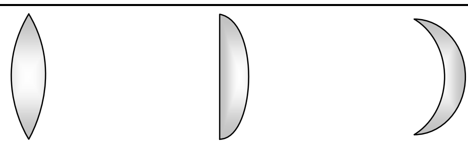
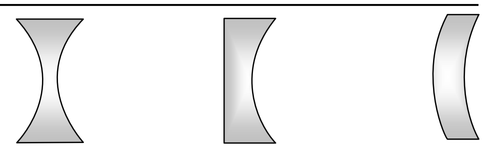
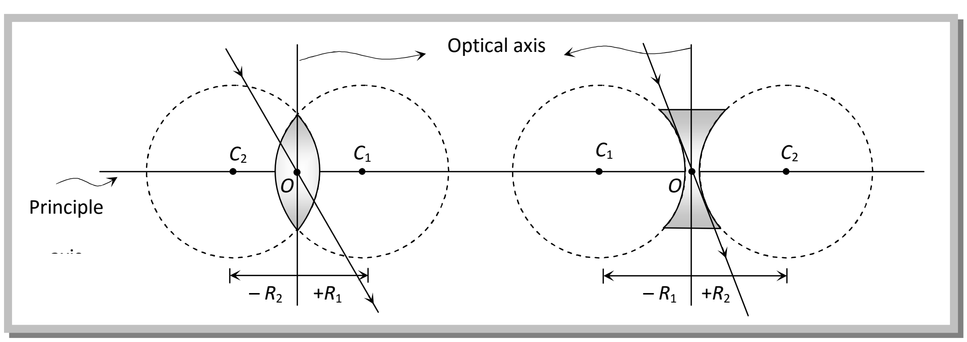
Lens.

Lens is a transparent medium bounded by two refracting surfaces, such that at least one surface is spherical.

(1) Type of lenses

Convex lens (Converges the light rays)	Concave lens (Diverges the light rays)
 Double convex Plano convex Concavo convex	 Double concave Plane concave Convexo concave
Thick at middle	Thin at middle
It forms real and virtual images both	It forms only virtual images

(2) Some definitions

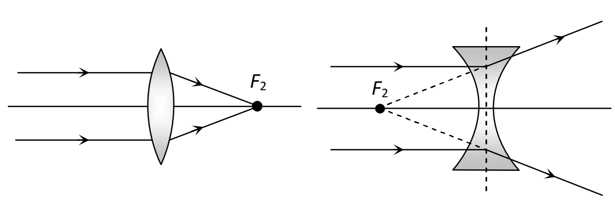


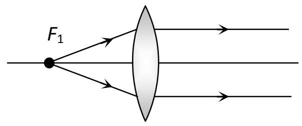
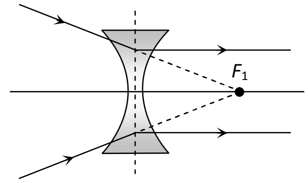
C_1, C_2 – Centre of curvature,

R_1, R_2 – Radii of curvature

(i) Optical center (O): A point for a given lens through which light ray passes undeviated (Light ray passes undeviated through optical center).

(ii) Principle focus

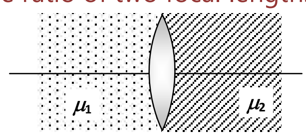


First principle focus	Second principle focus
	

Note: Second principle focus is the principle focus of the lens.

When medium on two sides of lens is same then $|F_1| = |F_2|$.

If medium on two sides of lens are not same then the ratio of two focal lengths $\frac{f_1}{f_2} = \frac{\mu_1}{\mu_2}$



(iii) Focal length (f): Distance of second principle focus from optical center is called focal length

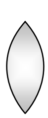
$f_{\text{convex}} \rightarrow$ Positive, $f_{\text{concave}} \rightarrow$ negative, $f_{\text{plane}} \rightarrow \infty$

(iv) Aperture: Effective diameter of light transmitting area is called aperture.

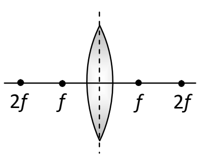
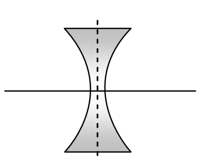
$\text{Intensity of image} \propto (\text{Aperture})^2$

(v) Power of lens (P): Means the ability of a lens to converge the light rays. Unit of power is Diopter (D).

$$P = \frac{1}{f(m)} = \frac{100}{f(cm)};\ P_{\text{convex}} \rightarrow \text{positive},\ P_{\text{concave}} \rightarrow \text{negative},\ P_{\text{plane}} \rightarrow \text{zero}.$$

Note:	Thick lens	thin lens
	P ↑ f ↓ R ↓	P ↓ f ↑ R ↑
		

(3) Image formation by lens

Lens	Location of the object	Location of the image	Nature of image		
			Magnification	Real virtual	Erect inverted
Convex	At infinity i.e. $u = \infty$	At focus i.e. $v = f$	$m < 1$ diminished	Real	Inverted
	Away from 2f i.e. $(u > 2f)$	Between f and 2f i.e. $f < v < 2f$	$m < 1$ diminished	Real	Inverted
	At 2f or $(u = 2f)$	At 2f i.e. $(v = 2f)$	$m = 1$ same size	Real	Inverted
	Between f and 2f i.e. $f < u < 2f$	Away from 2f i.e. $(v > 2f)$	$m > 1$ magnified	Real	Inverted
	At focus i.e. $u = f$	At infinity i.e. $v = \infty$	$m = \infty$ magnified	Real	Inverted
	Between optical center and focus, $u < f$	At a distance greater than that of object $v > u$	$m > 1$ magnified	Virtual	Erect
Concave	At infinity i.e. $u = \infty$	At focus i.e. v = f	$m < 1$ diminished	Virtual	Erect
	Anywhere between infinity and optical center	Between optical center and focus	$m < 1$ diminished	Virtual	Erect

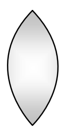
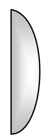
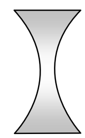
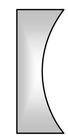
Note: Minimum distance between an object and its real image formed by a convex lens is 4f.

Maximum image distance for concave lens is its focal length.

(4) Lens maker's formula

The relation between f, μ, R1 and R2 is known as lens maker's formula and it is

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Equiconvex lens	Plano convex lens	Equi concave lens	Plano concave lens
$R_1 = R$ and $R_2 = -R$ $f = \frac{R}{2(\mu - 1)}$  for $\mu = 1.5$, $f = R$	$R_1 = \infty$, $R_2 = -R$ $f = \frac{R}{(\mu - 1)}$  for $\mu = 1.5$, $f = 2R$	$R_1 = -R$, $R_2 = +R$ $f = -\frac{R}{2(\mu - 1)}$  for $\mu = 1.5$ $f = -R$	$R_1 = \infty$, $R_2 = R$ $f = \frac{R}{2(\mu - 1)}$  for $\mu = 1.5,\ f = -2R$

(5) Lens in a liquid

Focal length of a lens in a liquid (f_l) can be determined by the following formula

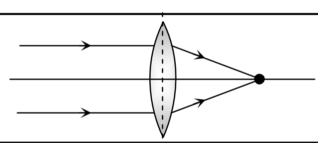
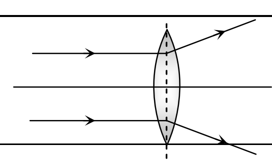
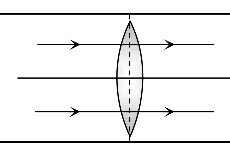
$$\frac{f_l}{f_a} = \frac{({}_a\mu_g - 1)}{({}_l\mu_g - 1)}$$ (Lens is supposed to be made of glass).

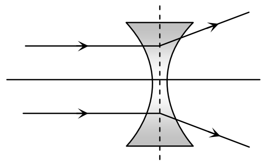
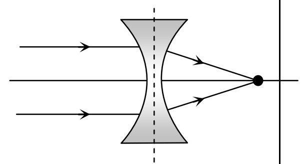
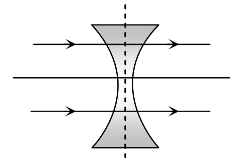
Note: Focal length of a glass lens $(\mu = 1.5)$ is f in air then inside the water its focal length is 4f.

In liquids focal length of lens increases (↑) and its power decreases (↓).

(6) Opposite behavior of a lens

In general refractive index of lens $(\mu_L) >$ refractive index of medium surrounding it (μ_M).

μL >μM	μL <μM	μL = μM
		

(7) Lens formula and magnification of lens

(i) Lens formula: $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$; (use sign convention)

(ii) Magnification: The ratio of the size of the image to the size of object is called magnification.

(a) Transverse magnification: $m=\frac{I}{O}=\frac{v}{u}=\frac{f}{f+u}=\frac{f-v}{f}$ (use sign convention while solving the problem)

(b) Longitudinal magnification: $m=\frac{I}{O}=\frac{v_2-v_1}{u_2-u_1}$. For very small object $m=\frac{dv}{du}=\left(\frac{v}{u}\right)^2=\left(\frac{f}{f+u}\right)^2=\left(\frac{f-v}{f}\right)^2$

(c) Areal magnification: $m_s=\frac{A_i}{A_o}=m^2=\left(\frac{f}{f+u}\right)^2$, (Ai = Area of image, Ao = Area of object)

(8) Relation between object and image speed

If an object move with constant speed (V_o) towards a convex lens from infinity to focus, the image will move slower in the beginning and then faster. Also $V_i=\left(\frac{f}{f+u}\right)^2.V_o$

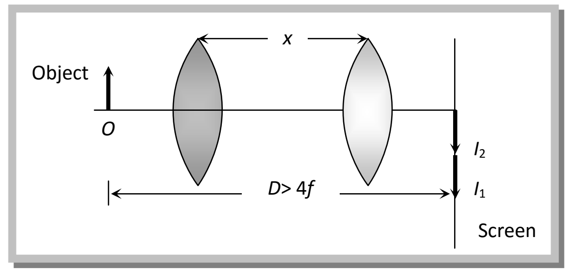
(9) Focal length of convex lens by displacement method

(i) For two different positions of lens two images $(I_1 \text{ and } I_2)$ of an object is formed at the same location.

(ii) Focal length of the lens $f = \frac{D^2 - x^2}{4D} = \frac{x}{m_1 - m_2}$

Where $m_1 = \frac{I_1}{O}$ and $m_2 = \frac{I_2}{O}$

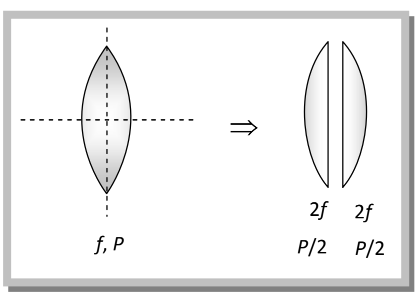
(iii) Size of object $O = \sqrt{I_1 . I_2}$

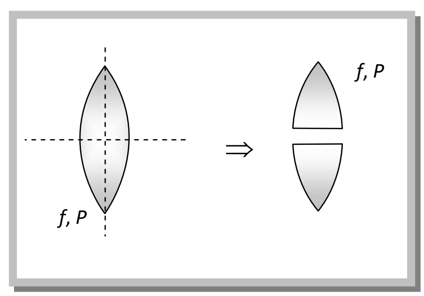


(10) Cutting of lens

(i) A symmetric lens is cut along optical axis in two equal parts. Intensity of image formed by each part will be same as that of complete lens.

(ii) A symmetric lens is cut along principle axis in two equal parts. Intensity of image formed by each part will be less compared as that of complete lens. (Aperture of each part is $\frac{1}{\sqrt{2}}$ times that of complete lens)





(11) Combination of lens

(i) For a system of lenses, the net power, net focal length and magnification given as follows:

$P = P_1 + P_2 + P_3 ....$, $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + ...$,

$m = m_1 \times m_2 \times m_3 \times ....$

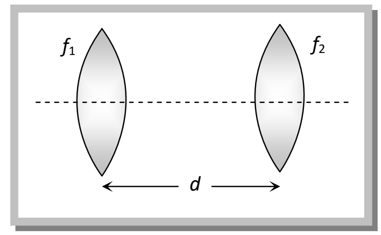
(ii) In case when two thin lens are in contact: Combination will behave as a lens, which have more power or lesser focal length.

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}\Rightarrow F=\frac{f_1 f_2}{f_1+f_2} \text{ and } P=P_1+P_2$$

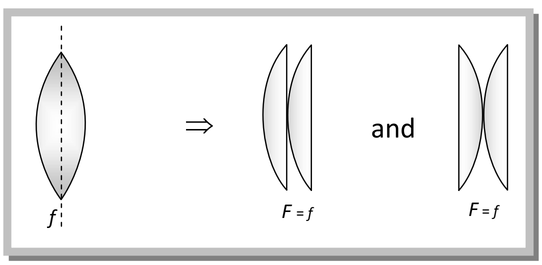
(iii) If two lens of equal focal length but of opposite nature are in contact then combination will behave as a plane glass plate and $F_{\text{combination}}=\infty$

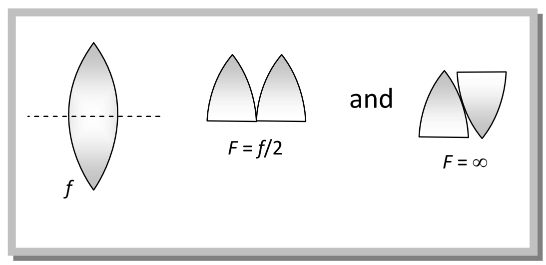
(iv) When two lenses are placed co-axially at a distance d from each other than equivalent focal length (F).

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2} \text{ and } P=P_1+P_2-dP_1P_2$$



(v) Combination of parts of a lens:





(12) Silvering of lens

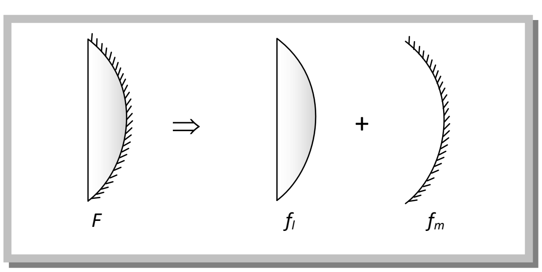
On silvering the surface of the lens it behaves as a mirror. The focal length of the silvered lens is

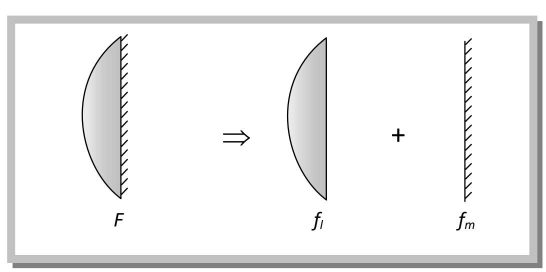
$$\frac{1}{F}=\frac{2}{f_l}+\frac{1}{f_m}$$

Where $f_l=$ focal length of lens from which refraction takes place (twice)

$f_m=$ Focal length of mirror from which reflection takes place.

(i) Plano convex is silvered





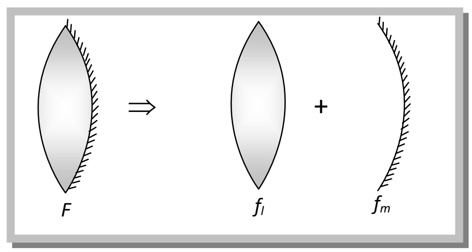
$f_m = \frac{R}{2}, f_l = \frac{R}{(\mu - 1)}$ so $F = \frac{R}{2\mu}$

$f_m = \infty, f_l = \frac{R}{(\mu - 1)}$ so $F = \frac{R}{2(\mu - 1)}$

(ii) Double convex lens is silvered

Since $f_l = \frac{R}{2(\mu - 1)}, f_m = \frac{R}{2}$

So $F = \frac{R}{2(2\mu - 1)}$



Note: Similar results can be obtained for concave lenses.

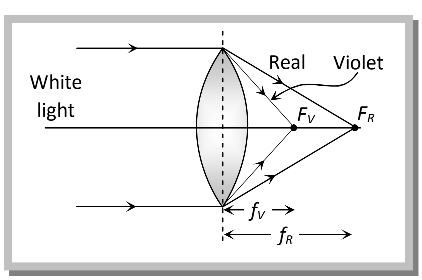
(13) Defects in lens

(i) Chromatic aberration: Image of a white object is colored and blurred because μ (hence f) of lens is different for different colors. This defect is called chromatic aberration.

$\mu_V > \mu_R$ so $f_R > f_V$

Mathematically chromatic aberration =

$f_R - f_V = \omega f_y$



ω = Dispersion power of lens.

fy = Focal length for mean color $= \sqrt{f_R f_V}$

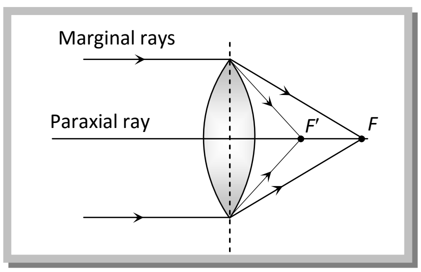
Removal: To remove this defect i.e. for Achromatism we use two or more lenses in contact in place of single lens.

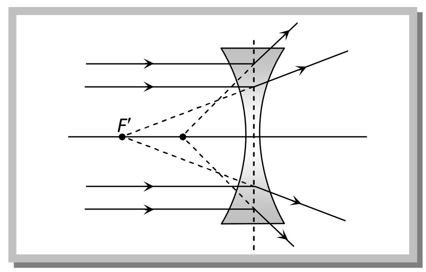
Mathematically condition of Achromatism is: $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$ or $\omega_1 f_2 = -\omega_2 f_1$

Note: Component lenses of an achromatic doublet cemented by canada blasam because it is transparent and has a refractive index almost equal to the refractive of the glass.

(ii) Spherical aberration: Inability of a lens to form the point image of a point object on the axis is called Spherical aberration.

In this defect all the rays passing through a lens are not focused at a single point and the image of a point object on the axis is blurred.





Removal: A simple method to reduce spherical aberration is to use a stop before and in front of the lens. (But this method reduces the intensity of the image as most of the light is cut off). Also by using plano-convex lens, using two lenses separated by distance d = F – F ', using crossed lens.

Note: Marginal rays: The rays farthest from the principal axis.

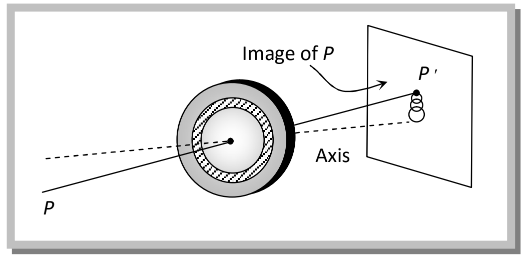
Paraxial rays: The rays close to the principal axis.

Spherical aberration can be reduced by either stopping paraxial rays or marginal rays, which can be done by using a circular annular mask over the lens.

Parabolic mirrors are free from spherical aberration.

(iii) Coma: When the point object is placed away from the principle axis and the image is received on a screen perpendicular to the axis, the shape of the image is like a comet. This defect is called Coma.

It refers to spreading of a point object in a plane ⊥ to principle axis.

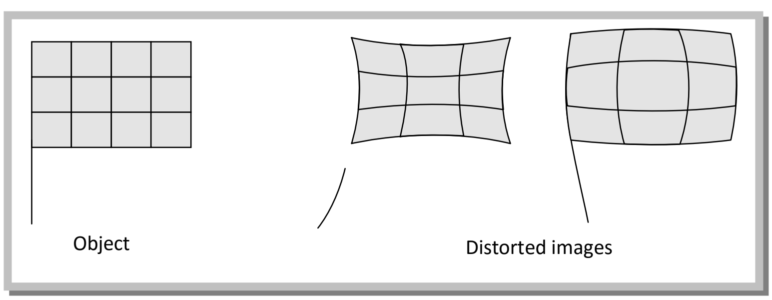


Removal: It can be reduced by properly designing radii of curvature of the lens surfaces. It can also be reduced by appropriate stops placed at appropriate distances from the lens.

(iv) Curvature: For a point object placed off the axis, the image is spread both along and perpendicular to the principal axis. The best image is, in general, obtained not on a plane but on a curved surface. This defect is known as Curvature.

Removal: Astigmatism or the curvature may be reduced by using proper stops placed at proper locations along the axis.

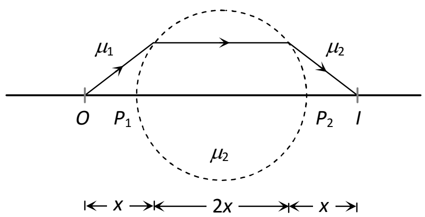
(v) Distortion: When extended objects are imaged, different portions of the object are in general at different distances from the axis. The magnification is not the same for all portions of the extended object. As a result a line object is not imaged into a line but into a curve.

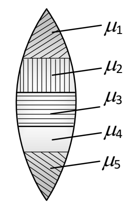


(vi) Astigmatism: The spreading of image (of a point object placed away from the principal axis) along the principal axis is called Astigmatism.

Concepts

If a sphere of radius R made of material of refractive index μ_2 is placed in a medium of refractive index μ_1, Then if the object is placed at a distance $\left(\frac{\mu_1}{\mu_2 - \mu_1}\right)R$ from the pole, the real image formed is equidistant from the sphere.





The lens doublets used in telescope are achromatic for blue and red colors, while these used in camera are achromatic for violet and green colors. The reason for this is that our eye is most sensitive between blue and red colors, while the photographic plates are most sensitive between violet and green colors.

Position of optical center

Equiconvex and equiconcave	Exactly at center of lens
Convexo-concave and concavo-convex	Outside the glass position
Plano convex and plano concave	On the pole of curved surface

Composite lens : If a lens is made of several materials then

Number of images formed = Number of materials used

Here no. of images = 5