## Electric Dipole.

(1) General information:System of two equal and opposite charges separated by a small fixed distance is called a dipole.

(i) Dipole axis:Line joining negative charge to positive charge of a dipole is called its axis. It may also be termed as its longitudinal axis.
(ii) Equatorial axis: Perpendicular bisector of the dipole is called its equatorial or transverse axis as it is perpendicular to length.
(iii) Dipole length: The distance between two charges is known as dipole length ( $L=2 \mathrm{l}$ )
(iv) Dipole moment: It is a quantity which gives information about the strength of dipole. It is a vector quantity and is directed from negative charge to positive charge along the axis. It is denoted as $\vec{p}$ and is defined as the product of the magnitude of either of the charge and the dipole length.
i.e.

$$
\vec{p}=q(2 \vec{l})
$$

Its S.I. unit is coulomb-meter or Debye ( 1 Debye $=3.3 \times 10^{-30} \mathrm{C} \times \mathrm{m}$ ) and its dimensions are $M^{0} L^{1} T^{1} A^{1}$ 。

Note: A region surrounding a stationary electric dipole has electric field only.
When a dielectric is placed in an electric field, its atoms or molecules are considered as tiny dipoles.


Water $\left(\mathrm{H}_{2} \mathrm{O}\right)$, Chloroform $\left(\mathrm{CHCl}_{3}\right)$, Ammonia $\left(\mathrm{NH}_{3}\right), \mathrm{HCl}, \mathrm{CO}$ molecules are some example of permanent electric dipole.

(2) Electric field and potential due to an electric dipole:It is better to understand electric dipole with magnetic dipole.

| S.No. | Electric dipole | Magnetic dipole |
| :---: | :---: | :---: |
| (i) | System of two equal and opposite charges separated by a small fixed distance. | System of two equal and opposite magnetic poles (Bar magnet) separated by a small fixed distance. |
| (ii) | Electric dipole moment: $\vec{p}=q(2 \vec{l})$, directed from $-q$ to $+q$. Its S.I. unit is coulomb $\times$ meter or Debye. | Magnetic dipole moment: $\vec{M}=m(2 \vec{l})$, directed from $S$ to N. Its S.I. unit is ampere $\times$ meter $^{2}$. |
| (iii) | Intensity of electric field <br> If $a, e$ and $g$ are three points on axial, equatorial and general position at a distance $r$ from the center of dipole on axial point $E_{a}=\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{2 p}{r^{3}}$ (directed from -q to $+q)$ on equatorial point $E_{e}=\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{p}{r^{3}}$ (directed from +q to $-q$ ) | Intensity of magnetic field <br> If $a, e$ and $g$ are three points on axial, equatorial and general position at a distance $r$ from the center of dipole on axial point $B_{a}=\frac{\mu_{0}}{4 \pi} \cdot \frac{2 M}{r^{3}}$ (directed from S to N) on equatorial point $B_{e}=\frac{\mu_{0}}{4 \pi} \cdot \frac{M}{r^{3}}$ (directed from N to S) |

$$
\begin{aligned}
& \text { on general point } E_{a}=\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{p}{r^{3}} \sqrt{\left(3 \cos ^{2} \theta+1\right)} \\
& \text { Angle between }-\vec{E}_{a} \text { and } \vec{p} \text { is } 0^{\circ}, \vec{E}_{e} \text { and } \vec{p} \text { is } \\
& 180^{\circ}, \vec{E} \text { and } \vec{p} \text { is }(\theta+\alpha)\left(\text { where } \tan \alpha=\frac{1}{2} \tan \theta\right) \\
& \text { Electric Potential }- \text { At a } V_{a}=\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{p}{r^{2}} \text {, At e } \\
& V=0 \\
& \text { At } g V=\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{p \cos \theta}{r^{2}}
\end{aligned}
$$

on general point $B_{a}=\frac{\mu_{0}}{4 \pi} \cdot \frac{M}{r^{3}} \sqrt{\left(3 \cos ^{2} \theta+1\right)}$
Angle between $-\vec{B}_{a}$ and $\vec{M}$ is $0^{\circ}, \vec{B}_{e}$ and $\vec{M}$ is $180^{\circ}, \vec{B}$ and $\vec{M}$ is $(\theta+\alpha)$ (where $\left.\tan \alpha=\frac{1}{2} \tan \theta\right)$

## (3) Dipole (electric/magnetic) in uniform field (electric/magnetic)

(i)Torque: If a dipole is placed in an uniform field such that dipole (i.e. $\vec{p}$ or $\vec{M}$ ) makes an angle $\theta$ with direction of field then two equal and opposite force acting on dipole constitute a couple whose tendency is to rotate the dipole hence a torque is developed in it and dipole tries to align itself in the direction of field.

Consider an electric dipole in placed in an uniform electric field such that dipole (i.e. $\vec{p}$ ) makes an angle $\theta$ with the direction of electric field as shown

(a) Net force on electric dipole $F_{\text {net }}=0$
(b) Produced torque $\tau=\mathrm{pE} \sin \theta(\vec{\tau}=\vec{P} \times \vec{E})$

A magnetic dipole of magnetic moment M is placed in uniform magnetic field $B$ by making an angle $\theta$ as shown

(a) Net force on magnetic dipole $F_{\text {net }}=0$
(b) torque $\tau=\mathrm{MB} \sin \theta(\vec{\tau}=\vec{M} \times \vec{B})$
(ii) Work:From the above discussion it is clear that in a uniform electric/magnetic field dipole tries to align itself in the direction of electric field (i.e. equilibrium position). To change its angular position some work has to be done.
Suppose an electric/magnetic dipole is kept in an uniform electric/magnetic field by making an angle $\theta_{1}$ with the field, if it is again turn so that it makes an angle $\theta_{2}$ with the field, work done in this process is given by the formula

| $W=p E\left(\cos \theta_{1}-\cos \theta_{2}\right)$ | $W=M B\left(\cos \theta_{1}-\cos \theta_{2}\right)$ |
| :---: | :---: |
| If $\theta_{1}=0^{\circ}$ and $\theta_{2}=\theta$ i.e. initially dipole is kept along the field then it turn through $\theta$ so work done $W=p E(1-\cos \theta)$ | If $\theta_{1}=0^{\circ}$ and $\theta_{2}=\theta$ then $\mathrm{W}=\mathrm{MB}(1-\cos \theta)$ |

(iii) Potential energy:In case of a dipole (in a uniform field), potential energy of dipole is defined as work done in rotating a dipole from a direction perpendicular to the field to the given direction i.e. if $\theta_{1}=90^{\circ}$ and $\theta_{2}=\theta$ then -

(iv) Equilibrium of dipole:We know that, for any equilibrium net torque and net force on a particle (or system) should be zero.

We already discussed when a dipole is placed in a uniform electric/magnetic field net force on dipole is always zero. But net torque will be zero only when $\theta=0^{\circ}$ or $180^{\circ}$
When $\theta=0^{\circ}$ i.e. dipole is placed along the electric field it is said to be in stable equilibrium, because after turning it through a small angle, dipole tries to align itself again in the direction of electric field.
When $\theta=180^{\circ}$ i.e. dipole is placed opposite to electric field, it is said to be in unstable equilibrium.

(v) Angular SHM: In a uniform electric/magnetic field (intensity $E / B$ ) if a dipole (electric/magnetic) is slightly displaced from its stable equilibrium position it executes angular SHM having period of oscillation. If I = moment of inertia of dipole about the axis passing through itscenter and perpendicular to its length.
For electric dipole: $T=2 \pi \sqrt{\frac{I}{p E}}$ and For Magnetic dipole: $T=2 \pi \sqrt{\frac{I}{M B}}$
(vi) Dipole-point charge interaction:If a point charge/isolated magnetic pole is placed in dipole field at a distance $r$ from the midpoint of dipole then force experienced by point charge/pole varies according to the relation $F \propto \frac{1}{r^{3}}$
(vii) Dipole-dipole interaction:When two dipoles placed closed to each other, they experiences a force due to each other. If suppose two dipoles (1) and (2) are placed as shown in figure then Both the dipoles are placed in the field of one another hence potential energy dipole (2) is

$$
U_{2}=-p_{2} E_{1} \cos 0=-p_{2} E_{1}=-p_{2} \times \frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{2 p_{1}}{r^{3}}
$$

Then by using $F=-\frac{d U}{d r}$, Force on dipole (2) is $F_{2}=-\frac{d U_{2}}{d r}$

$\Rightarrow F_{2}=-\frac{d}{d r}\left\{\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{2 p_{1} p_{2}}{r^{3}}\right\}=-\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{6 p_{1} p_{2}}{r^{4}}$
Similarly force experienced by dipole (1) $F_{1}=-\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{6 p_{1} p_{2}}{r^{4}}$ so $F_{1}=F_{2}=-\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{6 p_{1} p_{2}}{r^{4}}$
Negative sign indicates that force is attractive. $|F|=\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{6 p_{1} p_{2}}{r^{4}}$ and $\boldsymbol{F} \propto \frac{\mathbf{1}}{\boldsymbol{r}^{\mathbf{4}}}$


Note: Same result can also be obtained for magnetic dipole.
(4) Electric dipole in non-uniform electric field:When an electric dipole is placed in a nonuniform field, the two charges of dipole experiences unequal forces, therefore the net force on the dipole is not equal to zero. The magnitude of the force is given by the negative derivative of the potential energy w.r.t. distance along the axis of the dipole i.e. $\vec{F}=-\frac{d U}{d r}=-\vec{p} \cdot \frac{d \vec{E}}{d r}$.

Due to two unequal forces, a torque is produced which rotate the dipole so as to align it in the direction of field. When the dipole gets aligned with the field, the torque becomes zero and then the unbalanced force acts on the dipole and the dipole then moves linearly along the direction of field from weaker portion of the field to the stronger portion of the field. So in non-uniform electric field

(i) Motion of the dipole is translatory and rotatory
(ii) Torque on it may be zero.

## Concepts

(t) For a short dipole, electric field intensity at a point on the axial line is double than at a point on the equatorial line on electric dipole i.e. $\mathrm{E}_{\text {axial }}=2 \mathrm{E}_{\text {equatorial }}$
$\square$ It is interesting to note that dipole field $E \propto \frac{1}{r^{3}}$ decreases much rapidly as compared to the field of a point $\operatorname{charge}\left(E \propto \frac{1}{r^{2}}\right)$.

