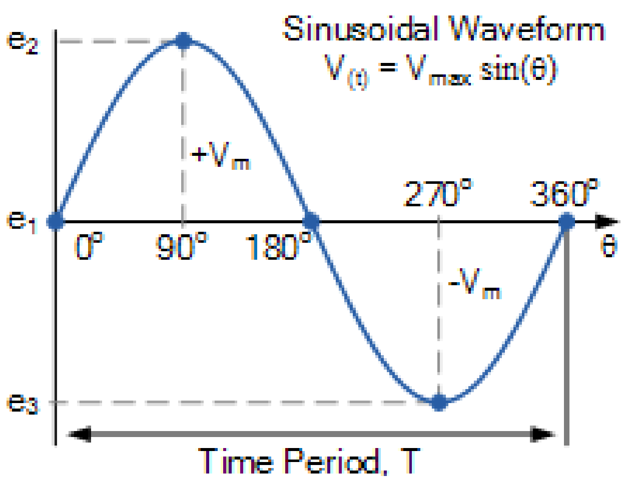
Power in AC Circuits:

In an AC circuit, the instantaneous values of the voltage, current and therefore power are constantly changing being influenced by the supply. So we can not calculate the power in AC circuits in the same manner as we can in DC circuits, but we can still say that power (p) is equal to the voltage (v) times the amperes (i).

Another important point is that AC circuits contain reactance, so there is a power component as a result of the magnetic and/or electric fields created by the components. The result is that unlike a purely resistive component, this power is stored and then returned back to the supply as the sinusoidal waveform goes through one complete periodic cycle.



$$e_1 = V_m \sin(0^\circ) = V_m \text{ x } 0 = 0\text{v}$$
or
$$e_1 = V_m \sin(180^\circ) = V_m \text{ x } 0 = 0\text{v}$$
or
$$e_1 = V_m \sin(360^\circ) = V_m \text{ x } 0 = 0\text{v}$$
$$e_2 = V_m \sin(90^\circ) = V_m \text{ x } +1 = +V_m$$
$$e_3 = V_m \sin(270^\circ) = V_m \text{ x } -1 = -V_m$$

Thus, the average power absorbed by a circuit is the sum of the power stored and the power returned over one complete cycle. So a circuits average power consumption will be the average of the instantaneous power over one full cycle with the instantaneous power, **p** defined as the multiplication of the instantaneous voltage, **v** by the instantaneous current, **i**. Note that as the sine function is periodic and continuous, the average power given over all time will be exactly the same as the average power given over a single cycle.

As the instantaneous power is the power at any instant of time, then:

$$p = v \times i$$

where:

$$v = V_m \sin(\omega t + \theta_v)$$

$$i = I_m \sin(\omega t + \theta_i)$$

$$p = \left[V_m \sin(\omega t + \theta_v) \right] \times \left[I_m \sin(\omega t + \theta_i) \right]$$

$$\therefore p = V_m I_m \left[\sin(\omega t + \theta_v) \sin(\omega t + \theta_i) \right]$$

Applying the trigonometric product-to-sum identity of:

$$\sin A \sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B) \right]$$

and **θ** = θ_v – θ_i (the phase difference between the voltage and the current waveforms) into the above equation gives:

$$p = \frac{V_m I_m}{2}\left[\cos\theta - \cos(2\omega t + \theta) \right]$$

$$\text{As: } \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} = V_{rms} \times I_{rms} \quad (W)$$

Where **V** and **I** are the root-mean-squared (rms) values of the sinusoidal waveforms, *v* and *i* respectively, and θ is the phase difference between the two waveforms. Therefore we can express the instantaneous power as being:

Instantaneous AC Power Equation

$$p = VI\cos\theta - VI\cos(2\omega t + \theta)$$

This equation shows us that the instantaneous AC power has two different parts and is therefore the sum of these two terms. The second term is a time varying sinusoid whose frequency is equal to twice the angular frequency of the supply due to the 2ω part of the term. The first term however is a constant whose value depends only on the phase difference, θ between the voltage, (V) and the current, (I).

As the instantaneous power is constantly changing with the profile of the sinusoid over time, this makes it difficult to measure. It is therefore more convenient, and easier on the maths to use the average or mean value of the power. So over a fixed number of cycles, the average value of the instantaneous power of the sinusoid is given simply as:

$$p = V \times I\cos\theta$$

Where **V** and **I** are the sinusoids rms values, and **θ** (Theta) is the phase angle between the voltage and the current. The units of power are in watts (W).

The AC Power dissipated in a circuit can also be found from the impedance, (Z) of the circuit using the voltage, V_{rms} or the current, I_{rms} flowing through the circuit as shown.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\theta = \cos^{-1} = \frac{R}{Z}, \quad \text{or} \quad \sin^{-1} = \frac{X_L}{Z}, \quad \text{or} \quad \tan^{-1} = \frac{X_L}{R}$$

$$\therefore P = \frac{V^2}{Z}\cos(\theta) \quad \text{or} \quad P = I^2 Z\cos(\theta)$$

Power Factor:

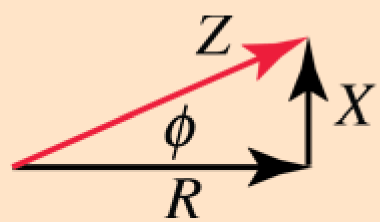
In AC circuits, the power factor is the ratio of the real power that is used to do work and the apparent power that is supplied to the circuit.
The power factor can get values in the range from 0 to 1.
When all the power is reactive power with no real power (usually inductive load) - the power factor is 0.
When all the power is real power with no reactive power (resistive load) - the power factor is 1.

For a DC circuit the power is P=VI, and this relationship also holds for the instantaneous power in an AC circuit. However, the average power in an AC circuit expressed in terms of the rms voltage and current is

$$P_{avg} = VI\cos\phi$$

where φ is the phase angle between the voltage and current. The additional term is called the power factor

$$\text{POWER FACTOR} = \cos\phi = \frac{R}{Z}$$



From the phasor diagram for AC impedance, it can be seen that the power factor is R/Z. For a purely resistive AC circuit, R=Z and the power factor = 1.

A power factor of one or "unity power factor" is the goal of any electric utility company since if the power factor is less than one, they have to supply more current to the user for a given amount of power use. In so doing, they incur more line losses. They also must have larger capacity equipment in place than would be otherwise necessary. As a result, an industrial facility will be charged a penalty if its power factor is much different from 1.

Industrial facilities tend to have a "lagging power factor", where the current lags the voltage (like an inductor). This is primarily the result of having a lot of electric induction motors - the windings of motors act as inductors as seen by the power supply. Capacitors have the opposite effect and can compensate for the inductive motor windings. Some industrial sites will have large banks of capacitors strictly for the purpose of correcting the power factor back toward one to save on utility company charges.

Wattless Current:

In an ac circuit where $R=0$

$\Rightarrow \cos\phi=0$

so $P_{av}=0$

i.e. in resistance less circuit the power consumed is zero. Such a circuit is called the wattless circuit and the current flowing is called the wattless current.

or

The component of current which does not contribute to the average power dissipation is called wattless current

(i) The average of wattless component over one cycle is zero

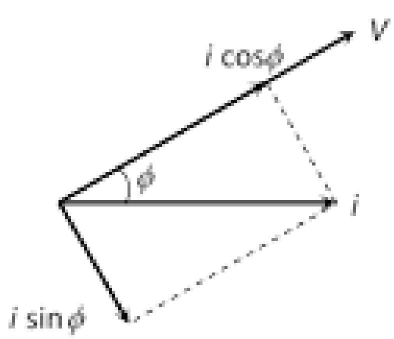
(ii) Amplitude of wattless current

$$=i_0\sin\phi$$

and r.m.s. value of wattless current

$$=i_{rms}\sin\varphi = i_0 2\surd\sin\phi$$

.



It is quadrature (90^o) with voltage.