Equilibrium of Charge.

(1) **Definition:** A charge is said to be in equilibrium, if net force acting on it is zero. A system of charges is said to be in equilibrium if each charge is separately in equilibrium.

(2) **Type of equilibrium:** Equilibrium can be divided in following type:

(i) **Stable equilibrium:** After displacing a charged particle from its equilibrium position, if it returns back then it is said to be in stable equilibrium. If U is the potential energy then in case of stable equilibrium $\frac{d^2U}{dx^2}$ is positive i.e., U is minimum.

(ii) **Unstable equilibrium :** After displacing a charged particle from its equilibrium position, if it never returns back then it is said to be in unstable equilibrium and in unstable equilibrium $\frac{d^2U}{dx^2}$ is negative i.e., U is maximum.

(iii) **Neutral equilibrium :** After displacing a charged particle from its equilibrium position if it neither comes back, nor moves away but remains in the position in which it was kept it is said to be in neutral equilibrium and in neutral equilibrium $\frac{d^2U}{dx^2}$ is zero i.e., U is constant

(3) **Guidelines to check the equilibrium**

(i) Identify the charge for which equilibrium is to be analyzed.

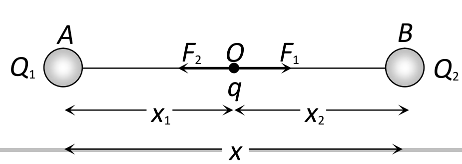
(ii) Check, how many forces acting on that particular charge.

(iii)There should be at least two forces acts oppositely on that charge.

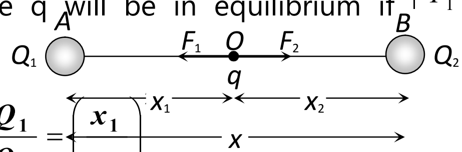
(iv)If magnitude of these forces are equal then charge is said to be in equilibrium then identify the nature of equilibrium.

(v)If all the charges of system are in equilibrium then system is said to be in equilibrium

(4) **Different cases of equilibrium of charge**

Case – 1 : Suppose three similar charge Q_1, q and Q_2 are placed along a straight line as shown below 	**Case – 2 :** Two similar charge Q_1 and Q_2 are placed along a straight line at a distance x from each other and a third dissimilar charge q is placed in between them as shown below

Charge q will be in equilibrium if $|F_1| = |F_2|$



i.e., $\dfrac{Q_1}{Q_2} = \left(\dfrac{x_1}{x_2}\right)$; This is the condition of equilibrium of charge q. After following the guidelines we can say that charge q is in stable equilibrium and this system is not in equilibrium

Note : $x_1 = \dfrac{x}{1+\sqrt{Q_2/Q_1}}$

and $x_2 = \dfrac{x}{1+\sqrt{Q_1/Q_2}}$

e.g. if two charges +4μC and +16 μC are separated by a distance of 30 cm from each other then for equilibrium a third charge should be placed between them at a distance $x_1 = \dfrac{30}{1+\sqrt{16/4}} = 10\ cm$ or $x_2 = 20\ cm$

Charge q will be in equilibrium if $|F_1| = |F_2|$

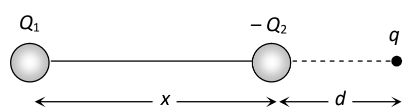
i.e., $\dfrac{Q_1}{Q_2} = \left(\dfrac{x_1}{x_2}\right)^2$.

Note : ❑ Same short trick can be used here to find the position of charge q as we discussed in Case–1 i.e.,

$x_1 = \dfrac{x}{1+\sqrt{Q_2/Q_1}}$ and $x_2 = \dfrac{x}{1+\sqrt{Q_1/Q_2}}$

❑ It is very important to know that magnitude of charge q can be determined if one of the extreme charge (either Q_1 or Q_2) is in equilibrium i.e. if Q_2 is in equilibrium then $|q| = Q_1\left(\dfrac{x_2}{x}\right)^2$ and if Q_1 is in equilibrium then $|q| = Q_2\left(\dfrac{x_1}{x}\right)^2$ (It should be remember that sign of q is opposite to that of Q_1 (or Q_2))

<u>Case – 3:</u> Two dissimilar charge Q_1 and Q_2 are placed along a straight line at a distance x from each other, a third charge q should be placed outside the line joining Q_1 and Q_2 for it to experience zero net force.



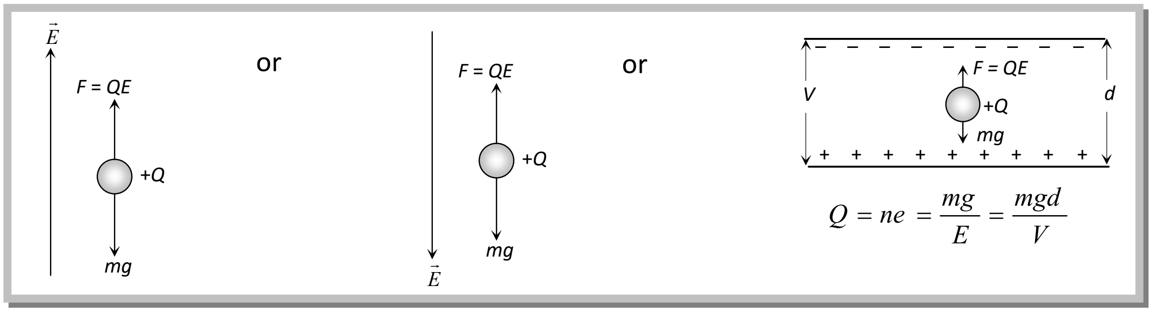
(Let $|Q_2| < |Q_1|$)

Short Trick :

For its equilibrium. Charge q lies on the side of charge which is smallest in magnitude and $d = \dfrac{x}{\sqrt{Q_1/Q_2}-1}$

(5) **Equilibrium of suspended charge in an electric field**

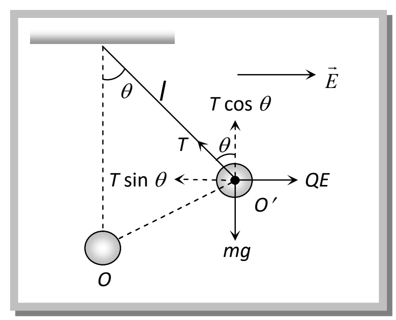
(i) **Freely suspended charged particle :** To suspend a charged a particle freely in air under the influence of electric field it's downward weight should be balanced by upward electric force for example if a positive charge is suspended freely in an electric field as shown then



In equilibrium $QE = mg \Rightarrow \boldsymbol{E = \frac{mg}{Q}}$

Note: In the above case if direction of electric field is suddenly reversed in any figure then acceleration of charge particle at that instant will be a = 2g.

(ii) **Charged particle suspended by a massless insulated string** (like simple pendulum): Consider a charged particle (like Bob) of mass m, having charge Q is suspended in an electric field as shown under the influence of electric field. It turned through an angle (sayθ) and comes in equilibrium.



So, in the position of equilibrium (O′ position)

$T \sin\theta = QE$ ....(i)

$T \cos\theta = mg$ ....(ii)

By squaring and adding equation (i) and (ii) $T = \sqrt{(QE)^2 + (mg)^2}$

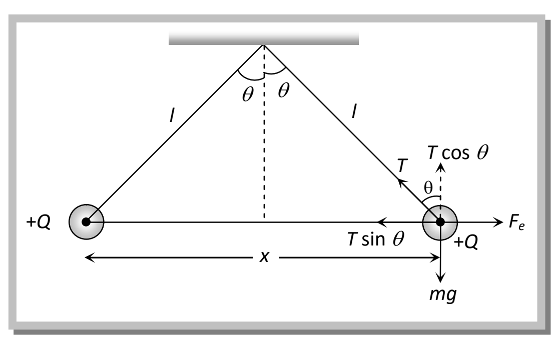
Dividing equation (i) by (ii) $\tan\theta = \frac{QE}{mg} \Rightarrow \theta = \tan^{-1}\frac{QE}{mg}$

(iii) **Equilibrium of suspended point charge system:**Suppose two small balls having charge +Q on each are suspended by two strings of equal length l. Then for equilibrium position as shown in figure.

$T \sin\theta = F_e$ ....(I)

$T \cos\theta = mg$ ....(ii)

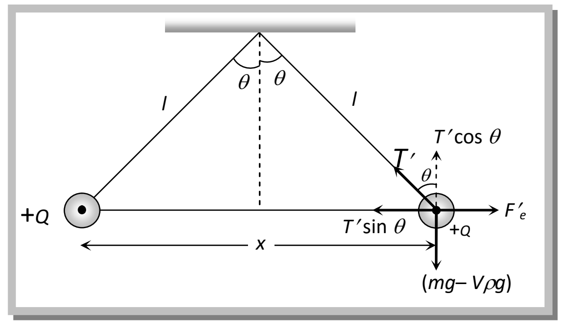
$T^2 = (F_e)^2 + (mg)^2$



and $\tan\theta = \dfrac{F_e}{mg}$; here $F_e = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q^2}{x^2}$ and $\dfrac{x}{2} = l\sin\theta$

(iv) **Equilibrium of suspended point charge system in a liquid:** In the previous discussion if point charge system is taken into a liquid of density ρ such that θ remain same then

In equilibrium $Fe' = T'\sin\theta$ and $(mg - V\rho g) = T'\cos\theta$



$\therefore$ $\tan\theta = \dfrac{Fe'}{(mg - V\rho g)} = \dfrac{Q^2}{4\pi\varepsilon_0 K(mg - V\rho g)x^2}$

When this system was in air $\tan\theta = \dfrac{Fe}{mg} = \dfrac{Q^2}{4\pi\varepsilon_0\, mgx^2}$

$\therefore$ So equating these two gives us

$$\frac{1}{m} = \frac{1}{k(m - V\rho)} \Rightarrow K = \frac{m}{m - V\rho} = \frac{1}{\left(1 - \frac{V}{m}\rho\right)}$$

If σ is the density of material of ball then

$$K = \frac{1}{\left(1 - \frac{\rho}{\sigma}\right)}$$