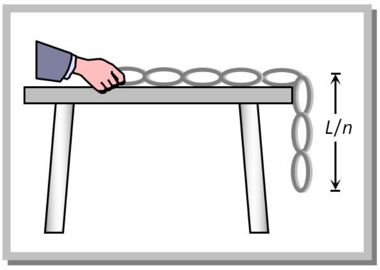
Work Done in Pulling the Chain against Gravity.

A chain of length L and mass M is held on a frictionless table with $(1/n)^{th}$ of its length hanging over the edge.



Let $m = \frac{M}{L} =$ mass per unit length of the chain and y is the length of the chain hanging over the edge. So the mass of the chain of length y will be ym and the force acting on it due to gravity will be mgy.

The work done in pulling the dy length of the chain on the table.

dW = F(– dy) [As y is decreasing]

i.e. dW = mgy (– dy)

So the work done in pulling the hanging portion on the table.

$$W = -\int_{L/n}^{0} mgy\,dy = mg\left[\frac{y^2}{2}\right]_{L/n}^{0} = \frac{mgL^2}{2n^2}$$

$\therefore$ $W = \frac{MgL}{2n^2}$ [As m = M/L]

Alternative method:

If point mass m is pulled through a height h then work done W = mgh

Similarly for a chain we can consider its centre of mass at the middle point of the hanging part i.e. at a height of L/(2n) from the lower end and mass of the hanging part of chain $= \frac{M}{n}$

So work done to raise the centre of mass of the chain on the table is given by

$W = \frac{M}{n} \times g \times \frac{L}{2n}$ [As W = mgh]

or $W = \frac{MgL}{2n^2}$