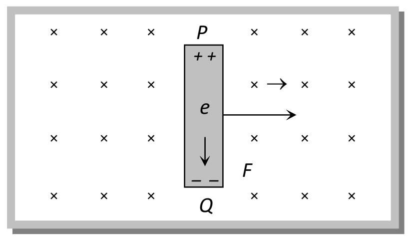
Dynamic (Motional) EMI Due to Translatory Motion.

When a conducting rod moves in a magnetic field, it cuts the magnetic field lines, this process is called flux cutting. Due to this a potential difference developed across the ends of the rod called Dynamic (motional) emf.

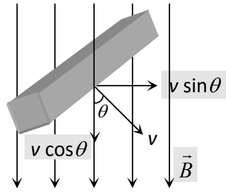
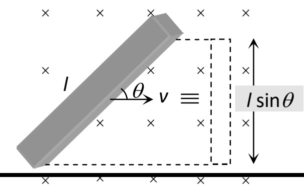
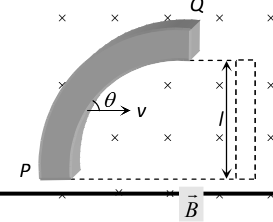
Consider a conducting rod of length l moving with a uniform velocity $\vec{v}$ perpendicular to a uniform magnetic field $\vec{B}$, directed into the plane of the paper. Let the rod be moving to the right as shown in figure. The conducting electrons also move to the right as they are trapped within the rod.



Conducting electrons experiences a magnetic force $\vec{F}_m = -e(\vec{v} \times \vec{B})$. In the present situation they experiences force towards Q, so they move from P to Q within the rod. The end P of the rod becomes positively charged while end Q becomes negatively charged, hence an electric field is set up within the rod which opposes the further downward movement of electrons i.e. an equilibrium is reached and in equilibrium electric force = magnetic force i.e. eE = evB or E = vB$\Rightarrow$ **Induced emf** $\boldsymbol{e = El = Bvl}$ $[E = \frac{V}{l}]$

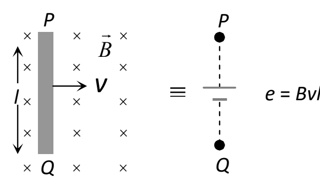
Important cases

If the rod does not translate in a plane perpendicular to the magnetic field or in other words rod is moving in a direction which is making an angle θ with the direction of magnetic field	If the rod is moving perpendicular to the magnetic field but it's direction of motion is making an angle θ with it's length.	An arbitrary shaped conducting rod translating in a uniform magnetic field.

		
This situation is equivalent to a straight conductor moving perpendicular to the magnetic field with a induced emf $e = B(v\sin\theta)l$ $\Rightarrow \boldsymbol{e = Bvl\sin\theta}$	This situation is equivalent to a straight rod of length l sinθ perpendicular to it's direction of motion so induced emf across the rod $e = Bv(l\sin\theta) \Rightarrow \boldsymbol{e = Bvl\sin\theta}$	This rod can be replaced by a straight conductor whose length is equal to the projected length of the conductor on to a plane perpendicular to the direction of motion (dotted rod) so induced emf between P and Q $\boldsymbol{e = Bvl}$

Note:Vector form of motional emf $: e = (\vec{v} \times \vec{B}).\vec{l}$

While solving the problems, flux cutting conducting rod can be treated as a single cell.



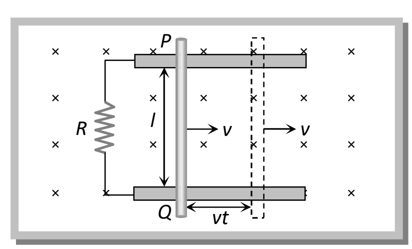
(1) Induced current

If conducting rod moves on two parallel conducting rails as shown in following figure then phenomenon of induced emf can also be understand by the concept of generated area (The area swept of conductor in magnetic field, during its motion)

As shown in figure in time t distance travelled by conductor = vt

Area generated A = lvt

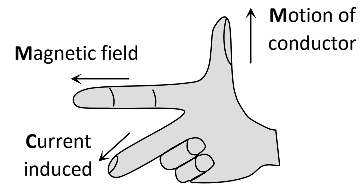
Flux linked with this area ϕ = BA = Blvt



Hence induced emf $|e| = \frac{d\phi}{dt} = Bvl$ induced current $i = \frac{e}{R}$; $\boldsymbol{i = \frac{Bvl}{R}}$

Direction of induced current can be found with the help of Flemings right hand rule.

Fleming's right hand rule: According to this law, if we stretch the right hand thumb and two nearby fingers perpendicular to one another and first finger points in the direction of magnetic field and the thumb in the direction of motion of the conductor then the central finger will point in the direction of the induced current.

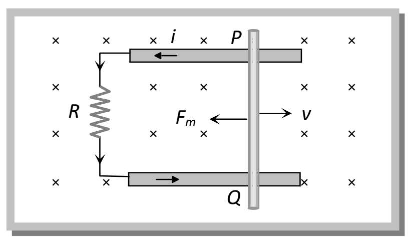


Note: Here it is worthy to note that the rod PQ is acting as a source of emf and inside a source of emf direction of current is from lower potential to higher potential; so the point P of the rod is at higher potential than Q though the current in the rod PQ is from Q to P.

(2) Magnetic force on conductor

Now current is set up in circuit (conductor). As we know when a current carrying conductor moves in a magnetic field, it experiences a force Fm = Bil (maximum) whose direction can be find with the help of Flemings left hand rule.

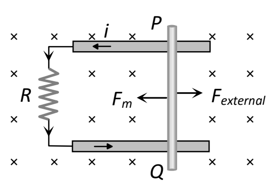
So, here conductor PQ experiences a magnetic force Fm = Bil in opposite direction of it's motion and $F_m = Bil = B\left(\frac{Bvl}{R}\right)l$; $F_m = \frac{B^2vl^2}{R}$



(As a result of this force (Fm) speed of rod decreases as time passes.)

Note: To move the rod with uniform velocity some external mechanical force is required and this is Fext = – Fm

$\Rightarrow |F_{ext}| = \frac{B^2vl^2}{R}$



(3) Power dissipated in moving the conductor

For uniform motion of rod PQ, the rate of doing mechanical work by external agent or mech.

Power delivered by external source is given as $P_{mech} = P_{ext} = \frac{dW}{dt} = F_{ext}.v = \frac{B^2vl^2}{R} \times v$ $\Rightarrow$

$$P_{mech} = \frac{B^2v^2l^2}{R}$$

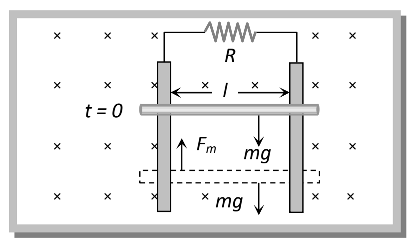
Also electrical power dissipated in resistance or rate of heat dissipation across resistance is given as

$$P_{thermal} = \frac{H}{t} = i^2R = \left(\frac{Bvl}{R}\right)^2.R \; ; \quad P_{thermal} = \frac{B^2v^2l^2}{R}$$

Note: It is clear that Pmech. = Pthermal which is consistent with the principle of conservation of energy.

(4) Motion of conductor rod in a vertical plane : If conducting rod released from rest (at t = 0) as shown in figure then with rise in it's speed (v), induces emf (e), induced current (i), magnetic force (Fm) increases but it's weight remains constant.

Rod will achieve a constant maximum (terminal) velocity vT if $F_m = mg$



So $$\frac{B^2v_T^2l^2}{R} = mg$$

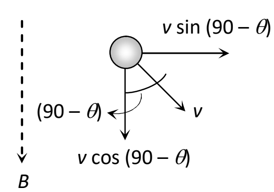
$\Rightarrow$ $$v_T = \frac{mgR}{B^2l^2}$$

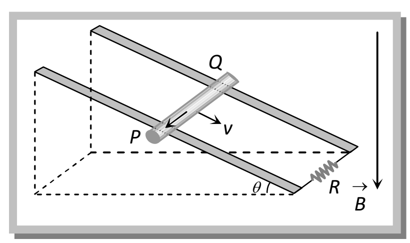
(5) Motion of conducting rod on an inclined plane: When conductor start sliding from the top of an inclined plane as shown, it moves perpendicular to it's length but at an angle (90 – θ) with the direction of magnetic field. Hence induced emf across the ends of conductor

$$e = Bv\sin(90-\theta)l = Bvl\cos\theta$$

So induced current $i = \frac{Bvl\cos\theta}{R}$

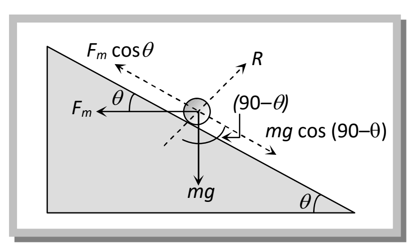
(directed from Q to P).





The forces acting on the bar are shown in following figure. The rod will move down with constant velocity only if

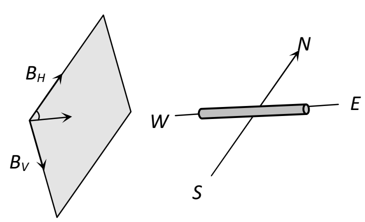
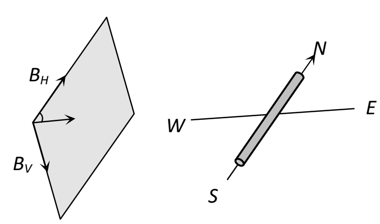
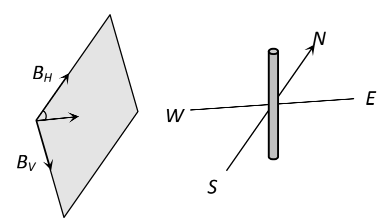
$$F_m\cos\theta = mg\cos(90-\theta) = mg\sin\theta$$



$$Bil\cos\theta = mg\sin\theta$$

$$B\left(\frac{Bv_T l\cos\theta}{R}\right)l\cos\theta = mg\sin\theta \Rightarrow v_T = \frac{mgR\sin\theta}{B^2l^2\cos^2\theta}$$

(6) Motion of a conducting rod in earth's magnetic field: Suppose a conducting rod of length l, executes translatory motion with speed v in earth's magnetic field with

Position I			Position II			Position III		
When conductor is held horizontal with it's length along E-W direction and then it moves – 			When conductor is held horizontal with it's length along N-S direction and then it moves – 			When conductor is held vertical and then it moves – 		
Towards East or West	Towards North or South	Vertically up or down	Towards East or West	Towards North or South	Vertically up or down	Towards East or West	Towards North or South	Vertically up or down
In this condition conductor is moving along it's length, so generated area A	Vertical Component (BV) is cut by the conductor perpendicularly so	Conductor cuts, perpendicularly horizontal component (BH) so $e = B_H vl$	Conductor cuts, the vertical component perpendicularly so $e = B_V vl$	Conductor is moving along its length so $e = 0$	Conductor moves in magnetic meridian i.e. No component is cut by the	Conductor cut's the horizontal component perpendicularly so $e = B_H vl$	Conductor moves in magnetic meridian so $e = 0$	Conductor moves along it'slength so $e = 0$

= 0 hence e = 0	$e = B_V v l$				conductor so e = 0			

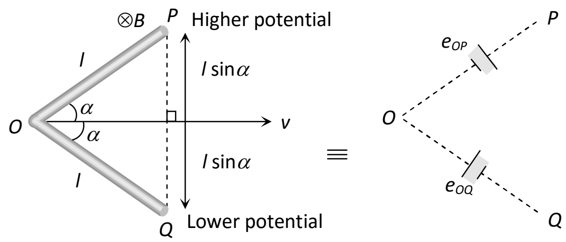
(7) Movement of train in earth's magnetic field: When a train moves on rails, then a potential difference between the ends of the axle of the wheels is induced because the axle of the wheels of the train cuts the vertical component BV of earth's magnetic field and so the magnetic flux linked with it changes and the potential difference or emf is induced. e = BVlv where l is the length of the axle and v is the speed of the train.

(8) Motion of aeroplane in earth's magnetic field: A potential difference or emf across the wings of an aeroplane flying horizontally at a definite height is also induced because aeroplane cuts the vertical component BV of earth's magnetic field. Thus induced emf e = BVlv volt where l is the length of the wings of an aeroplane and v is the speed of the aeroplane.

(9) Orbital satellite: If the orbital plane of an artificial satellite of metallic surface is coincident with equatorial plane of the earth, then no emf will be induced. If orbital plane makes an angle with the equatorial plane, then emf will be induced on it.

(10) Translatory motion of metallic frame in uniform/non-uniform magnetic field

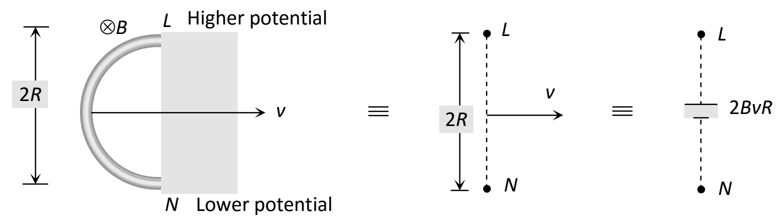
(i) Metal frame of different shape moves in uniform magnetic field



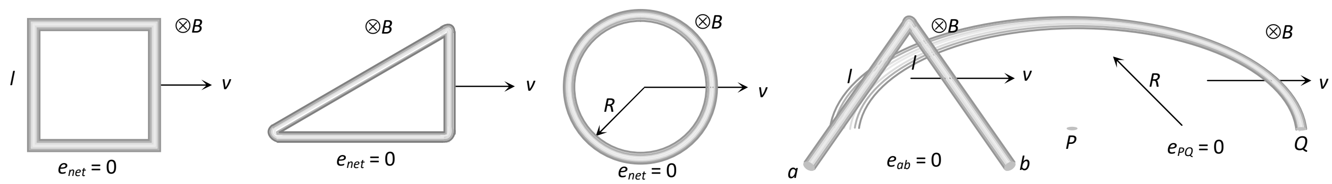
For part OP eOP = VP – VO = Bv$(l \sin \alpha)$

For part QO eQO = VO – VQ = Bv$(l \sin \alpha)$

$$e_{QP} = V_P - V_Q = 2Bv\,(l \sin \alpha)$$

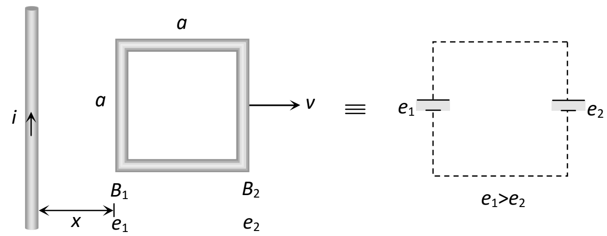


$$\boldsymbol{e_{LN} = 2\,BvR}$$



(ii) Moving metal frame in non-uniform magnetic field

$e_1 = B_1 av$ $e_2 = B_2 av$ For loop enet = (e1 –



e2)

$$e_{net} = av(B_1 - B_2)$$

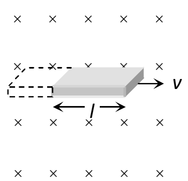
Now $B_1 = \frac{\mu_0 i}{2\pi x}$ and $B_2 = \frac{\mu_0 i}{2\pi(x+a)}$

$$e_{net} = \frac{\mu_0 iav}{2\pi}\left[\frac{1}{x} - \frac{1}{x+a}\right]$$

$$= \frac{\boldsymbol{\mu_0 ia^2 v}}{\boldsymbol{2\pi(x)(x+a)}}$$

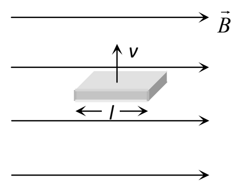
Concepts

In motional emf $\vec{B}, \vec{v}$ and $\vec{l}$ are three vectors. If any two vector are parallel – No flux cutting.



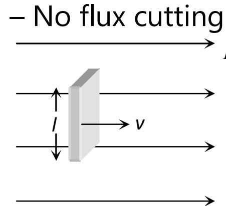
$\vec{v} \; || \; \vec{l}$ so $e = 0$

or Generated area $A = 0$



$\vec{B} \parallel \vec{l}$ so $e = 0$

or Normal to generated area makes an angle 90° with $\vec{B}$



$\vec{v} \parallel \vec{B}$ so $e = 0$

or Normal to generated area makes an angle 90° with $\vec{B}$

A piece of metal and a piece of non-metal are dropped from the same height near the surface of the earth. The non-metallic piece will reach the ground first because there will be no induced current in it.

If an aeroplane is landing down or taking off and its wings are in the east-west direction, then the potential difference or emf will be induced across the wings. If an aeroplane is landing down or taking off and its wings are in the north-south direction, then no potential difference or emf will be induced.

When a conducting rod moving horizontally on equator of earth no emf induces because there is no vertical component of earth's magnetic field. But at poles BV is maximum so maximum flux cutting hence emf induces.

When a conducting rod falling freely in earth's magnetic field such that it's length lies along East - West direction then induced emf continuously increases w.r.t. time and induced current flows from West - East.