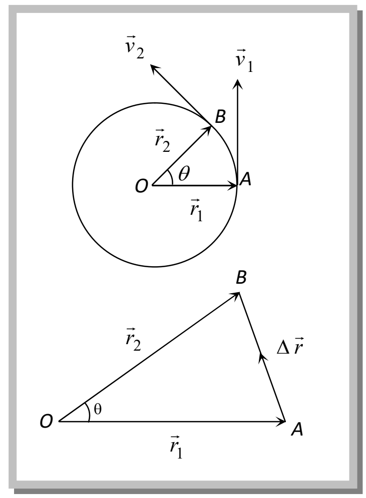
Variables of Circular Motion.

(1) **Displacement and distance:** When particle moves in a circular path describing an angle θ during time *t* (as shown in the figure) from the position *A* to the position *B*, we see that the magnitude of the position vector $\vec{r}$ (that is equal to the radius of the circle) remains constant. *i.e.,* $|\vec{r}_1| = |\vec{r}_2| = r$ and the direction of the position vector changes from time to time.

(i) Displacement**:** The change of position vector or the displacement $\Delta\vec{r}$ of the particle from position *A* to the position *B* is given by referring the figure.



$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\Rightarrow \Delta r = |\Delta\vec{r}| = |\vec{r}_2 - \vec{r}_1| \; \Delta r = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos\theta}$$

Putting $r_1 = r_2 = r$ we obtain

$$\Delta r = \sqrt{r^2 + r^2 - 2r.r\cos\theta}$$

$$\Rightarrow \Delta r = \sqrt{2r^2(1-\cos\theta)} = \sqrt{2r^2\left(2\sin^2\frac{\theta}{2}\right)}$$

$$\Delta r = 2r\sin\frac{\theta}{2}$$

(ii) Distance**:**The distanced covered by the particle during the time t is given as

d = length of the arc *AB* = $r\theta$

(iii)Ratioofdistanceanddisplacement**:** $\dfrac{d}{\Delta r} = \dfrac{r\theta}{2r\sin\theta/2} = \dfrac{\theta}{2}\text{cosec}\,(\theta/2)$

(2) **Angular displacement (θ):**The angle turned by a body moving on a circle from some reference line is called angular displacement.

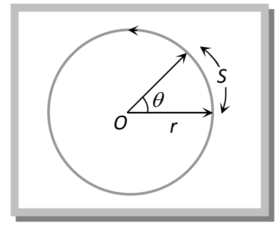
(i) Dimension = $[M^0L^0T^0]$ (as $\theta = arc/\text{radius}$).

(ii) Units = Radian or Degree. It is sometimes also specified in terms of fraction or multiple of revolution.

(iii) $2\pi\text{ rad} = 360^o = 1\text{ Revolution}$

(iv) Angular displacement is a axial vector quantity.

Its direction depends upon the sense of rotation of the object and can be given by Right Hand Rule; which states that if the curvature of the fingers of right hand represents the sense of rotation of the object, then the thumb, held perpendicular to the curvature of the fingers, represents the direction of angular displacement vector.



(v) Relation between linear displacement and angular displacement $\vec{s} = \vec{\theta} \times \vec{r}$

or $s = r\theta$

(3) **Angular velocity (ω):**Angular velocity of an object in circular motion is defined as the time rate of change of its angular displacement.

(i) Angular velocity $\omega = \frac{\text{angle traced}}{\text{time taken}} = \underset{\Delta t \to 0}{Lt} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$

$$\therefore \qquad \omega = \frac{d\theta}{dt}$$

(ii) Dimension: $[M^0 L^0 T^{-1}]$

(iii) Units: Radians per second ($rad.s^{-1}$) or Degree per second.

(iv) Angular velocity is an axial vector.

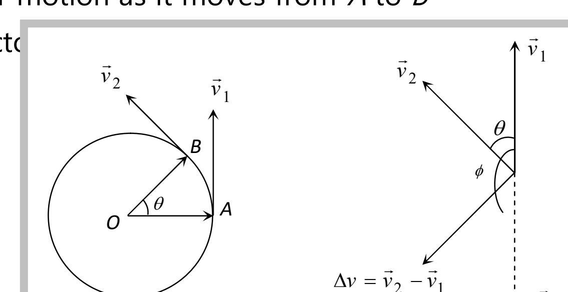
(v) Relation between angular velocity and linear velocity $\vec{v} = \vec{\omega} \times \vec{r}$

Its direction is the same as that of $\Delta\theta$. For anticlockwise rotation of the point object on the circular path, the direction of ω, according to Right hand rule is along the axis of circular path directed upwards. For clockwise rotation of the point object on the circular path, the direction of ω is along the axis of circular path directed downwards.

Note: It is important to note that nothing actually moves in the direction of the angular velocity vector $\vec{\omega}$. The direction of $\vec{\omega}$ simply represents that the rotational motion is taking place in a plane perpendicular to it.

(vi) For uniform circular motion ω remains constant whereas for non-uniform motion ω varies with respect to time.

(4) **Change in velocity:**We want to know the magnitude and direction of the change in velocity of the particle which is performing uniform circular motion as it moves from A to B during time t as shown in figure. The change in velocity vect



$$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$$

or $$|\Delta\vec{v}| = |\vec{v}_2 - \vec{v}_1| \Rightarrow \Delta v = \sqrt{v_1^2 + v_2^2 - 2v_1v_2\cos\theta}$$

For uniform circular motion $v_1 = v_2 = v$

So $$\Delta v = \sqrt{2v^2(1-\cos\theta)} = 2v\sin\frac{\theta}{2}$$

The direction of $\Delta\vec{v}$ is shown in figure that can be given as

$$\phi = \frac{180^o - \theta}{2} = (90^o - \theta/2)$$

Note: Relation between linear velocity and angular velocity.

In vector form $\vec{v} = \vec{\omega} \times \vec{r}$

(5) **Time period (*T*):** In circular motion, the time period is defined as the time taken by the object to complete one revolution on its circular path.

(i) Units: second.

(ii) Dimension: $[M^0L^0T]$

(iii) Time period of second's hand of watch = 60 *second*.

(iv) Time period of minute's hand of watch = 60 *minute*

(v) Time period of hour's hand of watch = 12 *hour*

(6) **Frequency (*n*):** In circular motion, the frequency is defined as the number of revolutions completed by the object on its circular path in a unit time.

(i) Units: s^{-1} or hertz (*Hz*).

(ii) Dimension: $[M^0L^0T^{-1}]$

Note: Relation between time period and frequency: If *n* is the frequency of revolution of an object in circular motion, then the object completes *n* revolutions in 1 second. Therefore, the object will complete one revolution in $1/n$ second.

$$\therefore T = 1/n$$

Relation between angular velocity, frequency and time period: Consider a point object describing a uniform circular motion with frequency *n* and time period *T*. When the object completes one revolution, the angle traced at its axis of circular motion is 2π radians. It means, when time $t = T$, $\theta = 2\pi$ radians. Hence, angular velocity $\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi n$ $(\because T = 1/n)$

$$\omega = \frac{2\pi}{T} = 2\pi n$$

If two particles are moving on same circle or different coplanar concentric circles in same direction with different uniform angular speeds ω_A and ω_B respectively, the angular velocity of B relative to A will be

$$\omega_{\text{rel}} = \omega_B - \omega_A$$

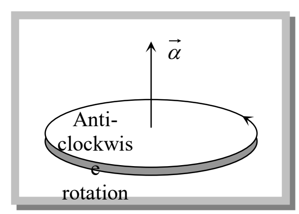
So the time taken by one to complete one revolution around O with respect to the other (i.e., time in which B complete one revolution around O with respect to the other (*i.e.*, time in which B completes one more or less revolution around O than A)

$$T = \frac{2\pi}{\omega_{\text{rel}}} = \frac{2\pi}{\omega_2 - \omega_1} = \frac{T_1 T_2}{T_1 - T_2} \qquad \left[\text{as } T = \frac{2\pi}{\omega}\right]$$

Special case: If $\omega_B = \omega_A, \omega_{\text{rel}} = 0$ and so $T = \infty$., particles will maintain their position relative to each other. This is what actually happens in case of geostationary satellite ($\omega_1 = \omega_2$ = constant)

(7) **Angular acceleration (α):** Angular acceleration of an object in circular motion is defined as the time rate of change of its angular velocity.

(i) If $\Delta\omega$ be the change in angular velocity of the object in time interval t and $t + \Delta t$, while moving on a circular path, then angular acceleration of the object will be



$$\alpha = \underset{\Delta t \to 0}{Lt} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

(ii) Units: *rad. s*$^{-2}$

(iii) Dimension: $[M^0 L^0 T^{-2}]$

(iv) Relation between linear acceleration and angular acceleration $\vec{a} = \vec{\alpha} \times \vec{r}$

(v) For uniform circular motion since ω is constant so $\alpha = \frac{d\omega}{dt} = 0$

(vi) For non-uniform circular motion $\alpha \neq 0$

Note: Relation between linear (tangential) acceleration and angular acceleration $\vec{a} = \vec{\alpha} \times \vec{r}$

a. For uniform circular motion angular acceleration is zero, so tangential acceleration also is equal to zero.

b. For non-uniform circular motion $a \neq 0$ (because $\alpha \neq 0$).