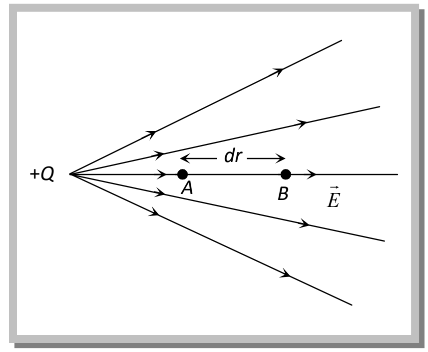
Relation between Electric Field and Potential.

In an electric field rate of change of potential with distance is known as **potential gradient.** It is a vector quantity and its direction is opposite to that of electric field. Potential gradient relates with electric field according to the following relation $\boldsymbol{E = -\frac{dV}{dr}}$; this relation gives another unit of electric field is $\boldsymbol{\frac{volt}{meter}}$. In the above relation negative sign indicates that in the direction of electric field potential decreases.



In space around a charge distribution we can also write $\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$

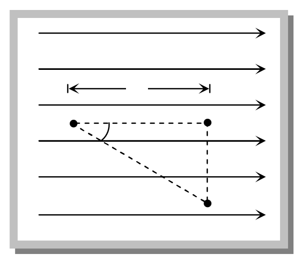
Where $E_x = -\frac{dV}{dx}$, $E_y = -\frac{dV}{dy}$ and $E_z = -\frac{dV}{dz}$

With the help of formula $E = -\frac{dV}{dr}$, potential difference between any two points in an electric field can be determined by knowing the boundary conditions $dV = -\int_{r_1}^{r_2} \vec{E}.\vec{dr} = -\int_{r_1}^{r_2} \vec{E}.\vec{dr}\cos\theta$.

For example: Suppose A, B and C are three points in a uniform electric field as shown in figure.

(i)Potential difference between point A and B is

$$\boldsymbol{V_B - V_A = -\int_A^B \vec{E}\cdot\vec{dr}}$$

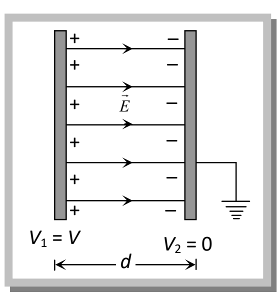


Since displacement is in the direction of electric field, hence θ = 0°

So, $$V_B - V_A = -\int_A^B E\cdot dr\cos 0 = -\int_A^B E\cdot dr = -Ed$$

In general we can say that in an uniform electric field $E = -\frac{V}{d}$ or $|E| = \frac{V}{d}$

Another example



(ii) Potential difference between points A and C is:

$$V_C - V_A = -\int_A^C E\,dr\cos\theta = -E(AC)\cos\theta = -E(AB) = -Ed$$

Above relation proves that potential difference between A and B is equal to the potential difference between A and C i.e. points B and C are at same potential.

Concept

☞ Negative of the slope of the V-r graph denotes intensity of electric field i.e. $\tan\theta = \frac{V}{r} = -E$