Application of Dimensional Analysis.

(1) **To find the unit of a physical quantity in a given system of units:** Writing the definition or formula for the physical quantity we find its dimensions. Now in the dimensional formula replacing M, L and T by the fundamental units of the required system we get the unit of physical quantity. However, sometimes to this unit we further assign a specific name, e.g., Work = Force × Displacement

So $[W] = [MLT^{-2}] \times [L] = [ML^2T^{-2}]$

So its units in C.G.S. system will be g cm^2/s^2 which is called erg while in M.K.S. system will be kg m^2/s^2 which is called joule.

(2) **To find dimensions of physical constant or coefficients** :As dimensions of a physical quantity are unique, we write any formula or equation incorporating the given constant and then by substituting the dimensional formulae of all other quantities, we can find the dimensions of the required constant or coefficient.

(i) Gravitational constant: According to Newton's law of gravitation

$F = G\frac{m_1 m_2}{r^2}$ or $G = \frac{Fr^2}{m_1 m_2}$

Substituting the dimensions of all physical quantities

$$[G] = \frac{[MLT^{-2}][L^2]}{[M][M]} = [M^{-1}L^3T^{-2}]$$

(ii) Plank constant: According to Planck $E = h\nu$ or $h = \frac{E}{\nu}$

Substituting the dimensions of all physical quantities

$$[h] = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$$

(iii) Coefficient of viscosity: According to Poiseuille's formula

$$\frac{dV}{dt} = \frac{\pi pr^4}{8\eta l} \text{ or } \eta = \frac{\pi pr^4}{8l(dV/dt)}$$

Substituting the dimensions of all physical quantities

$$[\eta] = \frac{[ML^{-1}T^{-2}][L^4]}{[L][L^3/T]} = [ML^{-1}T^{-1}]$$

(3) **To convert a physical quantity from one system to the other:** The measure of a physical quantity is nu = constant

If a physical quantity X has dimensional formula $[M^aL^bT^c]$ and if (derived) units of that physical quantity in two systems are $[M_1^a L_1^b T_1^c]$ and $[M_2^a L_2^b T_2^c]$ respectively and n_1 and n_2 be the numerical values in the two systems respectively, then $n_1[u_1] = n_2[u_2]$

$$\Rightarrow n_1[M_1^a L_1^b T_1^c] = n_2[M_2^a L_2^b T_2^c]$$

$$\Rightarrow n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

Where M_1, L_1 and T_1 are fundamental units of mass, length and time in the first (known) system and M_2, L_2 and T_2 are fundamental units of mass, length and time in the second (unknown) system. Thus knowing the values of fundamental

units in two systems and numerical value in one system, the numerical value in other system may be evaluated.

Example: (1) conversion of Newton into Dyne.

The Newton is the S.I. unit of force and has dimensional formula $[MLT^{-2}]$.

So 1 N = 1 kg-m/sec^2

By using $n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c = 1\left[\frac{kg}{gm}\right]^1\left[\frac{m}{cm}\right]^1\left[\frac{sec}{sec}\right]^{-2}$

$$= 1\left[\frac{10^3\,gm}{gm}\right]^1\left[\frac{10^2\,cm}{cm}\right]^1\left[\frac{sec}{sec}\right]^{-2} = 10^5$$

$\therefore$ 1 N = 10^5 Dyne

(2) Conversion of gravitational constant (G) from C.G.S. to M.K.S. system

The value of G in C.G.S. system is 6.67×10^{-8} C.G.S. units while its dimensional formula is $[M^{-1}L^3T^{-2}]$

So G = 6.67×10^{-8} $cm^3/g\ s^2$

By using $n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c = 6.67\times 10^{-8}\left[\frac{gm}{kg}\right]^{-1}\left[\frac{cm}{m}\right]^3\left[\frac{sec}{sec}\right]^{-2}$

$$= 6.67\times 10^{-8}\left[\frac{gm}{10^3\,gm}\right]^{-1}\left[\frac{cm}{10^2\,cm}\right]^3\left[\frac{sec}{sec}\right]^{-2} = 6.67\times 10^{-11}$$

$\therefore$ G = 6.67×10^{-11} M.K.S. units

(4) **To check the dimensional correctness of a given physical relation :** This is based on the 'principle of homogeneity'. According to this principle the dimensions of each term on both sides of an equation must be the same.

If $X = A \pm (BC)^2 \pm \sqrt{DEF}$,

Then according to principle of homogeneity [X] = [A] = $[(BC)^2]$ $= [\sqrt{DEF}]$

If the dimensions of each term on both sides are same, the equation is dimensionally correct, otherwise not. A dimensionally correct equation may or may not be physically correct.

Example: (1) $F = mv^2 / r^2$

By substituting dimension of the physical quantities in the above relation –

$$[MLT^{-2}] = [M][LT^{-1}]^2 /[L]^2$$

i.e. $[MLT^{-2}] = [MT^{-2}]$

As in the above equation dimensions of both sides are not same; this formula is not correct dimensionally, so can never be physically.

(2) $s = ut - (1/2)at^2$

By substituting dimension of the physical quantities in the above relation –

$$[L] = [LT^{-1}][T] - [LT^{-2}][T^2]$$

i.e. $[L] = [L] - [L]$

As in the above equation dimensions of each term on both sides are same, so this equation is dimensionally correct. However, from equations of motion we know that $s = ut + (1/2)at^2$

(5) **As a research tool to derive new relations:**If one knows the dependency of a physical quantity on other quantities and if the dependency is of the product type, then using the method of dimensional analysis, relation between the quantities can be derived.

Example: (i) Time period of a simple pendulum.

Let time period of a simple pendulum is a function of mass of the bob (m), effective length (l), acceleration due to gravity (g) then assuming the function to be product of power function of m, l and g

i.e., $T = Km^x l^y g^z$; where K = dimensionless constant

If the above relation is dimensionally correct then by substituting the dimensions of quantities –

$$[T] = [M]^x [L]^y [LT^{-2}]^z$$

or $[M^0L^0T^1] = [M^xL^{y+z}T^{-2z}]$

Equating the exponents of similar quantities x = 0, y = 1/2 and z = – 1/2

So the required physical relation becomes $T = K\sqrt{\frac{l}{g}}$

The value of dimensionless constant is found (2π) through experiments so

$$T = 2\pi\sqrt{\frac{l}{g}}$$

(ii) Stoke's law : When a small sphere moves at low speed through a fluid, the viscous force F, opposing the motion, is found experimentally to depend on the radius r, the velocity of the sphere v and the viscosity η of the fluid.

So F = f (η, r, v)

If the function is product of power functions of η, r and v, $F = K\eta^x r^y v^z$; where K is dimensionless constant.

If the above relation is dimensionally correct

$$[MLT^{-2}] = [ML^{-1}T^{-1}]^x [L]^y [LT^{-1}]^z$$

or $[MLT^{-2}] = [M^x L^{-x+y+z} T^{-x-z}]$

Equating the exponents of similar quantities x = 1; – x + y + z = 1 and – x – z = – 2

Solving these for x, y and z, we get x = y = z = 1

So eq^n (i) becomes F = Kηrv

On experimental grounds, K = 6π; so F = $6\pi\eta$rv

This is the famous Stoke's law.