Capacitance.

(1) **Definition:** We know that charge given to a conductor increases its potential i.e. $Q \propto V \Rightarrow Q = CV$

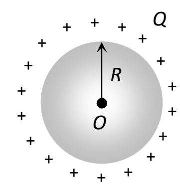
Where C is a proportionality constant, called capacity or capacitance of conductor. Hence capacitance is the ability of conductor to hold the charge.

(2) **Unit and dimensional formula**: S.I. unit is $\frac{Coulomb}{Volt} = Farad\ (F)$

Smaller S.I. units are mF, μF, nF and pF ($1mF = 10^{-3}F$, $1\mu F = 10^{-6}F$, $1nF = 10^{-9}F$, $1pF = 1\mu\mu F = 10^{-12}F$)

C.G.S. unit is Stat Farad $1F = 9\times 10^{11}$ $Stat\ \ Farad$. Dimension: $[C] = [M^{-1}L^{-2}T^4A^2]$.

(3) **Capacity of an isolated spherical conductor:** When charge Q is given to a spherical conductor of radius R, then potential at the surface of sphere is



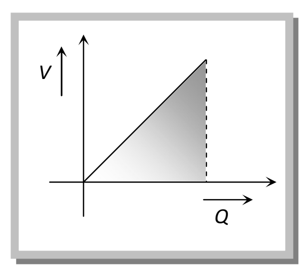
$$V = k.\frac{Q}{R} \qquad \left\{k = \frac{1}{4\pi\varepsilon_0}\right\}$$

Hence its capacity $C = \frac{Q}{V} = 4\pi\varepsilon_0 R \quad \Rightarrow C = 4\pi\varepsilon_0 R = \frac{1}{9\times 10^9}.R$

In C.G.S. $\qquad C = R$

Note: If earth is assumed to be spherical having radius $R = 6400\ km$. It's theoretical capacitance $C = \frac{1}{9\times 10^9}\times 6400\times 10^3 = 711\ \mu F$. But for all practical purpose capacitance of earth is taken infinity.

(4) **Energy of a charged conductor:** When a conductor is charged its potential increases from 0 to V as shown in the graph; and work is done against repulsion, between charge stored in the conductor and charge coming from the source (battery). This work is stored as "electrostatic potential energy"



From graph: Work done = Area of graph $= \frac{1}{2}QV$

Hence potential energy $U = \frac{1}{2}QV$; by using $Q = CV$, we can write

$$U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

(5) **Combination of drops:** Suppose we have n identical drops each having – Radius – r, Capacitance – c, Charge – q, Potential – v and Energy – u.

If these drops are combined to form a big drop of – Radius – R, Capacitance – C, Charge – Q, Potential – V and Energy – U then –

(i) **Charge on big drop:** $Q = nq$

(ii) **Radius of big drop:** Volume of big drop = n × volume of a single drop i.e., $\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$, $R = n^{1/3} r$

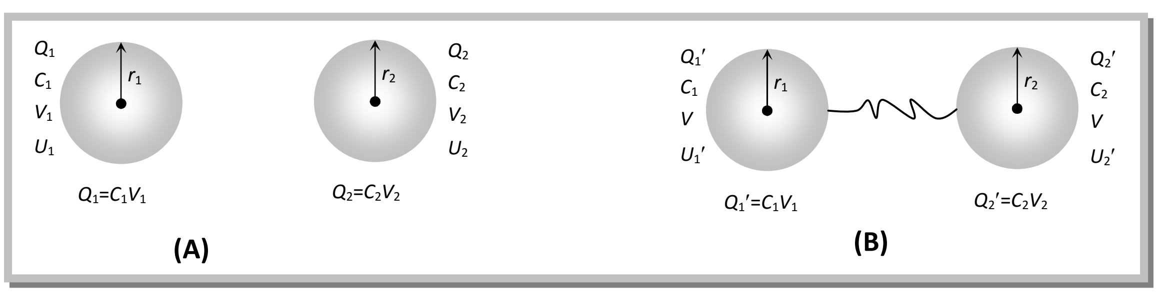
(iii) **Capacitance of big drop:** $C = n^{1/3} c$

(iv) **Potential of big drop:** $V = \frac{Q}{C} = \frac{nq}{n^{1/3} c}$ $V = n^{2/3} v$

(v) **Energy of big drop:** $U = \frac{1}{2} CV^2 = \frac{1}{2}(n^{1/3} c)(n^{2/3} v)^2$ $U = n^{5/3} u$

(6) **Sharing of charge :** When two conductors joined together through a conducting wire, charge begins to flow from one conductor to another till both have the same potential, due to flow of charge, loss of energy also takes place in the form of heat.

Suppose there are two spherical conductors of radii r_1 and r_2, having charge Q_1 and Q_2, potential V_1 and V_2, energies U_1 and U_2 and capacitance C_1 and C_2 respectively, as shown in figure. If these two spheres are connected through a conducting wire, then alteration of charge, potential and energy takes place.



(i) **New charge:** According to the conservation of charge $Q_1 + Q_2 = Q_1' + Q_2' = Q$ (say), also $\frac{Q_1'}{Q_2'} = \frac{C_1 V}{C_2 V} = \frac{4\pi\varepsilon_0 r_1}{4\pi\varepsilon_0 r_2}$, $\frac{Q_1'}{Q_2'} = \frac{r_1}{r_2} \Rightarrow 1 + \frac{Q_1'}{Q_2'} = 1 + \frac{r_1}{r_2} \Rightarrow \frac{Q_1' + Q_2'}{Q_2'} = \frac{r_1 + r_2}{r_2}$

$\Rightarrow Q_2' = Q\left[\frac{r_2}{r_1 + r_2}\right]$ and similarly $Q_1' = Q\left[\frac{r_1}{r_1 + r_2}\right]$

(ii) **Common potential:** Common potential $(V) = \frac{\text{Total charge}}{\text{Total capacity}} = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{Q_1' + Q_2'}{C_1 + C_2} \Rightarrow$ $V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$

(iii) **Energy loss:** As electrical energy stored in the system before and after connecting the spheres is

$U_i = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2$ and $U_f = \frac{1}{2}(C_1 + C_2).V^2 = \frac{1}{2}(C_1 + C_2)\left(\frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}\right)^2$

So energy loss $\Delta U = U_i - U_f = \frac{C_1 C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$

Concept

☞ Capacity of a conductor is a constant term, it does not depend upon the charge Q, and potential (V) and nature of the material of the conductor.