Electric Potential.

(1)**Definition:**Potential at a point in a field is defined as the amount of work done in bringing a unit positive test charge, from infinity to that point along any arbitrary path (infinity is point of zero potential). Electric potential is a scalar quantity, it is denoted by V; $V = \frac{W}{q_0}$

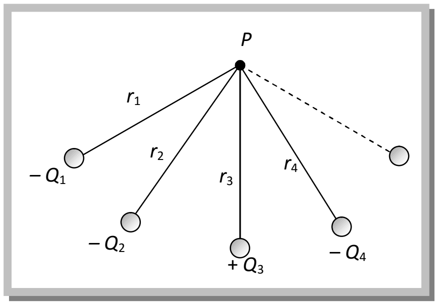
(2) **Unit and dimensional formula:**S. I. unit – $\frac{Joule}{Coulomb} = volt$ C.G.S. unit – Stat volt (e.s.u.); **1 volt** $= \frac{1}{300}$ **Stat volt** Dimension – $[V] = [ML^2T^{-3}A^{-1}]$

(3) **Types of electric potential:**According to the nature of charge potential is of two types

(i) Positive potential: Due to positive charge.

(ii) Negative potential: Due to negative charge.

(4) **Potential of a system of point charges:**Consider P is a point at which net electric potential is to be determined due to several charges. So net potential at P



$$V = k\frac{Q_1}{r_1} + k\frac{Q_2}{r_2} + k\frac{Q_3}{r_3} + k\frac{(-Q_4)}{r_4} + \ldots$$

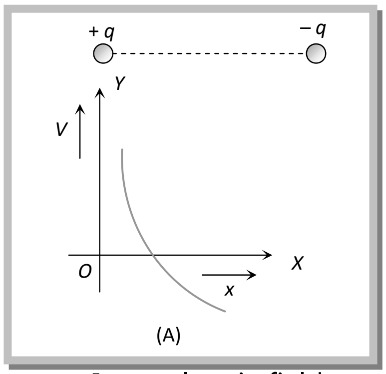
In general $V = \sum_{i=1}^{X} \frac{kQ_i}{r_i}$

Note: At the center of two equal and opposite charge V = 0 but $E \neq 0$

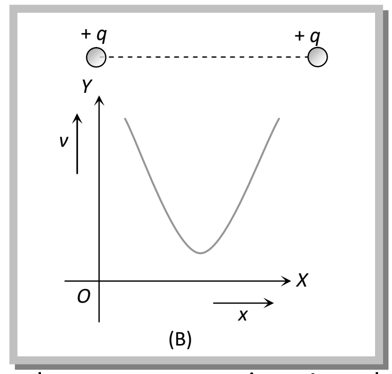
At the center of the line joining two equal and similar charge $V \neq 0, E = 0$

(5) **Electric potential due to a continuous charge distribution :** The potential due to a continuous charge distribution is the sum of potentials of all the infinitesimal charge elements in which the distribution may be divided i.e., $V = \int dV, \; = \int \frac{dQ}{4\pi\varepsilon_0 r}$

(6) **Graphical representation of potential:** When we move from a positive charge towards an equal negative charge along the line joining the two then initially potential decreases in magnitude and at center become zero, but this potential is throughout positive because when we are nearer to positive charge, overall potential must be positive. When we move from center towards the negative charge then though potential remain always negative but increases in magnitude fig. (A). As one move from one charge to other when both charges are like, the potential first decreases, at center become minimum and then increases Fig. (B).



(A)



(B)

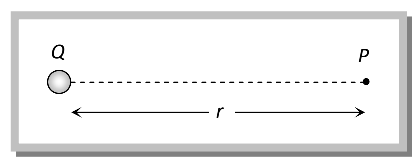
(7) **Potential difference :** In an electric field potential difference between two points A and B is defined as equal to the amount of work done (by external agent) in moving a unit positive charge from point A to point B.

i.e., $V_B - V_A = \frac{W}{q_0}$ in general $W = Q.\Delta V$; $\Delta V =$ Potential difference through which charge Q moves.

(8) **Electric Field and Potential Due to Various Charge Distribution.**

(1) **Point charge:** Electric field and potential at point P due to a point charge Q is

$$\boldsymbol{E} = \boldsymbol{k}\frac{\boldsymbol{Q}}{\boldsymbol{r}^2} \text{ or } \vec{E} = k\frac{Q}{r^2}\hat{r}\left(k = \frac{1}{4\pi\varepsilon_0}\right), \quad V = k\frac{Q}{r}$$

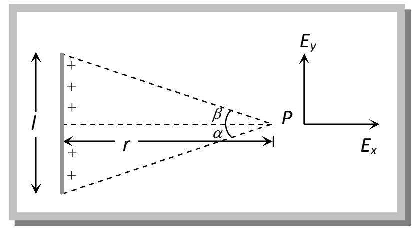


Note: Electric field intensity and electric potential due to a point charge q, at a distance $t_1 + t_2$ where t_1 is thickness of medium of dielectric constant K_1 and t_2 is thickness of medium of dielectric constant K_2 are:

$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0}\frac{\boldsymbol{Q}}{(t_1\sqrt{K_1} + t_2\sqrt{K_2})^2}; \qquad \boldsymbol{V} = \frac{1}{4\pi\varepsilon_0}\frac{\boldsymbol{Q}}{(t_1\sqrt{K_1} + t_2\sqrt{K_2})}$$

(2) **Line charge**

(i) **Straight conductor:**Electric field and potential due to a charged straight conducting wire of length l and charge density λ



(a) **Electric field:** $E_x = \frac{k\lambda}{r}(\sin\alpha + \sin\beta)$ and $E_y = \frac{k\lambda}{r}(\cos\beta - \cos\alpha)$

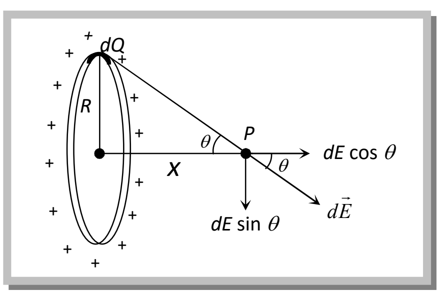
If α= β; $E_x = \frac{2k\lambda}{r}\sin\alpha$ and $E_y = 0$

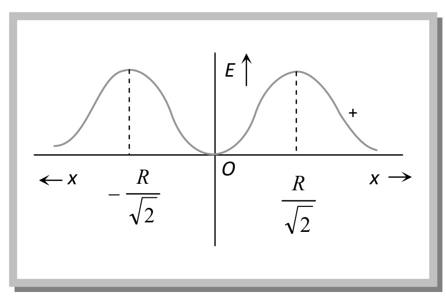
If l→∞ i.e. α = β= $\frac{\pi}{2}$; $E_x = \frac{2k\lambda}{r}$ and $E_y = 0$ so $E_{net} = \frac{\lambda}{2\pi\varepsilon_0 r}$

If α = 0, $\beta = \frac{\pi}{2}$; $|E_x| = |E_y| = \frac{k\lambda}{r}$ so $E_{net} = \sqrt{E_x^2 + E_y^2} = \frac{\sqrt{2}k\lambda}{r}$

(b)**Potential:** $V = \frac{\lambda}{2\pi\varepsilon_0}\log_e\left[\frac{\sqrt{r^2+l^2}-1}{\sqrt{r^2+l^2}+1}\right]$ for infinitely long conductor $V = \frac{-\lambda}{2\pi\varepsilon_0}\log_e r + c$

(ii) **Charged circular ring:**Suppose we have a charged circular ring of radius R and charge Q. On its axis electric field and potential is to be determined, at a point 'x' away from the center of the ring.





(a) **Electric field:**Consider an element carrying charge dQ. Its electric field $dE = \frac{KdQ}{(R^2 + x^2)}$ directed as shown. Its component along the axis is $dE\cos\theta$ and perpendicular to the axis is $dE\sin\theta$. By symmetry $\int dE\sin\theta = 0$, hence $E = \int dE\cos\theta = \int \frac{kdQ}{(R^2+x^2)} \cdot \frac{x}{(R^2+x^2)^{1/2}}$

$$E = \frac{kQx}{(R^2 + x^2)^{3/2}}$$ Directed away from the center if Q is positive

(b)**Potential:** $$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{\sqrt{x^2 + R^2}}$$

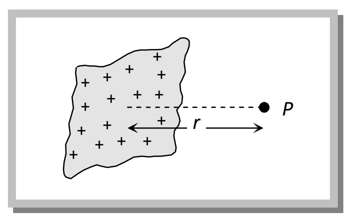
Note: At center x = 0 so E_{centre}= 0 and $V_{centre} = \frac{kQ}{R}$

At a point on the axis such that x >> R $E = \frac{kQ}{x^2}$ and $V = \frac{kQ}{x}$

At a point on the axis if $x = \pm \frac{R}{\sqrt{2}}$, $E_{\max} = \frac{Q}{6\sqrt{3}\pi\varepsilon_0 a^2}$

(3) **Surface charge:**

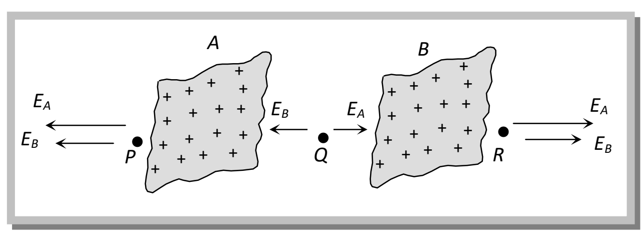
(i) **Infinite sheet of charge:** Electric field and potential at a point P as shown



$$E = \frac{\sigma}{2\varepsilon_0} \quad (E \propto r^o)$$

and $$V = -\frac{\sigma r}{2\varepsilon_0} + C$$

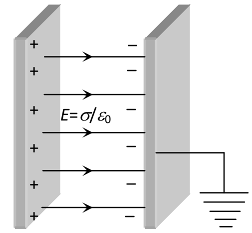
(ii) **Electric field due to two parallel plane sheet of charge:** Consider two large, uniformly charged parallel. Plates A and B, having surface charge densities are σ_A and σ_B respectively. Suppose net electric field at points P, Q and R is to be calculated.



At P, $$E_P = (E_A + E_B) = \frac{1}{2\varepsilon_0}(\sigma_A + \sigma_B)$$

At Q, $$E_Q = (E_A - E_B) = \frac{1}{2\varepsilon_0}(\sigma_A - \sigma_B)$$; At R, $$E_R = -(E_A + E_B) = -\frac{1}{2\varepsilon_0}(\sigma_A + \sigma_B)$$

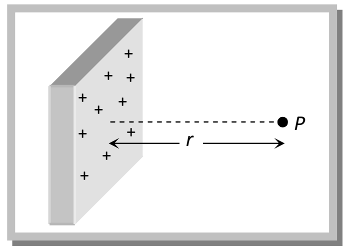
Note: If $\sigma_A = +\sigma$ and $\sigma_B = -\sigma$ then $E_p = 0, E_Q = \frac{\sigma}{\varepsilon_0}, E_R = 0$. Thus in case of two infinite plane sheets of charges having equal and opposite surface charge densities, the field is non-zero only in the space between the two sheets and is independent of the distance between them i.e., field is uniform in this region. It should be noted that this result will hold good for finite plane sheet also, if they are held at a distance much smaller than the dimensions of sheets i.e., parallel plate capacitor.



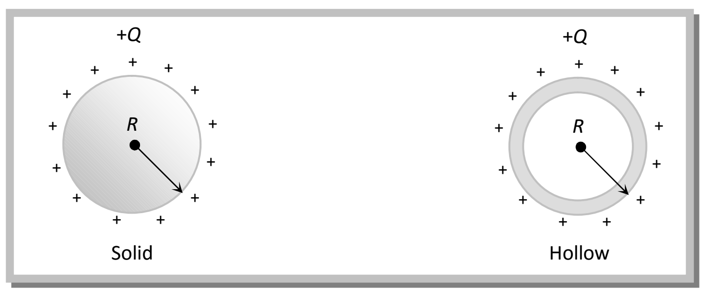
(iii) **Conducting sheet of charge:**

$$E = \frac{\sigma}{\varepsilon_0}$$

$$V = -\frac{\sigma r}{\varepsilon_0} + C$$



(iv) **Charged conducting sphere:** If charge on a conducting sphere of radius R is Q as shown in figure then electric field and potential in different situation are –



(a) **Outside the sphere:** P is a point outside the sphere at a distance r from the center at which electric field and potential is to be determined.

Electric field at P

$$E_{out} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r^2} = \frac{\sigma R^2}{\varepsilon_0 r^2} \text{ and } V_{out} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r} = \frac{\sigma R^2}{\varepsilon_0 r} \begin{cases} Q = \sigma \times A \\ \quad = \sigma \times 4\pi R^2 \end{cases}$$

(b) **At the surface of sphere:** At surface $r = R$

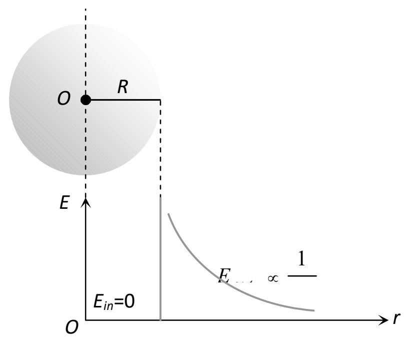
So, $$E_s = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R^2} = \frac{\sigma}{\varepsilon_0} \text{ and } V_s = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R} = \frac{\sigma R}{\varepsilon_0}$$

(c) **Inside the sphere:** Inside the conducting charge sphere electric field is zero and potential remains constant everywhere and equals to the potential at the surface.

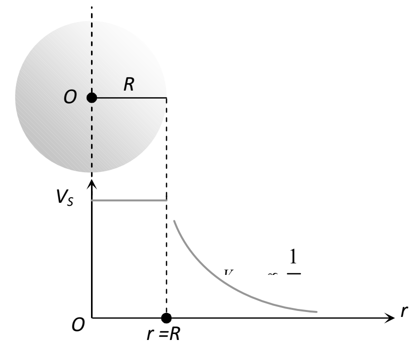
$E_{in} = 0$ and V_{in} = constant $= V_s$

Note: Graphical variation of electric field and potential of a charged spherical conductor with distance

E - *r* graph



V - *r* graph

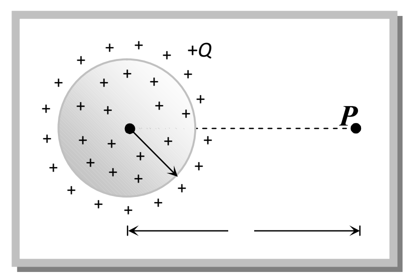


(4) **Volume charge (charged non-conducting sphere):**

Charge given to a non-conducting spheres spreads uniformly throughout its volume.

(i) **Outside the sphere at P**

$$E_{out} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r^2} \text{ and } V_{out} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r} \text{ by using } \rho = \frac{Q}{\frac{4}{3}\pi R^3}$$



$$E_{out} = \frac{\rho R^3}{3\varepsilon_0 r^2} \text{ and } V_{out} = \frac{\rho R^3}{3\varepsilon_0 r}$$

(ii) **At the surface of sphere:** At surface $r = R$

$$E_s = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R^2} = \frac{\rho R}{3\varepsilon_0} \text{ and } V_s = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R} = \frac{\rho R^2}{3\varepsilon_0}$$

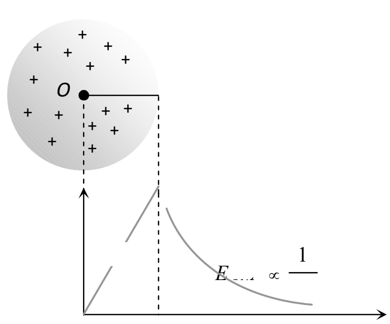
(iii) **Inside the sphere:**At a distance r from the center

$$E_{in} = \frac{1}{4\pi\varepsilon_0}.\frac{Qr}{R^3} = \frac{\rho r}{3\varepsilon_0} \; \{E_{in} \propto r\} \text{ and } V_{in} = \frac{1}{4\pi\varepsilon_0}\frac{Q[3R^2 - r^2]}{2R^3} = \frac{\rho(3R^2 - r^2)}{6\varepsilon_0}$$

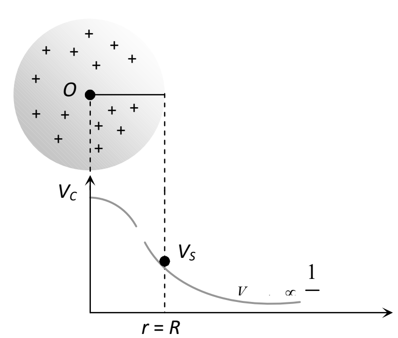
Note: At center $r = 0$ So, $V_{centre} = \frac{3}{2} \times \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R} = \frac{3}{2}V_s$ i.e., $V_{centre} > V_{surface} > V_{out}$

Graphical variation of electric field and potential with distance

E-*r*graph



V-*r* graph



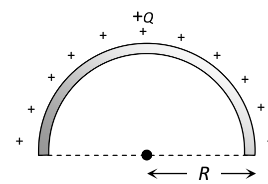
(5) **Electric field and potential in some other cases**

(i) **Uniformly charged semicircular ring :** $\lambda = \frac{\text{charge}}{\text{length}}$

At center :

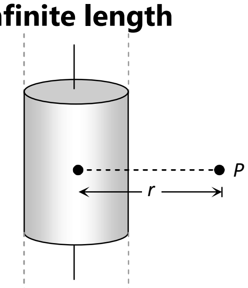
$$E = \frac{2K\lambda}{R} = \frac{Q}{2\pi^2 \varepsilon_0 R^2}$$

$$V = \frac{KQ}{R} = \frac{Q}{4\pi\varepsilon_0 R}$$

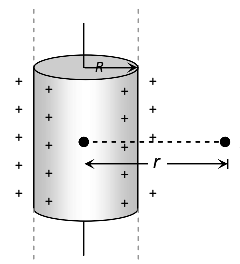


(iii) **Charged cylinder of infinite length**

(a) Conducting



(b) Non-conducting

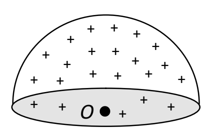


For both type of cylindrical charge distribution $E_{out} = \frac{\lambda}{2\pi\varepsilon_0 r}$, and $E_{suface} = \frac{\lambda}{2\pi\varepsilon_0 R}$ but for conducting $E_{in} = 0$ and for non-conducting $E_{\text{in}} = \frac{\lambda r}{2\pi\varepsilon_0 R^2}$. (We can also write formulae in form of ρ i.e., $E_{\text{out}} = \frac{\rho R^2}{2\varepsilon_0 r}$ etc.)

(ii) **Hemispherical charged body :**

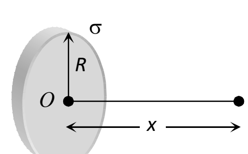
At center O, $E = \frac{\sigma}{4\varepsilon_0}$

$$V = \frac{\sigma R}{2\varepsilon_0}$$



(iv) **Uniformly charged disc**

At a distance x from center O on its axis



$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

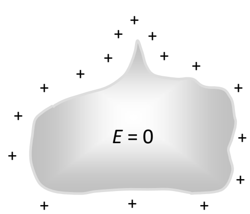
$$V = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{x^2 + R^2} - x\right]$$

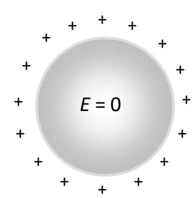
Note: Total charge on disc $Q = \sigma\pi R^2$

If $x \to 0$, $E \simeq \frac{\sigma}{2\varepsilon_0}$ i.e. for points situated near the disc, it behaves as an infinite sheet of charge.

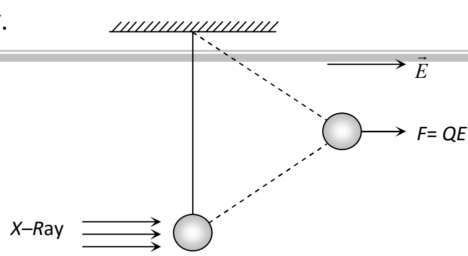
Concepts

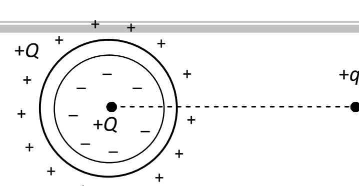
- No point charge produces electric field at its own position.
- Since charge given to a conductor resides on its surface hence electric field inside it is zero.





- The electric field on the surface of a conductor is directly proportional to the surface charge density at that point i.e, $E \propto \sigma$
- Two charged spheres having radii r_1 and r_2 charge densities σ_1 and σ_2 respectively, then the ratio of electric field on their surfaces will be $\frac{E_1}{E_2} = \frac{\sigma_1}{\sigma_2} = \frac{r_2^2}{r_1^2} \left\{\sigma = \frac{Q}{4\pi r^2}\right.$
- In air if intensity of electric field exceeds the value $3 \times 10^6\ N/C$ air ionizes.
- A small ball is suspended in a uniform electric field with the help of an insulated thread. If a high energy x–ray bean falls on the ball, x-rays knock out electrons from the ball so the ball is positively charged and therefore the ball is deflected in the direction of electric field.
- Electric field is always directed from higher potential to lower potential.
- A positive charge if left free in electric field always moves from higher potential to lower potential while a negative charge moves from lower potential to higher potential.
- The practical zero of electric potential is taken as the potential of earth and theoretical zero is taken at infinity.





- ☞ An electric potential exists at a point in a region where the electric field is zero and it's vice versa.
- ☞ A point charge +Q lying inside a closed conducting shell does not exert force another point charge q placed outside the shell as shown in figure

Actually the point charge +Q is unable to exert force on the charge +q because it cannot produce electric field at the position of +q. All the field lines emerging from the point charge +Q terminate inside as these lines cannot penetrate the conducting medium (properties of lines of force).

The charge q however experiences a force not because of charge +Q but due to charge induced on the outer surface of the shell.