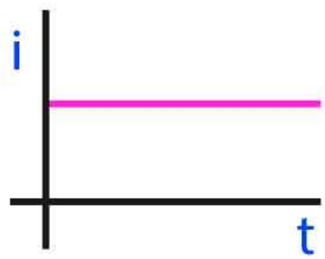
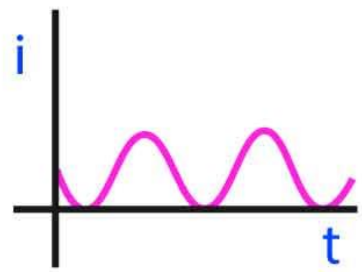
Alternating Current:

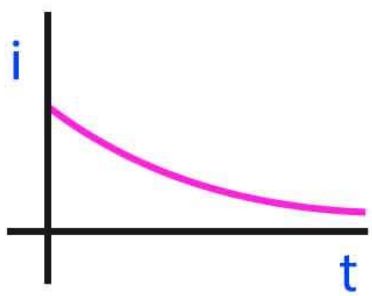
An Alternating Current or A.C changes its direction periodically whereas if the direction of current is constant then it is called direct current or D.C. Let us consider a sinusoidal varying function $i = I_m \sin(\omega t + \Phi)$. Here, the maximum current (peak current) is denoted by Im and 'i' is the instantaneous current. The factor $(\omega t + \Phi)$ is called phase and ω is termed as angular frequency. In terms of frequency, $\omega = 2\pi f$. Also frequency $f = \frac{1}{\text{TimePeriod}(T)}$.



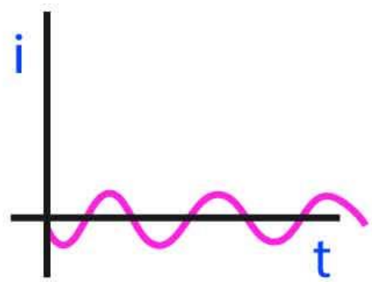
Constant DC



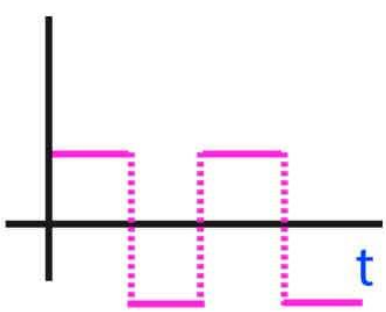
Periodic DC



Variable DC



AC



AC

Average Value:

It is the average of all the instantaneous values of an A.C (alternating current) and alternating voltages over 1 full cycle. The +ve half cycle will be exactly equal to the – ve half cycle in symmetrical waves like voltage or sinusoidal current waveform. Therefore, the average value of a complete cycle will always be 0. The work is done by both, + ve and – ve cycle. Therefore, the average value is calculated without taking its sign into consideration. Therefore, the only + ve half cycle is taken into consideration in determining the average value of alternating quantities (sinusoidal waves).

The average value for one alternation $0 \rightarrow \pi$ is

$$i_{avg} = \frac{i_1 + i_2 + i_3 + ... + i_n}{n}$$

The mean or average value of a given function over a specified time interval between $t_1 \text{ and } t_2$ is determined by:

$$F_{av} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} f(t) dt$$

The cycle of a periodic function is continuously repeated, irrespective of its shape. For example, sinusoidal function. The average value of such periodic function is determined by:

$$F_{av} = \frac{1}{T} \int_0^T f(t) dt$$

where T = time period of the periodic function.

For a symmetrical A.C or alternating current (either sinusoidal or nonsinusoidal) the average value is calculated by taking only the mean or average of one alternation or one-half cycle. Therefore, for all the symmetrical waveforms,

$$F_{av} = \frac{1}{\frac{T}{2}} \int_0^T f(t) dt$$

Root Mean Square (RMS) Value:

Split the base of 1 alternation into 'n' equal parts. Let $i_1, i_2, ..., i_n$ are the ordinates of 'n' equal parts then,

Heat generated during first interval = $i_1^2 R \times \frac{T}{n} J$

Heat generated during second interval = $i_1^2 R \times \frac{T}{n} J$

Heat generated during third interval = $i_1^2 R \times \frac{T}{n} J$

Heat generated during nth interval = $i_1^2 R \times \frac{T}{n} J$

Total heat generated in time T = $RT\left[\frac{i_1^2 + i_2^2 + i_3^2 + ... + i_n^2}{n}\right] J$

If Ieff is the effective current, then the heat generated by this current in time $T = I_{eff}^2 RT$ Joules. Also, by definition, both these expressions are equal.

i.e. $I^2_{eff}RT = RT\left[\frac{i^2_1+i^2_2+i^2_3+...+i^2_n}{n}\right]$

Or, $I^2_{eff} = \left[\frac{i^2_1+i^2_2+i^2_3+...+i^2_n}{n}\right]$

Or, $I_{eff} = \sqrt{\frac{i^2_1+i^2_2+i^2_3+...+i^2_n}{n}}$

Hence, the virtual or effective value of an alternating voltage or current is equal to the square root of the mean or average of the squares of the successive ordinates. Therefore, it is termed as the root mean square or RMS value.

The effective or RMS (root mean square) value over a time period (T) of an alternating quantity is determined by:

$$F_{rms} = \sqrt{\frac{1}{T}\int_0^T F^2(t)dt}$$

Power Consumed:

Let the potential difference $v = V_A - V_B = V_M \sin\omega t$

Let the electric current flowing through it be, $i = I\sin(\omega t + \Phi)$

Therefore, the Instantaneous power consumed by the device is given by

$$P = (V_m \sin\omega t)[I_m \sin(\omega t + \Phi)]$$

Average power consumed in a cycle:

$$\frac{\int_0^{\frac{2\pi}{\omega}} P\,dt}{\frac{2\pi}{\omega}} = \frac{1}{2} \times V_m I_m \cos\Phi$$

i.e. $\frac{V_m}{\sqrt{2}}.\frac{I_m}{\sqrt{2}}.\cos\Phi = V_{rms} I_{rms} \cos\Phi$

Here, $\cos\Phi = Power Factor$

RC Circuit in series with an A.C source:

$I_m = \frac{V_m}{\sqrt{X^2_c + R^2}}$

And, $\tan\Phi = \frac{I_m X_C}{I_m R} = \frac{X_C}{R}$

LR Circuit in series with an A.C source:

$V = \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{R^2 + X^2_L} = IZ$

Where,

Impedance $Z = \sqrt{R^2 + X^2_L}$

And, $\tan\Phi = \frac{IX_L}{IR} = \frac{X_L}{R}$

LC Circuit in series with an A.C source:

$V=I|X_L-X_C|=IZ \text{ and } \Phi=90$

RLC Circuit in series with an A.C source:

$V=\sqrt{(IR)^2+(IX_L-IX_C)^2}=I\sqrt{R^2+(X_L-X_C)^2}=IZ$

Where, $Z=\sqrt{R^2+(X_L-X_C)^2}$

And, $\tan\Phi=\frac{I(X_L-X_C)}{IR}=\frac{X_L-X_C}{R}$

Resonance:

The Amplitude of current in an RLC circuit is maximum for a given value of R and V if the impedance of the circuit is minimized. This is the condition for resonance.

Resonance occurs when **XL = XC**

Or, $2\pi fL=\frac{1}{2\pi fC}$

Or, $f^2=\frac{1}{2\pi L\times 2\pi C}=\frac{1}{4\pi^2 LC}$

Or, $f=\sqrt{\frac{1}{4\pi^2 LC}}=\frac{1}{2\pi\sqrt{LC}}$ Hz

Therefore, $\omega_r=\frac{1}{\sqrt{LC}}$ Radians.

Alternating Voltage:

Same will be the case with alternating voltage. i.e. $v=V_m \sin(\omega t+\Phi)$.