Non-Uniform Circular Motion.

If the speed of the particle in a horizontal circular motion changes with respect to time, then its motion is said to be non-uniform circular motion.

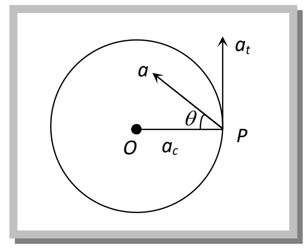
Consider a particle describing a circular path of radius *r* with center at *O*. Let at an instant the particle be at *P* and $\vec{\upsilon}$ be its linear velocity and $\vec{\omega}$ be its angular velocity.

Then, $\vec{\upsilon} = \vec{\omega} \times \vec{r}$ .....(i)

Differentiating both sides of *w.r.t.* time *t* we have

$$\frac{d\vec{\upsilon}}{dt} = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt} \quad .....(ii)$$

Here, $\frac{d\vec{v}}{dt} = \vec{a}$, (Resultant acceleration)



$$\vec{a} = \vec{\alpha} \times \vec{r} \quad + \quad \vec{\omega} \times \vec{\upsilon}$$

$\frac{d\vec{\omega}}{dt} = \vec{\alpha}$ (Angular acceleration)

$$\vec{a} = \quad \vec{a}_t \quad + \quad \vec{a}_c \quad .....(iii)$$

$\frac{d\vec{r}}{dt} = \vec{\upsilon}$ (Linear velocity)

Thus the resultant acceleration of the particle at *P* has two component accelerations

(1) **Tangential acceleration:** $\vec{a_t} = \vec{\alpha} \times \vec{r}$

It acts along the tangent to the circular path at *P* in the plane of circular path.

According to right hand rule since $\vec{\alpha}$ and $\vec{r}$ are perpendicular to each other, therefore, the magnitude of tangential acceleration is given by

$$|\vec{a}_t| = |\vec{\alpha} \times \vec{r}| = \alpha r \sin 90^o = \alpha r.$$

(2) **Centripetal (Radial) acceleration:** $\vec{a_c} = \vec{\omega} \times \vec{v}$

It is also called centripetal acceleration of the particle at *P*.

It acts along the radius of the particle at *P*.

According to right hand rule since $\vec{\omega}$ and $\vec{\upsilon}$ are perpendicular to each other, therefore, the magnitude of centripetal acceleration is given by

$$|\vec{a}_c| = |\vec{\omega} \times \vec{\upsilon}| = \omega \upsilon \sin 90^o = \omega \upsilon = \omega(\omega r) = \omega^2 r = \upsilon^2 / r$$

(3) **Tangential and centripetal acceleration in different motions**

Centripetal acceleration	Tangential acceleration	Net acceleration	Type of motion
$a_c = 0$	$a_t = 0$	$a = 0$	Uniform translatory motion
$a_c = 0$	$a_t \neq 0$	$a = a_t$	Accelerated translatory motion
$a_c \neq 0$	$a_t = 0$	$a = a_c$	Uniform circular motion
$a_c \neq 0$	$a_t \neq 0$	$a = \sqrt{a_c^2 + a_t^2}$	Non-uniform circular motion

Note: Here a_t governs the magnitude of $\vec{v}$ while $\vec{a}_c$ its direction of motion.

(4) **Force:**In non-uniform circular motion the particle simultaneously possesses two forces

Centripetal force: $F_c = ma_c = \frac{mv^2}{r} = mr\omega^2$

Tangential force: $F_t = ma_t$

Net force: $F_{\text{net}} = ma = m\sqrt{a_c^2 + a_t^2}$

Note: In non-uniform circular motion work done by centripetal force will be zero since $\vec{F}_c \perp \vec{v}$

a. In non-uniform circular motion work done by tangential of force will not be zero since $F_t \neq 0$
b. Rate of work done by net force in non-uniform circular = rate of work done by tangential force

i.e. $P = \frac{dW}{dt} = \vec{F}_t . \vec{v}$