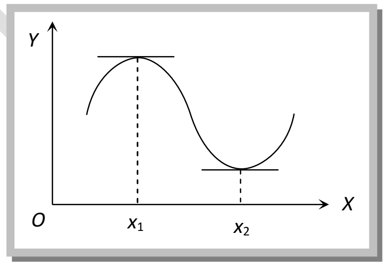
(1) **Fundamental formulae of differentiation:**

Function	**Differentiation**
If c is some constant	$\frac{d}{dx}(c) = 0$
If $y = cx$ where c is a constant	$\frac{dy}{dx} = \frac{d}{dx}(cx) = c\frac{dx}{dx} = c$

If $y = c\,u$ where c is a constant and u is a function of x	$\frac{dy}{dx} = \frac{d}{dx}(c\,u) = c\,\frac{du}{dx}$
If $y = x^n$ where n is a real number	$\frac{dy}{dx} = nx^{n-1}$
If $y = u^n$ where n is a real number and u is a function of x	$\frac{dy}{dx} = nu^{n-1}\,\frac{du}{dx}$
If $y = u + v$ where u and v are the functions of x	$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$
If $y = uv$ where u and v are functions of x (product formula)	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
If $y = \frac{u}{v}$ where u and v are the functions of x (quotient formula)	$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
If $y = f(u)$ and $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
If $y = (ax + b)^n$	$\frac{dy}{dx} = n\,(ax + b)^{n-1} \times \frac{d}{dx}(ax + b)$
If $y = \sin x$	$\frac{dy}{dx} = \frac{d}{dx}(\sin x) = \cos x$
If $y = \cos x$	$\frac{dy}{dx} = \frac{d}{dx}(\cos x) = -\sin x$
If $y = \tan x$	$\frac{dy}{dx} = \frac{d}{dx}(\tan x) = \sec^2 x$
If $y = \cot x$	$\frac{dy}{dx} = \frac{d}{dx}(\cot x) = -\text{cosec}^2 x$
If $y = \sec x$	$\frac{dy}{dx} = \frac{d}{dx}(\sec x) = \tan x \sec x$
If $y = \text{cosec } x$	$\frac{dy}{dx} = \frac{d}{dx}(\text{cosec } x) = -\cot x \text{ cosec } x$
If $y = \sin u$ where u is the function of x	$\frac{dy}{dx} = \frac{d}{dx}(\sin u) = \cos u\,\frac{d(u)}{dx}$
If $y = \cos u$ where u is the function of x	$\frac{dy}{dx} = \frac{d}{dx}(\cos u) = -\sin u\,\frac{d(u)}{dx}$
If $y = \tan u$ where u is the function of x	$\frac{dy}{dx} = \frac{d}{dx}(\tan u) = \sec^2 u\,\frac{d(u)}{dx}$
If $y = \cot u$ where u is the function of x	$\frac{dy}{dx} = \frac{d}{dx}(\cot u) = -\text{cosec}^2 u\,\frac{d(u)}{dx}$
If $y = \sec u$ where u is the function of x	$\frac{dy}{dx} = \frac{d}{dx}(\sec u) = \sec u \tan u\,\frac{d(u)}{dx}$

If $y =$ cosec u where u is the function of x	$\frac{dy}{dx} = \frac{d}{dx}(\text{cosec } u) = -\text{cosec } u \cot u \frac{d(u)}{dx}$
If $y = \log_a x$	$\frac{dy}{dx} = \frac{1}{x}\log_a e$

(2) **Maxima and minima:** If a quantity y depends on another quantity x in a manner shown in figure. It becomes maximum at x_1 and minimum at x_2.



At these points the tangent to the curve is parallel to X-axis and hence its slope is $\tan\theta = 0$. but the slope of the curve equals the rate of change $\frac{dy}{dx}$. Thus, at a maximum or minimum $\frac{dy}{dx} = 0$

Just before the maximum the slope is positive, at the maximum it is zero and just after the maximum it is negative. Thus $\frac{dy}{dx}$ decreases at a maximum and hence the rate of change of $\frac{dy}{dx}$ is negative at a maximum. *i.e.*, $\frac{d}{dx}\left(\frac{dy}{dx}\right) < 0$ at a maximum.

Hence the condition of maxima: $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$ (Second derivative test)

Similarly, at a minimum the slope changes from negative to positive. The slope increases at such a point and hence $\frac{d}{dx}\left(\frac{dy}{dx}\right) > 0$

Hence the condition of minima: $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$. (Second derivative test)