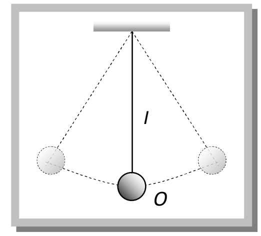
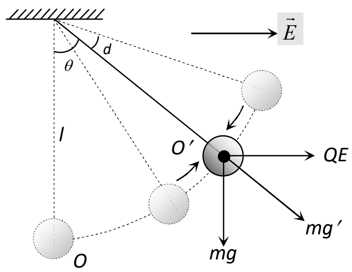
Time Period of Oscillation of a Charged Body.

(1) **Simple pendulum based:** If a simple pendulum having length l and mass of bob m oscillates about its mean position than its time period of oscillation $T = 2\pi\sqrt{\frac{l}{g}}$



Case – 1 : If some charge say +Q is given to bob and an electric field E is applied in the direction as shown in figure then equilibrium position of charged bob (point charge) changes from O to O′.



On displacing the bob from its equilibrium position 0′. It will oscillate under the effective acceleration g′, where

$$mg' = \sqrt{(mg)^2 + (QE)^2}$$

$$\Rightarrow g' = \sqrt{g^2 + (QE/m)^2}$$

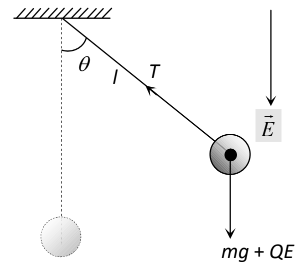
Hence the new time period is $T_1 = 2\pi\sqrt{\frac{l}{g'}}$

$$T_1 = 2\pi\sqrt{\frac{l}{\left(g^2 + (QE/m)^2\right)^{\frac{1}{2}}}}$$

Since g'>g, hence $\mathbf{T_1 < T}$

i.e. time period of pendulum will decrease.

Case – 2 : If electric field is applied in the downward direction then.



Effective acceleration

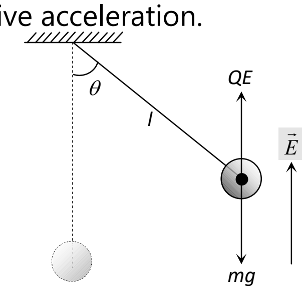
$$g' = g + QE/m$$

So new time period

$$T_2 = 2\pi\sqrt{\frac{l}{g + (QE/m)}}$$

$\mathbf{T_2 < T}$

Case – 3 : In case 2 if electric field is applied in upward direction then, effective acceleration.



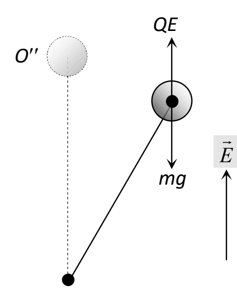
$$g' = g - QE/m$$

So new time period

$$T_3 = 2\pi\sqrt{\frac{l}{g - (QE/m)}}$$

$\mathbf{T_3 > T}$

Case – 4 : In the case 3,

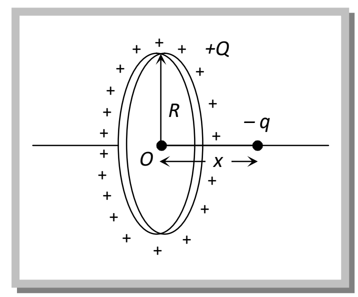


if $T_3 = \frac{T}{2}$ i.e., $2\pi\sqrt{\frac{l}{(g - QE/m)}}$ $= \frac{1}{2}2\pi\sqrt{\frac{l}{g}}$ $\Rightarrow$ QE = 3 mg

i.e., effective vertical force (gravity + electric) on the bob = mg – 3 mg = – 2 mg, hence the

	equilibrium position O′′ of the bob will be above the point of suspension and bob will oscillate under on effective acceleration 2g directed upward. Hence new time period $T_4 = 2\pi\sqrt{\frac{l}{2g}}$, **$T_4 < T$**

(2) **Charged circular ring:** A thin stationary ring of radius R has a positive charge +Q unit. If a negative charge – q (mass m) is placed at a small distance x from the center. Then motion of the particle will be simple harmonic motion.



Electric field at the location of – q charge $E = \frac{1}{4\pi\varepsilon_0} . \frac{Qx}{(x^2 + R^2)^{\frac{3}{2}}}$

Since x<< R, So x^2 neglected hence $E = \frac{1}{4\pi\varepsilon_0} . \frac{Qx}{R^3}$

Force experienced by charge – q is $F = -q\frac{1}{4\pi\varepsilon_0} . \frac{Qx}{R^3}$

$\Rightarrow F \propto -x$ Hence motion is simple harmonic

Having time period $T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 mR^3}{Qq}}$

(3) **Spring mass system:** A block of mass m containing a negative charge – Q is placed on a frictionless horizontal table and is connected to a wall through an upstretched spring of spring constant k as shown. If electric field E applied as shown in figure the block experiences an electric force, hence spring compress and block comes in new position. This is called the equilibrium position of block under the influence of electric field. If block compressed further or stretched, it execute oscillation having time period $T = 2\pi\sqrt{\frac{m}{k}}$. Maximum compression in the spring due to electric field = $\frac{QE}{k}$