Kinetic Energy.

The energy possessed by a body by virtue of its motion is called kinetic energy.

Examples : (i) Flowing water possesses kinetic energy which is used to run the water mills.

(ii) Moving vehicle possesses kinetic energy.

(iii) Moving air (i.e. wind) possesses kinetic energy which is used to run wind mills.

(iv) The hammer possesses kinetic energy which is used to drive the nails in wood.

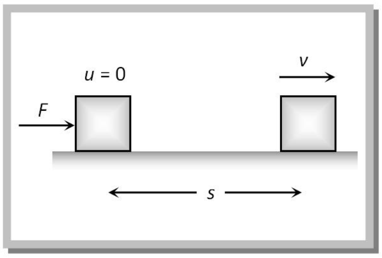
(v) A bullet fired from the gun has kinetic energy and due to this energy the bullet penetrates into a target.

(1) **Expression for kinetic energy:** Let

m = mass of the body, u = Initial velocity of the body (= 0)

F = Force acting on the body, a = Acceleration of the body

s = Distance travelled by the body, v = Final velocity of the body



From $v^2 = u^2 + 2as$

$$\Rightarrow \quad v^2 = 0 + 2as \quad \therefore s = \frac{v^2}{2a}$$

Since the displacement of the body is in the direction of the applied force, then work done by the force is

$$W = F \times s = ma \times \frac{v^2}{2a}$$

$$\Rightarrow \quad W = \frac{1}{2}mv^2$$

This work done appears as the kinetic energy of the body $KE = W = \frac{1}{2}mv^2$

(2) **Calculus method:** Let a body is initially at rest and force $\vec{F}$ is applied on the body to displace it through $d\vec{s}$ along its own direction then small work done

$$dW = \vec{F}.d\vec{s} = F\,ds$$

$$\Rightarrow \quad dW = m\,a\,ds \qquad \text{[As F = ma]}$$

$$\Rightarrow \quad dW = m\frac{dv}{dt}ds \qquad \left[As\,a = \frac{dv}{dt}\right]$$

$$\Rightarrow \quad dW = mdv.\frac{ds}{dt}$$

$$\Rightarrow \quad dW = m\,v\,dv \qquad ....(i) \qquad \left[As\,\frac{ds}{dt} = v\right]$$

Therefore work done on the body in order to increase its velocity from zero to v is given by

$$W = \int_0^v mv\,dv = m\int_0^v v\,dv = m\left[\frac{v^2}{2}\right]_0^v = \frac{1}{2}mv^2$$

This work done appears as the kinetic energy of the body $KE = \frac{1}{2}mv^2$.

In vector form $KE = \frac{1}{2}m(\vec{v}.\vec{v})$

As m and $\vec{v}.\vec{v}$ are always positive, kinetic energy is always positive scalar i.e. kinetic energy can never be negative.

(3) **Kinetic energy depends on frame of reference:** The kinetic energy of a person of mass m, sitting in a train moving with speed v, is zero in the frame of train but $\frac{1}{2}mv^2$ in the frame of the earth.

(4) **Kinetic energy according to relativity:** As we know $E = \frac{1}{2}mv^2$.

But this formula is valid only for (v << c) If v is comparable to c (speed of light in free space = $3 \times 10^8\ m/s$) then according to Einstein theory of relativity

$$E = \frac{mc^2}{\sqrt{1-(v^2/c^2)}} - mc^2$$

(5) **Work-energy theorem:** From equation (i) $dW = mv\,dv$.

Work done on the body in order to increase its velocity from u to v is given by

$$W = \int_u^v mv\,dv = m\int_u^v v\,dv = m\left[\frac{v^2}{2}\right]_u^v$$

$$\Rightarrow \qquad W = \frac{1}{2}m[v^2 - u^2]$$

Work done = change in kinetic energy

$$W = \Delta E$$

This is work energy theorem, it states that work done by a force acting on a body is equal to the change produced in the kinetic energy of the body.

This theorem is valid for a system in presence of all types of forces (external or internal, conservative or non-conservative).

If kinetic energy of the body increases, work is positive i.e. body moves in the direction of the force (or field) and if kinetic energy decreases work will be negative and object will move opposite to the force (or field).

Examples : (i) In case of vertical motion of body under gravity when the body is projected up, force of gravity is opposite to motion and so kinetic energy of the body decreases and when it falls down, force of gravity is in the direction of motion so kinetic energy increases.

(ii) When a body moves on a rough horizontal surface, as force of friction acts opposite to motion, kinetic energy will decrease and the decrease in kinetic energy is equal to the work done against friction.

(6) **Relation of kinetic energy with linear momentum:** As we know

$$E = \frac{1}{2}mv^2 = \frac{1}{2}\left[\frac{P}{v}\right]v^2 \qquad [\text{As } P = mv]$$

$$\therefore \qquad E = \frac{1}{2}Pv$$

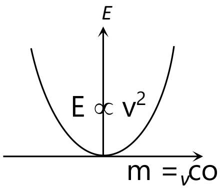
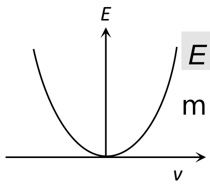
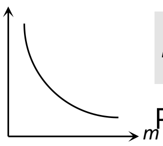
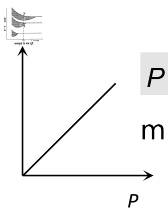
or $\qquad E = \dfrac{P^2}{2m} \qquad \left[\text{As } v = \dfrac{P}{m}\right]$

So we can say that kinetic energy $E = \frac{1}{2}mv^2 = \frac{1}{2}Pv = \frac{p^2}{2m}$

and $\qquad$ Momentum P $= \frac{2E}{v} = \sqrt{2mE}$.

From above relation it is clear that a body can not have kinetic energy without having momentum and vice-versa.

(7) **Various graphs of kinetic energy**

 $E \propto v^2$ m = constant	 $E \propto P^2$ m = constant
 $E \propto \frac{1}{m}$ P = constant	 $P \propto \sqrt{E}$ m = constant