# Law of Conservation of Linear Momentum.

If no external force acts on a system (called isolated) of constant mass, the total momentum of the system remains constant with time.

(1) According to this law for a system of particles $\vec{F} = \frac{d\vec{p}}{dt}$

In the absence of external force $\vec{F} = 0$ then $\vec{p} =$ constant

i.e., $\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + .... =$ constant.

Or $m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 + .... =$ constant

This equation shows that in absence of external force for a closed system the linear momentum of individual particles may change but their sum remains unchanged with time.

(2) Law of conservation of linear momentum is independent of frame of reference though linear momentum depends on frame of reference.

(3) Conservation of linear momentum is equivalent to Newton's third law of motion.

For a system of two particles in absence of external force by law of conservation of linear momentum.

$\vec{p}_1 + \vec{p}_2 =$ constant.

$\therefore$ $m_1\vec{v}_1 + m_2\vec{v}_2 =$ constant.

Differentiating above with respect to time

$$m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} = 0 \Rightarrow m_1\vec{a}_1 + m_2\vec{a}_2 = 0 \Rightarrow \vec{F}_1 + \vec{F}_2 = 0$$

$\therefore$ $\vec{F}_2 = -\vec{F}_1$

i.e. for every action there is equal and opposite reaction which is Newton's third law of motion.

(4) Practical applications of the law of conservation of linear momentum

(i) When a man jumps out of a boat on the shore, the boat is pushed slightly away from the shore.

(ii) A person left on a frictionless surface can get away from it by blowing air out of his mouth or by throwing some object in a direction opposite to the direction in which he wants to move.

(iii) **Recoiling of a gun:** For bullet and gun system, the force exerted by trigger will be internal so the momentum of the system remains unaffected


Let $m_G$ = mass of gun, $m_B$ = mass of bullet,

$v_G$ = Velocity of gun, $v_B$ = velocity of bullet

Initial momentum of system = 0

Final momentum of system $= m_G\vec{v}_G + m_B\vec{v}_B$

By the law of conservation linear momentum

$$m_G\vec{v}_G + m_B\vec{v}_B = 0$$

So recoil velocity $\vec{v}_G = -\dfrac{m_B}{m_G}\vec{v}_B$

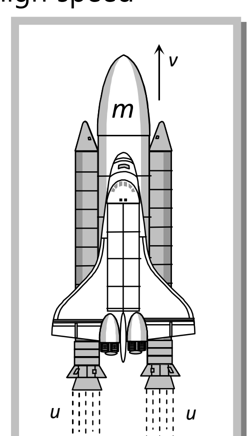
(a) Here negative sign indicates that the velocity of recoil $\vec{v}_G$ is opposite to the velocity of the bullet.

(b) $v_G \propto \dfrac{1}{m_G}$ i.e. higher the mass of gun, lesser the velocity of recoil of gun.

(c) While firing the gun must be held tightly to the shoulder, this would save hurting the shoulder because in this condition the body of the shooter and the gun behave as one body. Total mass become large and recoil velocity becomes too small.

$$v_G \propto \frac{1}{m_G + m_{\text{man}}}$$

(iv) **Rocketpropulsion:** The initial momentum of the rocket on its launching pad is zero. When it is fired from the launching pad, the exhaust gases rush downward at a high speed and to conserve momentum, the rocket moves upwards.




Let $m_0$ = initial mass of rocket,

m = mass of rocket at any instant 't' (instantaneous mass)

$m_r$ = residual mass of empty container of the rocket

u = velocity of exhaust gases,

v = velocity of rocket at any instant 't' (instantaneous velocity)

$\frac{dm}{dt}$ = rate of change of mass of rocket = rate of fuel consumption

= rate of ejection of the fuel.

(a) Thrust on the rocket: $F = -u\frac{dm}{dt} - mg$

Here negative sign indicates that direction of thrust is opposite to the direction of escaping gases.

$F = -u\frac{dm}{dt}$ (if effect of gravity is neglected)

(b) Acceleration of the rocket: $a = \frac{u}{m}\frac{dm}{dt} - g$

and if effect of gravity is neglected $a = \frac{u}{m}\frac{dm}{dt}$

(c) Instantaneous velocity of the rocket: $v = u\log_e\left(\frac{m_0}{m}\right) - gt$

and if effect of gravity is neglected $v = u\log_e\left(\frac{m_0}{m}\right) = 2.303\,u\log_{10}\left(\frac{m_0}{m}\right)$

(d) Burnt out speed of the rocket: $v_b = v_{\max} = u\log_e\left(\frac{m_0}{m_r}\right)$

The speed attained by the rocket when the complete fuel gets burnt is called burnt out speed of the rocket. It is the maximum speed acquired by the rocket.