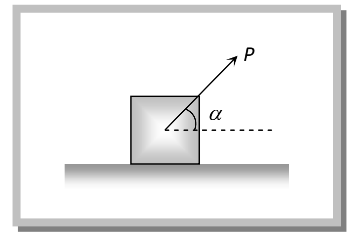
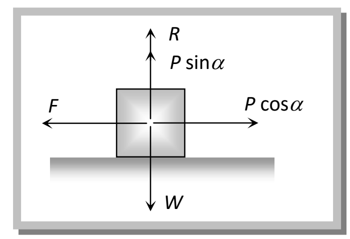
Calculation of Necessary Force in Different Conditions.

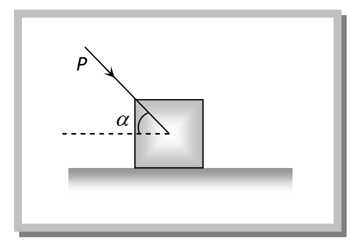
If W = weight of the body, θ = angle of friction, $\mu = \tan\theta$ = coefficient of friction

then we can calculate necessary force for different condition in the following manner :

(1) **Minimum pulling force P at an angle α from the horizontal**







By resolving P in horizontal and vertical direction (as shown in figure)

For the condition of equilibrium

$$F = P\cos\alpha \quad \text{and} \quad R = W - P\sin\alpha$$

By substituting these value in $F = \mu R$

$$P\cos\alpha = \mu(W - P\sin\alpha)$$

$$\Rightarrow P\cos\alpha = \frac{\sin\theta}{\cos\theta}(W - P\sin\alpha) \quad [\text{As } \mu = \tan\theta]$$

$$\Rightarrow P = \frac{W\sin\theta}{\cos(\alpha - \theta)}$$

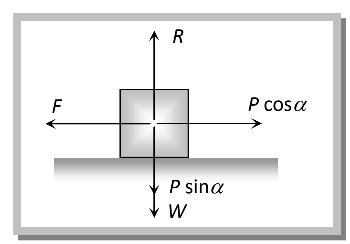
(2) **Minimum pushing force P at an angle α from the horizontal**

By Resolving P in horizontal and vertical direction (as shown in the figure)

For the condition of equilibrium

$$F = P\cos\alpha \quad \text{and} \quad R = W + P\sin\alpha$$

By substituting these value in $F = \mu R$



$$\Rightarrow P\cos\alpha = \mu(W + P\sin\alpha)$$

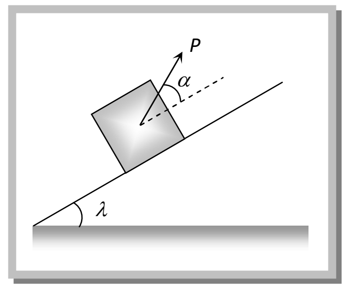
$$\Rightarrow P\cos\alpha = \frac{\sin\theta}{\cos\theta}(W + P\sin\alpha) \quad [\text{As } \mu = \tan\theta]$$

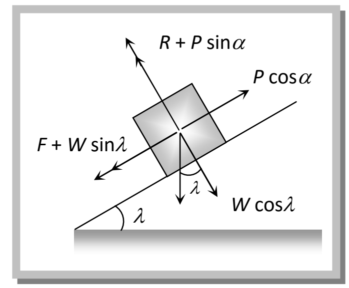
$$\Rightarrow P = \frac{W\sin\theta}{\cos(\alpha + \theta)}$$

(3) **Minimum pulling force P to move the body up an inclined plane**

By Resolving P in the direction of the plane and perpendicular to the plane (as shown in the figure)

For the condition of equilibrium





$$R + P\sin\alpha = W\cos\lambda$$

$\therefore \; R = W\cos\lambda - P\sin\alpha$

and $$F + W\sin\lambda = P\cos\alpha$$

$\therefore \; F = P\cos\alpha - W\sin\lambda$

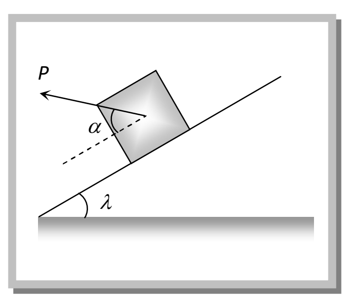
By substituting these values in $F = \mu R$ and solving we get

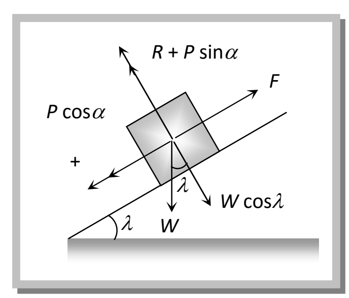
$$P = \frac{W\sin(\theta + \lambda)}{\cos(\alpha - \theta)}$$

(4) **Minimum force on body in downward direction along the surface of inclined plane to start its motion**

By Resolving P in the direction of the plane and perpendicular to the plane (as shown in the figure

For the condition of equilibrium





$$R + P\sin\alpha = W\cos\lambda$$

$\therefore \; R = W\cos\lambda - P\sin\alpha$

and $$F = P\cos\alpha + W\sin\lambda$$

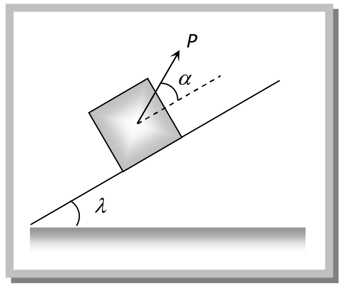
By substituting these values in $F = \mu R$ and solving we get

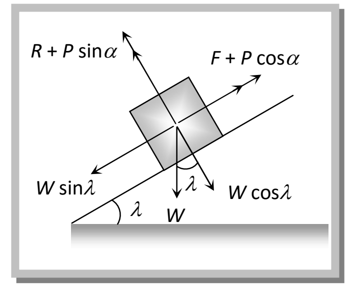
$$P = \frac{W\sin(\theta - \lambda)}{\cos(\alpha - \theta)}$$

(5) **Minimum force to avoid sliding a body down an inclined plane**

By Resolving P in the direction of the plane and perpendicular to the plane (as shown in the figure)

For the condition of equilibrium





$$R + P\sin\alpha = W\cos\lambda$$

$\therefore$ $$R = W\cos\lambda - P\sin\alpha$$

and $$P\cos\alpha + F = W\sin\lambda$$

$\therefore$ $$F = W\sin\lambda - P\cos\alpha$$

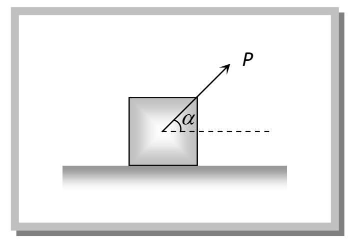
By substituting these values in $F = \mu R$ and solving we get

$$P = W\left[\frac{\sin(\lambda-\theta)}{\cos(\theta+\alpha)}\right]$$

(6) **Minimum force for motion and its direction**

Let the force P be applied at an angle α with the horizontal.

By resolving P in horizontal and vertical direction (as shown in figure)



For vertical equilibrium

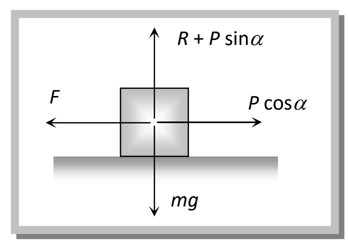
$$R + P\sin\alpha = mg$$

$\therefore$ $$R = mg - P\sin\alpha \qquad ....(i)$$

and for horizontal motion

$$P\cos\alpha \geq F$$

i.e. $P\cos\alpha \geq \mu R \qquad ....(ii)$



Substituting value of R from (i) in (ii)

$$P\cos\alpha \geq \mu(mg - P\sin\alpha)$$

$$P \geq \frac{\mu mg}{\cos\alpha + \mu\sin\alpha} \qquad ....(iii)$$

For the force P to be minimum $(\cos\alpha + \mu\sin\alpha)$ must be maximum *i.e.*

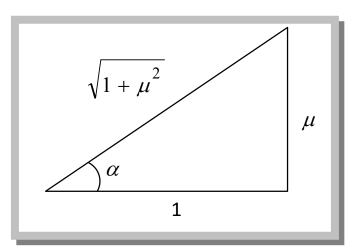
$$\frac{d}{d\alpha}[\cos\alpha + \mu\sin\alpha] = 0 \Rightarrow -\sin\alpha + \mu\cos\alpha = 0$$

$\therefore$ $$\tan\alpha = \mu$$

or $\alpha = \tan^{-1}(\mu) = \text{angle of friction}$

i.e. For minimum value of P its angle from the horizontal should be equal to angle of friction

As $\tan\alpha = \mu$ so from the figure $\sin\alpha = \dfrac{\mu}{\sqrt{1+\mu^2}}$ and $\cos\alpha = \dfrac{1}{\sqrt{1+\mu^2}}$



By substituting these value in equation (iii)

$$P \geq \frac{\mu mg}{\frac{1}{\sqrt{1+\mu^2}} + \frac{\mu^2}{\sqrt{1+\mu^2}}} \geq \frac{\mu mg}{\sqrt{1+\mu^2}}$$

$$\therefore \qquad P_{\min} = \frac{\mu mg}{\sqrt{1+\mu^2}}$$