

Acceleration.

The time rate of change of velocity of an object is called acceleration of the object.

(1) It is a vector quantity. Its direction is same as that of change in velocity (Not of the velocity)

(2) There are three possible ways by which change in velocity may occur

When only direction of velocity changes	When only magnitude of velocity changes	When both magnitude and direction of velocity changes
Acceleration perpendicular to velocity	Acceleration parallel or anti-parallel to velocity	Acceleration has two components one is perpendicular to velocity and another parallel or anti-parallel to velocity
e.g. Uniform circular motion	e.g. Motion under gravity	e.g. Projectile motion

(3) Dimension: $[M^0L^1T^{-2}]$

(4) Unit: meter/second² (S.I.); cm/second² (C.G.S.)

(5) Types of acceleration:

(i) **Uniform acceleration:** A body is said to have uniform acceleration if magnitude and direction of the acceleration remains constant during particle motion.

Note: If a particle is moving with uniform acceleration, this does not necessarily imply that particle is moving in straight line. e.g. Projectile motion.

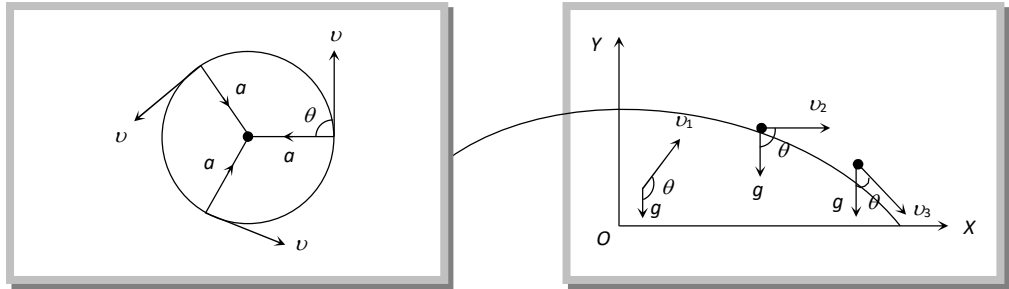
(ii) **Non-uniform acceleration:** A body is said to have non-uniform acceleration, if magnitude or direction or both, change during motion.

(iii) **Average acceleration:** $\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$

The direction of average acceleration vector is the direction of the change in velocity vector as $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$

(iv) **Instantaneous acceleration** = $\vec{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$

(v) For a moving body there is no relation between the direction of instantaneous velocity and direction of acceleration.



e.g.(a) In uniform circular motion $\theta = 90^\circ$ always

(b) In a projectile motion θ is variable for every point of trajectory.

(vi) If a force \vec{F} acts on a particle of mass m , by Newton's 2nd law, acceleration $\vec{a} = \frac{\vec{F}}{m}$

(vii) By definition $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}$ [As $\vec{v} = \frac{d\vec{x}}{dt}$]

i.e., if x is given as a function of time, second time derivative of displacement gives acceleration

(viii) If velocity is given as a function of position, then by chain rule

$$a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = v \cdot \frac{dv}{dx} \left[\text{as } v = \frac{dx}{dt} \right]$$

(ix) If a particle is accelerated for a time t_1 by acceleration a_1 and for time t_2 by acceleration

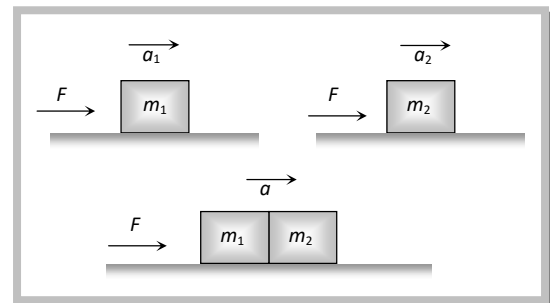
$$a_{av} = \frac{a_1 t_1 + a_2 t_2}{t_1 + t_2}$$

(x) If same force is applied on two bodies of different masses m_1 and m_2 separately then

it produces accelerations a_1 and a_2 respectively. Now these bodies are attached together and form a combined system and same force is applied on that system so that a be the acceleration of the combined system, then

$$F = (m_1 + m_2)a \Rightarrow \frac{F}{a} = \frac{F}{a_1} + \frac{F}{a_2}$$

$$\text{So, } \frac{1}{a} = \frac{1}{a_1} + \frac{1}{a_2} \Rightarrow a = \frac{a_1 a_2}{a_1 + a_2}$$



(xi) Acceleration can be positive, zero or negative. Positive acceleration means velocity increasing with time, zero acceleration means velocity is uniform constant while negative acceleration (retardation) means velocity is decreasing with time.

(xii) For motion of a body under gravity, acceleration will be equal to "g", where g is the acceleration due to gravity. Its normal value is 9.8 m/s^2 or 980 cm/s^2 or 32 feet/s^2 .