. Motion of Body under Gravity (Free Fall).

The force of attraction of earth on bodies, is called force of gravity. Acceleration produced in the body by the force of gravity, is called acceleration due to gravity. It is represented by the symbol g.

In the absence of air resistance, it is found that all bodies (irrespective of the size, weight or composition) fall with the same acceleration near the surface of the earth. This motion of a body falling towards the earth from a small altitude (h << R) is called free fall.

An ideal one-dimensional motion under gravity in which air resistance and the small changes in acceleration with height are neglected.

(1) If a body dropped from some height (initial velocity zero)

(i) Equation of motion: Taking initial position as origin and direction of motion (i.e., downward direction) as a positive, here we have

u = 0	[As body starts from rest]
a = +g	[As acceleration is in the direction of motion]
v = g t	(i)
$h=\frac{1}{2}gt^2$	(ii)
$v^2 = 2gh$	(iii)
$h_n = \frac{g}{2}(2n)$	-1)(iv)



(ii) Graph of distance velocity and acceleration with respect to time:



(iii) As $h = (1/2)gt^2$, i.e., $h \propto t^2$, distance covered in time t, 2t, 3t, etc., will be in the ratio of $1^2 : 2^2 : 3^2$, i.e., square of integers.

(iv) The distance covered in the nth sec, $h_n = \frac{1}{2}g(2n-1)$

So distance covered in I, II, III sec, etc., will be in the ratio of 1: 3: 5, i.e., odd integers only.

(2) If a body is projected vertically downward with some initial velocity

Equation of motion: v = u + g t

$$h = ut + \frac{1}{2}gt^{2}$$

$$v^{2} = u^{2} + 2gh$$

$$h_{n} = u + \frac{g}{2}(2n - 1)$$

(3) If a body is projected vertically upward

(i) Equation of motion: Taking initial position as origin and direction of motion (i.e., vertically up) as positive

a = -g [As acceleration is downwards while motion upwards]

So, if the body is projected with velocity u and after time t it reaches up to height h then

$$v = u - gt;$$
 $h = ut - \frac{1}{2}gt^{2};$ $v^{2} = u^{2} - 2gh; h_{n} = u - \frac{g}{2}(2n - 1)$

(ii) For maximum height v = 0

So from above equation

u = gt, $h = \frac{1}{2}gt^{2}$

and $u^2 = 2gh$



(iii) Graph of distance, velocity and acceleration with respect to time (for maximum height):



It is clear that both quantities do not depend upon the mass of the body or we can say that in absence of air resistance, all bodies fall on the surface of the earth with the same rate.

(4) In case of motion under gravity for a given body, mass, acceleration, and mechanical energy remain constant while speed, velocity, momentum, kinetic energy and potential energy change.

(5) The motion is independent of the mass of the body, as in any equation of motion, mass is not involved. That is why a heavy and light body when released from the same height, reach the ground simultaneously and with same velocity i.e., $t = \sqrt{(2h/g)}$ and $v = \sqrt{2gh}$.

(6) In case of motion under gravity time taken to go up is equal to the time taken to fall down through the same distance. Time of descent $(t_1) = time of ascent (t_2) = u/g$

 \therefore Total time of flight T = t₁ + t₂ = $\frac{2u}{g}$

(7) In case of motion under gravity, the speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.

As well as the magnitude of velocity at any point on the path is same whether the body is moving in upwards or downward direction.

(8) A ball is dropped from a building of height h and it reaches after t seconds on earth. From the same building if two ball are thrown (one upwards and other downwards) with the same velocity u and they reach the earth surface after t_1 and t_2 seconds respectively then

 $t = \sqrt{t_1 t_2}$

(9) A body is thrown vertically upwards. If air resistance is to be taken into account, then the time of ascent is less than the time of descent. $t_2 > t_1$

Let u is the initial velocity of body then time of ascent $t_1 = \frac{u}{g+a}$ and $h = \frac{u^2}{2(g+a)}$

Where g is acceleration due to gravity and a is retardation by air resistance and for upward motion both will work vertically downward.

For downward motion a and g will work in opposite direction because a always work in direction opposite to motion and g always work vertically downward.

So
$$h = \frac{1}{2}(g-a)t_2^2 \Rightarrow \frac{u^2}{2(g+a)} = \frac{1}{2}(g-a)t_2^2 \Rightarrow t_2 = \frac{u}{\sqrt{(g+a)(g-a)}}$$

Comparing t_1 and t_2 we can say that $t_2 > t_1$ since (g + a) > (g - a)

(10) A particle is dropped vertically from rest from a height. The time taken by it to fall through successive distance of 1m each will then be in the ratio of the difference in the square roots of the integer's i.e.

$$\sqrt{1}, (\sqrt{2} - \sqrt{1}), (\sqrt{3} - \sqrt{2}), \dots, (\sqrt{4} - \sqrt{3}), \dots, \dots$$

