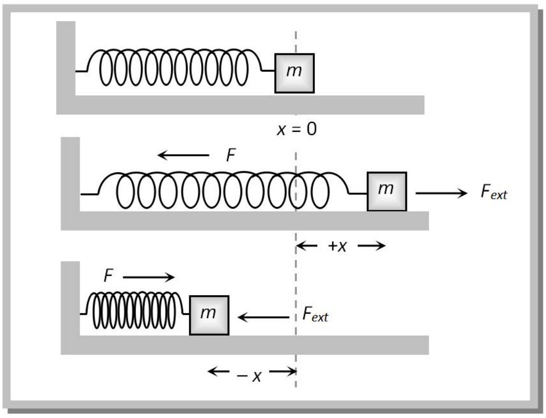
Elastic Potential Energy.

(1) **Restoring force and spring constant:** When a spring is stretched or compressed from its normal position (x = 0) by a small distance x, then a restoring force is produced in the spring to bring it to the normal position.



According to Hooke's law this restoring force is proportional to the displacement x and its direction is always opposite to the displacement.

i.e. $\vec{F} \propto -\vec{x}$

or $\vec{F} = -k\vec{x}$ .....(i)

Where k is called spring constant.

If x = 1, F = k (Numerically)

or k = F

Hence spring constant is numerically equal to force required to produce unit displacement (compression or extension) in the spring. If required force is more, then spring is said to be more stiff and vice-versa.

Actually k is a measure of the stiffness/softness of the spring.

Dimension: As $k = \frac{F}{x}$ $\therefore$ $[k] = \frac{[F]}{[x]} = \frac{[MLT^{-2}]}{L} = [MT^{-2}]$

Units: S.I. unit Newton/metre, C.G.S unit Dyne/cm.

Note: Dimension of force constant is similar to surface tension.

(2) **Expression for elastic potential energy:** When a spring is stretched or compressed from its normal position (x = 0), work has to be done by external force against restoring force.

$\vec{F}_{ext} = \vec{F}_{restoring} = k\vec{x}$

Let the spring is further stretched through the distance dx, then work done

$dW = \vec{F}_{ext}.d\vec{x} = F_{ext}.dx\cos 0^o = kx\,dx$ [As cos 0° = 1]

Therefore total work done to stretch the spring through a distance x from its mean position is given by

$$W = \int_0^x dW = \int_0^x kx\,dx = k\left[\frac{x^2}{2}\right]_0^x = \frac{1}{2}kx^2$$

This work done is stored as the potential energy of the stretched spring.

$\therefore$ Elastic potential energy $U = \frac{1}{2}kx^2$

$U = \frac{1}{2}Fx \qquad \left[\text{As } k = \frac{F}{x}\right]$

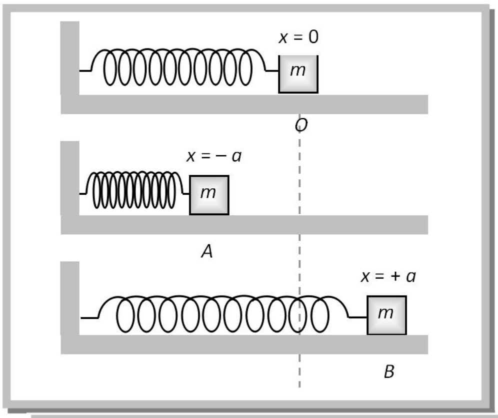
$U = \frac{F^2}{2k} \qquad \left[\text{As } x = \frac{F}{k}\right]$

$\therefore$ Elastic potential energy $U = \frac{1}{2}kx^2 = \frac{1}{2}Fx = \frac{F^2}{2k}$

Note: If spring is stretched from initial position x_1 to final position x_2 then work done

= Increment in elastic potential energy $= \frac{1}{2}k(x_2^2 - x_1^2)$

(3) **Energy graph for a spring:** If the mass attached with spring performs simple harmonic motion about its mean position then its potential energy at any position (x) can be given by



$U = \frac{1}{2}kx^2$ ....(i)

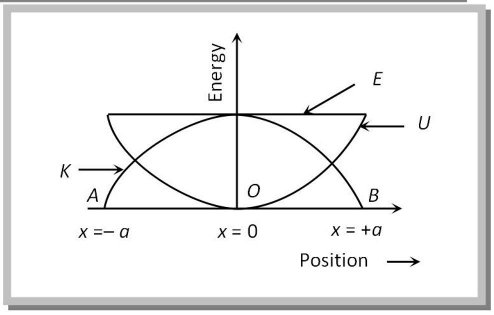
So for the extreme position

$U = \frac{1}{2}ka^2$ [As x = ± a for extreme]

This is maximum potential energy or the total energy of mass.

$\therefore$ Total energy $E = \frac{1}{2}ka^2$ ....(ii)

[Because velocity of mass = 0 at extreme $\therefore$ $K = \frac{1}{2}mv^2 = 0$]



Now kinetic energy at any position $K = E - U = \frac{1}{2}ka^2 - \frac{1}{2}kx^2$

$K = \frac{1}{2}k(a^2 - x^2)$ ....(iii)

From the above formula we can check that

$U_{max} = \frac{1}{2}ka^2$ [At extreme x = ± a] and $U_{min} = 0$ [At mean x = 0]

$K_{max} = \frac{1}{2}ka^2$ [At mean x = 0] and $K_{min} = 0$ [At extreme x = ± a]

$E = \frac{1}{2}ka^2 =$ constant (at all positions)

It mean kinetic energy changes parabolically w.r.t. position but total energy remain always constant irrespective to position of the mass.