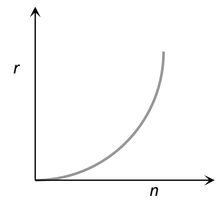
Bohr's Orbits (For Hydrogen and H2-Like Atoms).

(1) Radius of orbit - For an electron around a stationary nucleus the electrostatics force of attraction provides the necessary centripetal force

i.e. $\frac{1}{4\pi\varepsilon_0}\frac{(Ze)e}{r^2} = \frac{mv^2}{r}$ .... (i) Also $mvr = \frac{nh}{2\pi}$ ....(ii)

From equation (i) and (ii) radius of nth orbit



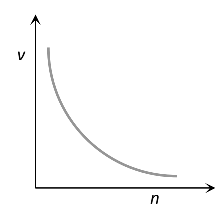
$$r_n = \frac{n^2h^2}{4\pi^2 kZme^2} = \frac{n^2h^2\varepsilon_0}{\pi mZe^2} = 0.53\frac{n^2}{Z}\mathring{A}\left[\text{where } k = \frac{1}{4\pi\varepsilon_0}\right]$$

$\Rightarrow \boldsymbol{r_n \propto \frac{n^2}{Z}}$

Note: The radius of the innermost orbit (n = 1) hydrogen atom (z = 1) is called Bohr's radius a0 i.e. $a_0 = 0.53\,\mathring{A}$.

(2) Speed of electron

From the above relations, speed of electron in nth orbit can be calculated as



$$v_n = \frac{2\pi kZe^2}{nh} = \frac{Ze^2}{2\varepsilon_0 nh} = \left(\frac{c}{137}\right).\frac{Z}{n} = 2.2\times10^6\frac{Z}{n}m/sec$$

Where (c = speed of light 3 × 108 m/s)

Note: The ratio of speed of an electron in ground state in Bohr's first orbit of hydrogen atom to velocity of light in air is equal to $\frac{e^2}{2\varepsilon_0 ch} = \frac{1}{137}$ (where c = speed of light in air)

(3) Some other quantities

For the revolution of electron in nth orbit, some other quantities are given in the following table

Quantity	Formula	Dependency on n and Z
(1) Angular speed	$\omega_n = \frac{v_n}{r_n} = \frac{\pi m z^2 e^4}{2\varepsilon_0^2 n^3 h^3}$	$\omega_n \propto \frac{Z^2}{n^3}$
(2) Frequency	$\nu_n = \frac{\omega_n}{2\pi} = \frac{m z^2 e^4}{4\varepsilon_0^2 n^3 h^3}$	$\nu_n \propto \frac{Z^2}{n^3}$
(3) Time period	$T_n = \frac{1}{\nu_n} = \frac{4\varepsilon_0^2 n^3 h^3}{m z^2 e^4}$	$T_n \propto \frac{n^3}{Z^2}$
(4) Angular momentum	$L_n = m v_n r_n = n\left(\frac{h}{2\pi}\right)$	$L_n \propto n$
(5) Corresponding current	$i_n = e\nu_n = \frac{m z^2 e^5}{4\varepsilon_0^2 n^3 h^3}$	$i_n \propto \frac{Z^2}{n^3}$
(6) Magnetic moment	$M_n = i_n A = i_n\left(\pi r_n^2\right)$ (where $\mu_0 = \frac{eh}{4\pi m} =$ Bohr magneton)	$M_n \propto n$
(7) Magnetic field	$B = \frac{\mu_0 i_n}{2r_n} = \frac{\pi m^2 z^3 e^7 \mu_0}{8\varepsilon_0^3 n^5 h^5}$	$B \propto \frac{Z^3}{n^5}$

(4) Energy

(i) Potential energy: An electron possesses some potential energy because it is found in the field of nucleus potential energy of electron in nth orbit of radius rn is given by

$$U = k.\frac{(Ze)(-e)}{r_n} = -\frac{kZe^2}{r_n}$$

(ii) Kinetic energy: Electron possess kinetic energy because of its motion. Closer orbits have greater kinetic energy than outer ones.

As we know $\frac{mv^2}{r_n} = \frac{k.(Ze)(e)}{r_n^2}$ $\Rightarrow$ Kinetic energy $K = \frac{kZe^2}{2r_n} = \frac{|U|}{2}$

(iii) Total energy: Total energy (E) is the sum of potential energy and kinetic energy i.e. E = K + U

$\Rightarrow$ $E = -\frac{kZe^2}{2r_n}$ Also $r_n = \frac{n^2 h^2 \varepsilon_0}{\pi m z e^2}$. Hence $E = -\left(\frac{me^4}{8\varepsilon_0^2 h^2}\right) \cdot \frac{z^2}{n^2} = -\left(\frac{me^4}{8\varepsilon_0^2 ch^3}\right) ch \frac{z^2}{n^2}$

$= -R\,ch \frac{Z^2}{n^2} = -13.6 \frac{Z^2}{n^2} eV$

Where $R = \frac{me^4}{8\varepsilon_0^2 ch^3}$ = Rydberg's constant = 1.09 × 107 per meter

Note: Each Bohr orbit has a definite energy

For hydrogen atom (Z = 1) $\Rightarrow$ $E_n = -\frac{13.6}{n^2} eV$

The state with n = 1 has the lowest (most negative) energy. For hydrogen atom it is E1 = – 13.6 eV.

Rch = Rydberg's energy $\simeq 2.17 \times 10^{-18} J \simeq 31.6\, eV$.

$E = -K = \frac{U}{2}$.

(iv) Ionization energy and potential: The energy required to ionize an atom is called ionization energy. It is the energy required to make the electron jump from the present orbit to the infinite orbit.

Hence $E_{ionisation} = E_\infty - E_n = 0 - \left(-13.6 \frac{Z^2}{n^2}\right) = +\frac{13.6 Z^2}{n^2} eV$

For H2-atom in the ground state $E_{ionisation} = \frac{+13.6(1)^2}{n^2} = 13.6\, eV$

The potential through which an electron need to be accelerated so that it acquires energy equal to the ionization energy is called ionization potential. $V_{ionisation} = \frac{E_{ionisation}}{e}$

(v) Excitation energy and potential: When the electron is given energy from external source, it jumps to higher energy level. This phenomenon is called excitation.

The minimum energy required to excite an atom is called excitation energy of the particular excited state and corresponding potential is called exciting potential.

$E_{Excitation} = E_{Final} - E_{Initial}$ and $V_{Excitation} = \frac{E_{excitation}}{e}$

(vi) Binding energy (B.E.): Binding energy of a system is defined as the energy released when its constituents are brought from infinity to form the system. It may also be defined as the energy needed to separate its constituents to large distances. If an electron and a proton are initially at rest and brought from large distances to form a hydrogen atom, 13.6 eV energy will be released. The binding energy of a hydrogen atom is therefore 13.6 eV.

Note: For hydrogen atom principle quantum number $n = \sqrt{\frac{13.6}{(\text{B.E.})}}$.

(5) Energy level diagram

The diagrammatic description of the energy of the electron in different orbits around the nucleus is called energy level diagram.

Energy level diagram of hydrogen/hydrogen like atom

Principle quantum number	Orbit	Excited state	Energy for H2 – atom	Energy for H2 – like atom	Ionization energy from this level (for H2 – atom)
n = ∞	Infinite	Infinite	E∞ = 0 eV	0 eV	0 eV
n = 4	Fourth	Third	E4 = – 0.85 eV	– 0.85 Z2	+ 0.85 eV
n = 3	Third	Second	E3 = – 1.51 eV	– 1.51 Z2	+ 1.51 eV
n = 2	Second	First	E2 = – 3.4 eV	– 3.4 Z2	+ 3.4 eV
n = 1	First	Ground	E1 = – 13.6 eV	– 13.6 Z2	+ 13.6 eV

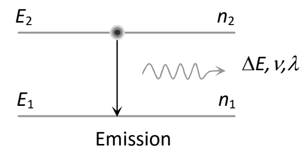
Note: In hydrogen atom excitation energy to excite electron from ground state to first excited state will be $-3.4-(-13.6)=10.2\ eV$ .and from ground state to second excited state it is [$-1.51-(-13.6)=12.09\ eV$].

In an H_2 atom when e^- makes a transition from an excited state to the ground state its kinetic energy increases while potential and total energy decreases.

(6) Transition of electron

When an electron makes transition from higher energy level having energy E2 (n2) to a lower energy level having energy E1 (n1) then a photon of frequency ν is emitted

(i) Energy of emitted radiation



$$\Delta E = E_2 - E_1 = \frac{-Rc\,h\,Z^2}{n_2^2} - \left(-\frac{Rch\,Z^2}{n_1^2}\right) = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

(ii) Frequency of emitted radiation

$$\Delta E = h\nu \Rightarrow \nu = \frac{\Delta E}{h} = \frac{E_2 - E_1}{h} = Rc\,Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

(iii) Wave number/wavelength

Wave number is the number of waves in unit length $\bar{\nu} = \frac{1}{\lambda} = \frac{\nu}{c}$

$$\Rightarrow \frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \frac{13.6Z^2}{hc}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

(iv) Number of spectral lines: If an electron jumps from higher energy orbit to lower energy orbit it emits radiations with various spectral lines.

If electron falls from orbit n2 to n1 then the number of spectral lines emitted is given by

$$N_E = \frac{(n_2 - n_1 + 1)(n_2 - n_1)}{2}$$

If electron falls from nth orbit to ground state (i.e. n2 = n and n1 = 1) then number of spectral lines emitted $\mathbf{N_E = \frac{n(n-1)}{2}}$

Note: Absorption spectrum is obtained only for the transition from lowest energy level to higher energy levels. Hence the number of absorption spectral lines will be (n – 1).

(v) Recoiling of an atom: Due to the transition of electron, photon is emitted and the atom is recoiled

Recoil momentum of atom = momentum of photon $= \frac{h}{\lambda} = hRZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

Also recoil energy of atom $= \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$ (where m = mass of recoil atom)

(7) Draw backs of Bohr's atomic model

(i) It is valid only for one electron atoms, e.g. : H, He+, Li+2, Na+1 etc.

(ii) Orbits were taken as circular but according to Sommerfield these are elliptical.

(iii) Intensity of spectral lines could not be explained.

(iv) Nucleus was taken as stationary but it also rotates on its own axis.

(v) It could not be explained the minute structure in spectrum line.

(vi) This does not explain the Zeeman Effect (splitting up of spectral lines in magnetic field) and Stark effect (splitting up in electric field)

(vii) This does not explain the doublets in the spectrum of some of the atoms like sodium (5890Å & 5896Å)