Motion of Charged Particle in an Electric Field.

(1) **When charged particle initially at rest is placed in the uniform field:**

Let a charge particle of mass m and charge Q be initially at rest in an electric field of strength E

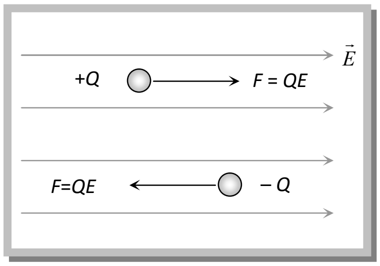


Fig. (A)

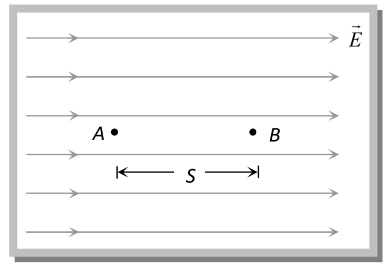


Fig. (B)

(i) **Force and acceleration:** The force experienced by the charged particle is $F = QE$. Positive charge experiences force in the direction of electric field while negative charge experiences force in the direction opposite to the field. [Fig. (A)]

Acceleration produced by this force is $a = \frac{F}{m} = \frac{QE}{m}$

Since the field E in constant the acceleration is constant, thus motion of the particle is uniformly accelerated.

(ii) **Velocity:** Suppose at point A particle is at rest and in time t, it reaches the point B [Fig. (B)]

V = Potential difference between A and B; S = Separation between A and B

(a) By using $v = u + at$, $\quad v = 0 + Q\frac{E}{m}t$, $\quad \Rightarrow v = \frac{QEt}{m}$

(b) By using $v^2 = u^2 + 2as$, $v^2 = 0 + 2 \times \frac{QE}{m} \times s \quad v^2 = \frac{2QV}{m} \left\{\because E = \frac{V}{s}\right\} \Rightarrow v = \sqrt{\frac{2QV}{m}}$

(iii) **Momentum:** Momentum p = mv, $p = m \times \frac{QEt}{m} = QEt$ or $p = m \times \sqrt{\frac{2QV}{m}} = \sqrt{2mQV}$

(iv) **Kinetic energy:** Kinetic energy gained by the particle in time t is $K = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{(QEt)^2}{m} = \frac{Q^2E^2t^2}{2m}$

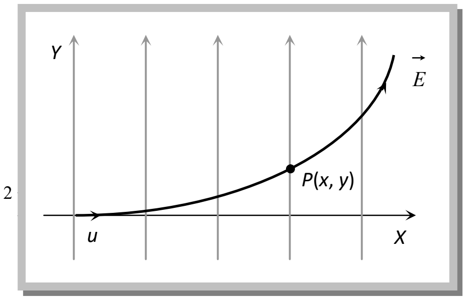
or $$K = \frac{1}{2} m \times \frac{2QV}{m} = QV$$

(2) **When a charged particle enters with an initial velocity at right angle to the uniform field:**

When charged particle enters perpendicularly in an electric field, it describe a parabolic path as shown

(i) Equation of trajectory: Throughout the motion particle has uniform velocity along x-axis and horizontal displacement (x) is given by the equation x = ut

Since the motion of the particle is accelerated along y–axis, we will use equation of motion for uniform acceleration to determine displacement y. From $S = ut + \frac{1}{2}at^2$



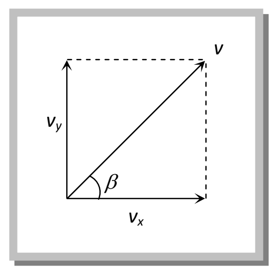
We have $u = 0$ (along y-axis) so $y = \frac{1}{2}at^2$

i.e., displacement along y-axis will increase rapidly with time (since $y \propto t^2$

From displacement along x-axis $t = \frac{x}{u}$

So $y = \frac{1}{2}\left(\frac{QE}{m}\right)\left(\frac{x}{u}\right)^2$; this is the equation of parabola which shows $y \propto x^2$

(ii) Velocity at any instant: At any instant t, $v_x = u$ and $v_y = \frac{QEt}{m}$



So $$v = |\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + \frac{Q^2E^2t^2}{m^2}}$$

If β is the angle made by v with x-axis than $\tan\beta = \frac{v_y}{v_x} = \frac{QEt}{mu}$.

Concepts

☞ An electric field is completely characterized by two physical quantities Potential and Intensity.Force characteristic of the field is intensity and work characteristic of the field is potential.

☞ If a charge particle (say positive) is left free in an electric field, it experiences a force $(F = QE)$ in the direction of electric field and moves in the direction of electric field (which is desired by electric field), so its kinetic energy increases, potential energy decreases, then work is done by the electric field and it is negative.