

Errors of Measurement.

The measuring process is essentially a process of comparison. In spite of our best efforts, the measured value of a quantity is always somewhat different from its actual value, or true value. This difference in the true value of a quantity is called error of measurement.

(1) **Absolute error:** Absolute error in the measurement of a physical quantity is the magnitude of the difference between the true value and the measured value of the quantity.

Let a physical quantity be measured n times. Let the measured value be $a_1, a_2, a_3, \dots, a_n$. The arithmetic mean of these values is $a_m = \frac{a_1 + a_2 + \dots + a_n}{n}$

Usually, a_m is taken as the true value of the quantity, if the same is unknown otherwise.

By definition, absolute errors in the measured values of the quantity are

$$\Delta a_1 = a_m - a_1$$

$$\Delta a_2 = a_m - a_2$$

.....

$$\Delta a_n = a_m - a_n$$

The absolute errors may be positive in certain cases and negative in certain other cases.

(2) **Mean absolute error:** It is the arithmetic mean of the magnitudes of absolute errors in all the measurements of the quantity. It is represented by $\overline{\Delta a}$. Thus

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + \dots + |\Delta a_n|}{n}$$

Hence the final result of measurement may be written as $a = a_m \pm \overline{\Delta a}$

This implies that any measurement of the quantity is likely to lie between $(a_m + \overline{\Delta a})$ and $(a_m - \overline{\Delta a})$.

(3) **Relative error or Fractional error:** The relative error or fractional error of measurement is defined as the ratio of mean absolute error to the mean value of the quantity measured.

Thus Relative error or Fractional error = $\frac{\text{mean absolute error}}{\text{mean value}} = \frac{\overline{\Delta a}}{a_m}$

4) **Percentage error:** When the relative/fractional error is expressed in percentage, we call it percentage error. Thus

Percentage error = $\frac{\overline{\Delta a}}{a_m} \times 100\%$