Integral calculus.

The process of integration is just the reverse of differentiation. The symbol $\int$ is used to denote integration.

If $f(x)$ is the differential coefficient of function $F(x)$ with respect to x, then by integrating $f(x)$ we can get $F(x)$ again.

(1) **Fundamental formulae of integration:**

$\int x^n dx = \frac{x^{n+1}}{n+1}$, provided $n \neq -1$	$\int \sec^2 x\, dx = \tan x$
$\int dx = \int x^0 dx = \frac{x^{0+1}}{0+1} = x$	$\int \cos ec^2 x\, dx = -\cot x$
$\int (u+v) dx = \int u\, dx + \int v\, dx$	$\int \sec x \tan x\, dx = \sec x$
$\int cu\, dx = c\int u\, dx$ where c is a constant and u is a function of x.	$\int \text{cosec}\, x \cot x\, dx = -\text{cosec}\, x$
$\int cx^n dx = c\frac{x^{n+1}}{n+1}$	$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1)\frac{d}{dx}(ax+b)} = \frac{(ax+b)^{n+1}}{a(n+1)}$
$\int x^{-1} dx = \int \frac{dx}{x} = \log_e x$	$\int \frac{a}{(ax+b)} dx = \frac{a \log_e (ax+b)}{\frac{d}{dx}(ax+b)} = \log_e (ax+b)$
$\int e^x dx = e^x$	$\int e^{ax+b} dx = \frac{e^{ax+b}}{\frac{d}{dx}(ax+b)} = \frac{e^{ax+b}}{a}$
$\int a^x dx = \frac{a^x}{\log_e a}$	$\int a^{cx+d} dx = \frac{a^{cx+d}}{\log_e a \frac{d}{dx}(cx+d)} = \frac{a^{cx+d}}{c \log_e a}$
$\int \sin x\, dx = -\cos x$	$\int \sec^2 (ax+b) dx = \frac{\tan(ax+b)}{\frac{d}{dx}(ax+b)} = \frac{\tan(ax+b)}{a}$
$\int \sin nx\, dx = \frac{-\cos nx}{n}$	$\int \text{cosec}^2 (ax+b) dx = \frac{-\cot(ax+b)}{\frac{d}{dx}(ax+b)} = \frac{-\cot(ax+v)}{a}$

$\int \cos x \, dx = \sin x$	$\int \sec(ax+b)\tan(ax+b)\,dx$ $= \frac{\sec(ax+b)}{\frac{d}{dx}(ax+b)} = \frac{\sec(ax+b)}{a}$
$\int \cos nx \, dx = \frac{\sin nx}{n}$	$\int \operatorname{cosec}(ax+b)\cot(ax+b)\,dx$ $= \frac{-\operatorname{cosec}(ax+b)}{\frac{d}{dx}(ax+b)} = \frac{-\operatorname{cosec}(ax+b)}{a}$

(2) **Method of integration:**Sometimes, we come across some functions which cannot be integrated directly by using the standard integrals. In such cases, the integral of a function can be obtained by using one or more of the following methods.

(i) Integration by substitution: Those functions which cannot be integrated directly can be reduced to standard integrand by making a suitable substitution and then can be integrated by using the standard integrals. To understand the method, we take the few examples.

(ii) Integration by parts: This method of integration is based on the following rule :

Integral of a product of two functions = first function × integral of second function – integral of (differential coefficient of first function × integral of second function).

Thus, if *u* and *v* are the functions of *x*, then $\int uv\,dx = u\int v\,dx - \int\left[\frac{du}{dx}\times\int v\,dx\right]dx$

(3) **Definite integrals:** When a function is integrated between definite limits, the integral is called definite integral. For example,

$\int_a^b f(x)dx$ is definite integral of $f(x)$ between the limits *a* and *b* and is written as

$\int_a^b f(x)dx = F(x)|_a^b = F(b) - F(a)$

Here *a* is called the lower limit and *b* is called the upper limit of integration.

Geometrically $\int_a^b f(x)\,dx$ equals to area of curve $F(x)$ between the limits a and b.