Potential Energy.

Potential energy is defined only for conservative forces. In the space occupied by conservative forces every point is associated with certain energy which is called the energy of position or potential energy. Potential energy generally are of three types: Elastic potential energy, Electric potential energy and Gravitational potential energy etc.

(1) **Change in potential energy:** Change in potential energy between any two points is defined in the terms of the work done by the associated conservative force in displacing the particle between these two points without any change in kinetic energy.

$$U_2 - U_1 = -\int_{r_1}^{r_2} \vec{F}.d\vec{r} = -W \qquad \text{......(i)}$$

We can define a unique value of potential energy only by assigning some arbitrary value to a fixed point called the reference point. Whenever and wherever possible, we take the reference point at infinite and assume potential energy to be zero there, i.e. if take $r_1 = \infty$ and $r_2 = r$ then from equation (i)

$$U = -\int_{\infty}^{r} \vec{F}.d\vec{r} = -W$$

In case of conservative force (field) potential energy is equal to negative of work done in shifting the body from reference position to given position.

This is why in shifting a particle in a conservative field (say gravitational or electric), if the particle moves opposite to the field, work done by the field will be negative and so change in potential energy will be positive i.e. potential energy will increase. When the particle moves in the direction of field, work will be positive and change in potential energy will be negative i.e. potential energy will decrease.

(2) **Three dimensional formula for potential energy:** For only conservative fields $\vec{F}$ equals the negative gradient $(-\vec{\nabla})$ of the potential energy.

So $\vec{F} = -\vec{\nabla} U$ ($\vec{\nabla}$ read as Del operator or Nabla operator and $\vec{\nabla} = \frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}$)

$$\Rightarrow \quad \vec{F} = -\left[\frac{dU}{dx}\hat{i} + \frac{dU}{dy}\hat{j} + \frac{dU}{dz}\hat{k}\right]$$

Where $\frac{dU}{dx} =$ Partial derivative of U w.r.t. x (keeping y and z constant)

$\frac{dU}{dy} =$ Partial derivative of U w.r.t. y (keeping x and z constant)

$\frac{dU}{dz} =$ Partial derivative of U w.r.t. z (keeping x and y constant)

(3) **Potential energy curve:** A graph plotted between the potential energy of a particle and its displacement from the centre of force is called potential energy curve.

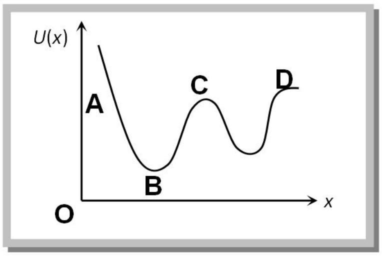


Figure shows a graph of potential energy function U(x) for one dimensional motion.

As we know that negative gradient of the potential energy gives force.

$\therefore \; -\frac{dU}{dx} = F$

(4) **Nature of force:**

(i) Attractive force: On increasing x, if U increases $\frac{dU}{dx} = \text{positive}$

Then F is negative in direction i.e. force is attractive in nature. In graph this is represented in region BC.

(ii) Repulsive force: On increasing x, if U decreases $\frac{dU}{dx} = \text{negative}$

Then F is positive in direction i.e. force is repulsive in nature. In graph this is represented in region AB.

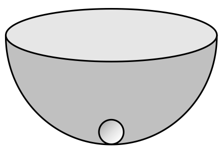
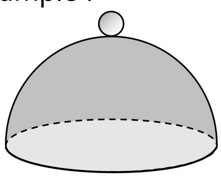
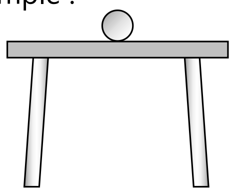
(iii) Zero force: On increasing x, if U does not changes $\frac{dU}{dx} = 0$

Then F is zero i.e. no force works on the particle. Point B, C and D represents the point of zero force or these points can be termed as position of equilibrium.

(5) **Types of equilibrium:** If net force acting on a particle is zero, it is said to be in equilibrium.

For equilibrium $\frac{dU}{dx} = 0$, but the equilibrium of particle can be of three types :

Stable	**Unstable**	**Neutral**
When a particle is displaced slightly from a position, then	When a particle is displaced slightly from a position, then	When a particle is slightly displaced from a position

a force acting on it brings it back to the initial position, it is said to be in stable equilibrium position.	a force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in unstable equilibrium.	then it does not experience any force acting on it and continues to be in equilibrium in the displaced position, it is said to be in neutral equilibrium.
Potential energy is minimum.	Potential energy is maximum.	Potential energy is constant.
$F = -\frac{dU}{dx} = 0$	$F = -\frac{dU}{dx} = 0$	$F = -\frac{dU}{dx} = 0$
$\frac{d^2U}{dx^2} = \text{positive}$ i.e. rate of change of $\frac{dU}{dx}$ is positive.	$\frac{d^2U}{dx^2} = \text{negative}$ i.e. rate of change of $\frac{dU}{dx}$ is negative.	$\frac{d^2U}{dx^2} = 0$ i.e. rate of change of $\frac{dU}{dx}$ is zero.
Example :  A marble placed at the bottom of a hemispherical bowl.	Example :  A marble balanced on top of a hemispherical bowl.	Example :  A marble placed on horizontal table.