Speed and Velocity.

(1) **Speed:** Rate of distance covered with time is called speed.

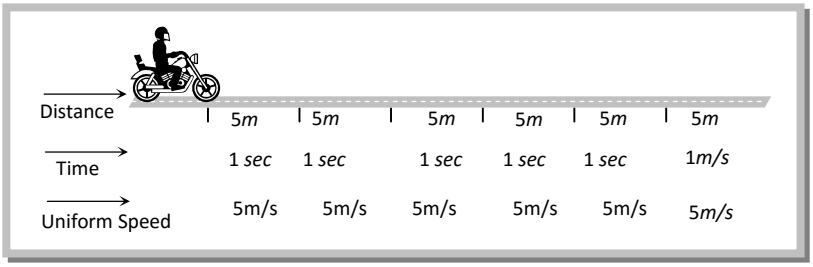
(i) It is a scalar quantity having symbol υ.

(ii) Dimension: $[M^0L^1T^{-1}]$

(iii) Unit: meter/second (S.I.), cm/second (C.G.S.)

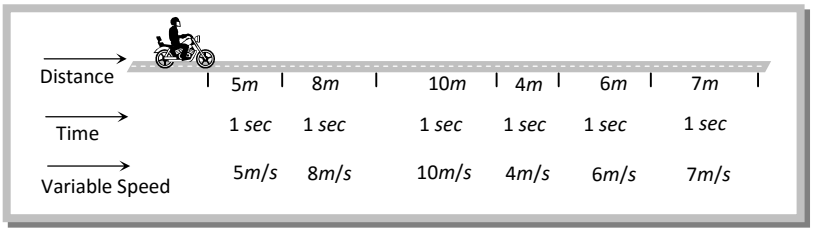
(iv) Types of speed:

(a) **Uniform speed:** When a particle covers equal distances in equal intervals of time, (no matter how small the intervals are) then it is said to be moving with uniform speed. In given illustration motorcyclist travels equal distance (= 5m) in each second. So we can say that particle is moving with uniform speed of 5 m/s.



(b) **Non-uniform (variable) speed:** In non-uniform speed particle covers unequal distances in equal intervals of time. In the given illustration motorcyclist travels 5m in 1st second, 8m in 2nd second, 10m in 3rd second, 4m in 4th second etc.

Therefore its speed is different for every time interval of one second. This means particle is moving with variable speed.



(c) **Average speed:** The average speed of a particle for a given 'Interval of time' is defined as the ratio of distance travelled to the time taken.

Average speed $= \frac{\text{Distance travelled}}{\text{Time taken}}$; $v_{av} = \frac{\Delta s}{\Delta t}$

❑ Time average speed: When particle moves with different uniform speed υ_1, υ_2, υ_3 ... etc. in different time intervals t_1, t_2, t_3, ... etc. respectively, its average speed over the total time of journey is given as

$$v_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{d_1 + d_2 + d_3 + ...}{t_1 + t_2 + t_3 + ...} = \frac{\upsilon_1 t_1 + \upsilon_2 t_2 + \upsilon_3 t_3 + ...}{t_1 + t_2 + t_3 + ...}$$

Special case: When particle moves with speed v_1 up to half time of its total motion and in rest time it is moving with speed v_2 then $v_{av} = \frac{v_1 + v_2}{2}$

❑ Distance averaged speed : When a particle describes different distances d_1, d_2, d_3, ... with different time intervals t_1, t_2, t_3, ... with speeds v_1, v_2, v_3..... respectively then the speed of particle averaged over the total distance can be given as

$$\upsilon_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{d_1 + d_2 + d_3 + ...}{t_1 + t_2 + t_3 + ...} = \frac{d_1 + d_2 + d_3 + ...}{\frac{d_1}{\upsilon_1} + \frac{d_2}{\upsilon_2} + \frac{d_3}{\upsilon_3} + ...}$$

❑When particle moves the first half of a distance at a speed of v_1 and second half of the distance at speed v_2 then

$$v_{av} = \frac{2v_1 v_2}{v_1 + v_2}$$

❑When particle covers one-third distance at speed v_1, next one third at speed v_2 and last one third at speed v_3, then

$$v_{av} = \frac{3\, v_1 v_2 v_3}{v_1 v_2 + v_2 v_3 + v_3 v_1}$$

(d) **Instantaneous speed:** It is the speed of a particle at particular instant. When we say "speed", it usually means instantaneous speed.

The instantaneous speed is average speed for infinitesimally small time interval (i.e., $\Delta t \to 0$). Thus

Instantaneous speed $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$

(2) **Velocity:** Rate of change of position i.e. rate of displacement with time is called velocity.

(i) It is a scalar quantity having symbol v.

(ii) Dimension: $[M^0L^1T^{-1}]$

(iii) Unit: meter/second (S.I.), cm/second (C.G.S.)

(iv) Types

(a) **Uniform velocity :** A particle is said to have uniform velocity, if magnitudes as well as direction of its velocity remains same and this is possible only when the particles moves in same straight line without reversing its direction.

(b) **Non-uniform velocity:** A particle is said to have non-uniform velocity, if either of magnitude or direction of velocity changes (or both changes).

(c) **Average velocity**: It is defined as the ratio of displacement to time taken by the body

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time taken}}; \quad \vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t}$$

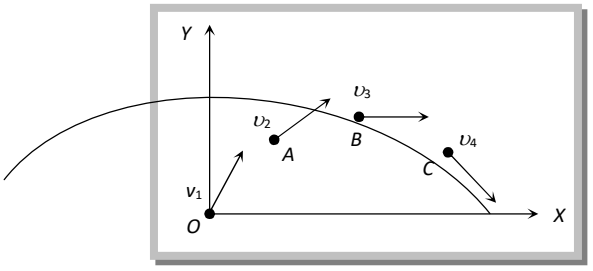
(d) **Instantaneous velocity:** Instantaneous velocity is defined as rate of change of position vector of particles with time at a certain instant of time.

Instantaneous velocity $\vec{v} = \lim_{t\to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$

(v) **Comparison between instantaneous speed and instantaneous velocity**

(a) Instantaneous velocity is always tangential to the path followed by the particle.

When a stone is thrown from point O then at point of projection the instantaneous velocity of stone is v_1, at point A the instantaneous velocity of stone is v_2, similarly at point B and C are v_3 and v_4 respectively.



Direction of these velocities can be found out by drawing a tangent on the trajectory at a given point.

(b) A particle may have constant instantaneous speed but variable instantaneous velocity.

Example: When a particle is performing uniform circular motion then for every instant of its circular motion its speed remains constant but velocity changes at every instant.

(c) The magnitude of instantaneous velocity is equal to the instantaneous speed.

(d) If a particle is moving with constant velocity then its average velocity and instantaneous velocity are always equal.

(e) If displacement is given as a function of time, then time derivative of displacement will give velocity.

Let displacement $\vec{x} = A_0 - A_1 t + A_2 t^2$

Instantaneous velocity $\vec{v} = \frac{d\vec{x}}{dt} = \frac{d}{dt}(A_0 - A_1 t + A_2 t^2)$

$$\vec{v} = -A_1 + 2A_2 t$$

For the given value of t, we can find out the instantaneous velocity.

E.g. for $t = 0$,Instantaneous velocity $\vec{v} = -A_1$ and Instantaneous speed $|\vec{v}| = A_1$

(vi) **Comparison between average speed and average velocity**

(a) Average speed is scalar while average velocity is a vector both having same units (m/s) and dimensions $[LT^{-1}]$.

(b) Average speed or velocity depends on time interval over which it is defined.

(c) For a given time interval average velocity is single valued while average speed can have many values depending on path followed.

(d) If after motion body comes back to its initial position then $\vec{v}_{av} = \vec{0}$ (as $\Delta\vec{r} = 0$) but $v_{av} > \vec{0}$ and finite as $(\Delta s > 0)$.

(e) For a moving body average speed can never be negative or zero (unless $t \to \infty$) while average velocity can be i.e. $v_{av} > 0$ while $\vec{v}_{av}$ = or < 0.