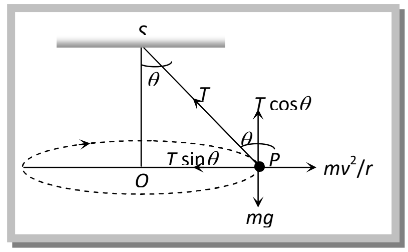
Conical Pendulum.

This is the example of uniform circular motion in horizontal plane.

A bob of mass m attached to a light and in-extensible string rotates in a horizontal circle of radius r with constant angular speed ω about the vertical. The string makes angle θ with vertical and appears tracing the surface of a cone. So this arrangement is called conical pendulum.

The force acting on the bob are tension and weight of the bob.

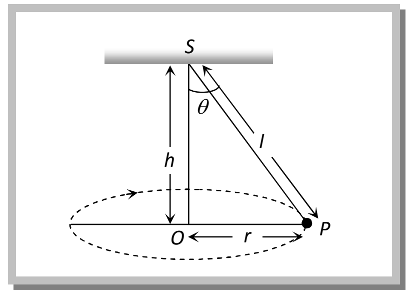


From the figure $T\sin\theta = \frac{mv^2}{r}$ ....(i)

and $T\cos\theta = mg$ ....(ii)

(1) Tension in the string: $T = mg\sqrt{1 + \left(\frac{v^2}{rg}\right)^2}$

$$T = \frac{mg}{\cos\theta} = \frac{mgl}{\sqrt{l^2 - r^2}} \qquad \left[\text{As } \cos\theta = \frac{h}{l} = \frac{\sqrt{l^2 - r^2}}{l}\right]$$



(2) Angle of string from the vertical: $\tan\theta = \frac{v^2}{rg}$

(3) Linear velocity of the bob: $v = \sqrt{gr\tan\theta}$

(4) Angular velocity of the bob: $\omega = \sqrt{\frac{g}{r}\tan\theta} = \sqrt{\frac{g}{h}} = \sqrt{\frac{g}{l\cos\theta}}$

(5) Time period of revolution: $T_P = 2\pi\sqrt{\frac{l\cos\theta}{g}} = 2\pi\sqrt{\frac{h}{g}} = 2\pi\sqrt{\frac{l^2 - r^2}{g}} = 2\pi\sqrt{\frac{r}{g\tan\theta}}$