

Projectile Motion on an Inclined Plane.

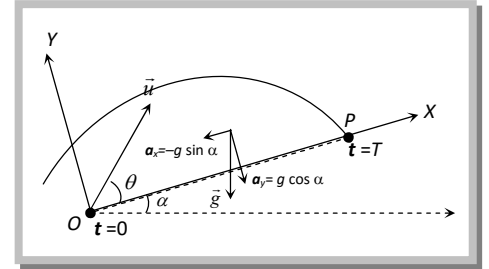
Let a particle be projected up with a speed u from an inclined plane which makes an angle α with the horizontal. The velocity of projection makes an angle θ with the inclined plane.

We have taken reference x-axis in the direction of plane.

Hence the component of initial velocity parallel and perpendicular to the plane are equal to $u \cos \theta$ and $u \sin \theta$ respectively i.e. $u_{\parallel} = u \cos \theta$ and $u_{\perp} = u \sin \theta$.

The component of g along the plane is $g \sin \alpha$ and perpendicular to the plane is $g \cos \alpha$ as shown in the figure i.e. $a_{\parallel} = -g \sin \alpha$ and $a_{\perp} = g \cos \alpha$.

Therefore the particle decelerates at a rate of $g \sin \alpha$ as it moves from O to P.



(1) **Time of flight:** We know for oblique projectile motion $T = \frac{2u \sin \theta}{g}$

or we can say $T = \frac{2u_{\perp}}{a_{\perp}}$

\therefore Time of flight on an inclined plane $T = \frac{2u \sin \theta}{g \cos \alpha}$

(2) **Maximum height:** We know for oblique projectile motion $H = \frac{u^2 \sin^2 \theta}{2g}$

or we can say $H = \frac{u_{\perp}^2}{2a_{\perp}}$

\therefore Maximum height on an inclined plane $H = \frac{u^2 \sin^2 \theta}{2g \cos \alpha}$

(3) **Horizontal range:** For one dimensional motion $s = ut + \frac{1}{2}at^2$

Horizontal range on an inclined plane $R = u_{\parallel} T + \frac{1}{2}a_{\parallel} T^2$

$$R = u \cos \theta T - \frac{1}{2} g \sin \alpha T^2$$

$$R = u \cos \theta \left(\frac{2u \sin \theta}{g \cos \alpha} \right) - \frac{1}{2} g \sin \alpha \left(\frac{2u \sin \theta}{g \cos \alpha} \right)^2$$

By solving $R = \frac{2u^2 \sin \theta \cos(\theta + \alpha)}{g \cos^2 \alpha}$

(i) Maximum range occurs when $\theta = \frac{\pi}{4} - \frac{\alpha}{2}$

(ii) The maximum range along the inclined plane when the projectile is thrown upwards is given by

$$R_{\max} = \frac{u^2}{g(1 + \sin \alpha)}$$

(iii) The maximum range along the inclined plane when the projectile is thrown downwards is given by

$$R_{\max} = \frac{u^2}{g(1 - \sin \alpha)}$$

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