Motion of Two Bodies One Resting on the Other.

When a body *A* of mass *m* is resting on a body *B* of mass *M* then two conditions are possible

(1) A force *F* is applied to the upper body, (2) A force *F* is applied to the lower body

We will discuss above two cases one by one in the following manner:

(1) **A force *F* is applied to the upper body, then following four situations are possible**

(i) **When there is no friction**

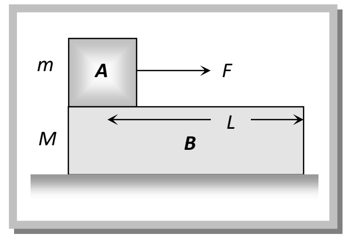
(a) The body *A* will move on body *B* with acceleration (*F*/*m*).

$$a_A = F/m$$

(b) The body *B* will remain at rest

$$a_B = 0$$

(c) If *L* is the length of *B* as shown in figure *A* will fall from *B* after time *t*



$$t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2mL}{F}} \qquad \left[\text{As } s = \frac{1}{2}a\,t^2 \text{ and } a = F/m\right]$$

(ii) **If friction is present between *A* and *B* only and applied force is less than limiting friction ($F < F_l$)**

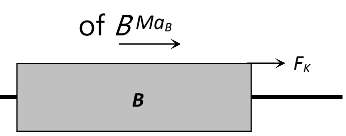
(F = Applied force on the upper body, F_l = limiting friction between *A* and *B*, F_k = Kinetic friction between *A* and *B*)

(a) The body *A* will not slide on body *B* till $F < F_l$ *i.e.* $F < \mu_s mg$

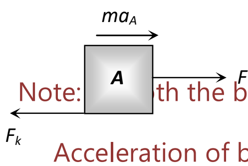
(b) Combined system (*m* + *M*) will move together with common acceleration $a_A = a_B = \dfrac{F}{M+m}$

(iii) **If friction is present between *A* and *B* only and applied force is greater than limiting friction ($F > F_l$)**

In this condition the two bodies will move in the same direction (*i.e.* of applied force) but with different acceleration. Here force of kinetic friction $\mu_k mg$ will oppose the motion of *A* while will cause the motion of *B*.

$F - F_k = m\,a_A$ *i.e.* $a_A = \dfrac{F - F_k}{m}$	Free body diagram of *A*	$F_k = M\,a_B$ *i.e.* $a_B = \dfrac{F_k}{M}$	Free body diagram of *B* 

$a_A = \dfrac{(F - \mu_k mg)}{m}$	$\therefore\ a_B = \dfrac{\mu_k mg}{M}$



Note: Both the bodies are moving in the same direction.

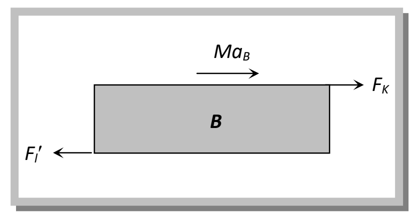
Acceleration of body *A* relative to *B* will be $a = a_A - a_B = \dfrac{MF - \mu_k mg\,(m+M)}{mM}$

So, A will fall from *B* after time $t = \sqrt{\dfrac{2L}{a}} = \sqrt{\dfrac{2\,m\,ML}{MF - \mu_k mg\,(m+M)}}$

(iv) **If there is friction between *B* and floor**

(where $F_l' = \mu'(M+m)g$ = limiting friction between *B* and floor, F_k = kinetic friction between *A* and *B*)

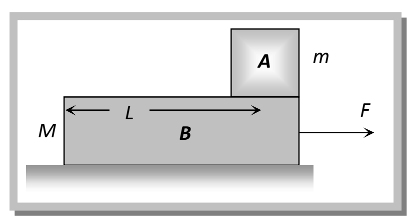
B will move only if $F_k > F_l'$ and then $F_k - F_l' = M\,a_B$



However if *B* does not move then static friction will work (not limiting friction) between body *B* and the floor *i.e.* friction force = applied force (= F_k) not F_l'.

(2) **A force *F* is applied to the lower body, then following four situations are possible**

(i) **When there is no friction**



(a) *B* will move with acceleration (*F*/*M*) while *A* will remain at rest (relative to ground) as there is no pulling force on *A*.

$$a_B = \left(\frac{F}{M}\right) \text{ and } a_A = 0$$

(b) As relative to *B*, *A* will move backwards with acceleration (*F*/*M*) and so will fall from it in time *t*.

$$\therefore t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2ML}{F}}$$

(ii) **If friction is present between *A* and *B* only and $F' < F_l$**

(where F' = Pseudo force on body *A* and F_l = limiting friction between body *A* and *B*)

(a) Both the body will move together with common acceleration $a = \dfrac{F}{M+m}$

(b) Pseudo force on the body *A*, $F' = ma = \dfrac{mF}{m+M}$ and $F_l = \mu_s mg$

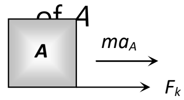
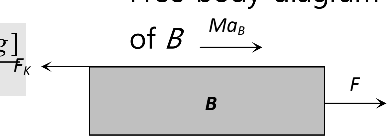
(c) $F' < F_l \Rightarrow \dfrac{mF}{m+M} < \mu_s mg \Rightarrow F < \mu_s (m+M) g$

So both bodies will move together with acceleration $a_A = a_B = \dfrac{F}{m+M}$ if $F < \mu_s [m+M] g$

(iii) **If friction is present between *A* and *B* only and $F > F_l'$**

(where $F_l' = \mu_s (m+M) g$ = limiting friction between body *B* and surface)

Both the body will move with different acceleration. Here force of kinetic friction $\mu_k mg$ will oppose the motion of *B* while will cause the motion of *A*.

$ma_A = \mu_k mg$ i.e. $a_A = \mu_k g$	Free body diagram of *A* 	$F - F_k = Ma_B$ i.e. $a_B = \dfrac{[F - \mu_k mg]}{M}$	Free body diagram of *B* 

Note: As both the bodies are moving in the same direction

Acceleration of body *A* relative to *B* will be

$$a = a_A - a_B = -\left[\frac{F - \mu_k g(m+M)}{M}\right]$$

Negative sign implies that relative to *B*, A will move backwards and will fall it after time

$$t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2ML}{F - \mu_k g(m+M)}}$$

(iv) **If there is friction between *B* and floor:** The system will move only if $F > F_l'$ then replacing *F* by $F - F_l'$. The entire case (iii) will be valid.

However if $F < F_l'$ the system will not move and friction between *B* and floor will be *F* while between *A* and *B* is zero.