Head on Inelastic Collision.

(1) **Velocity after collision:** Let two bodies A and B collide inelastically and coefficient of restitution is e.

Where $e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$ $\Rightarrow v_2 - v_1 = e(u_1 - u_2)$

:. $v_2 = v_1 + e(u_1 - u_2)$ (i)

From the law of conservation of linear momentum

 $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ (ii)

By solving (i) and (ii) we get

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right) u_1 + \left(\frac{(1 + e)m_2}{m_1 + m_2}\right) u_2$$

Cincillate
$$\left[(1 + e)m_1\right]_{1 \le 1} \left(m_2 - em_1\right)$$

Similarly

$$v_{2} = \left[\frac{(1+e)m_{1}}{m_{1}+m_{2}}\right]u_{1} + \left(\frac{m_{2}-em_{1}}{m_{1}+m_{2}}\right)u_{2}$$

By substituting e = 1, we get the value of v_1 and u_2 for perfectly elastic head on collision.

(2) Ratio of velocities after inelastic collision : A sphere of mass m moving with velocity u

hits inelastically with another stationary sphere of same mass.

$$\therefore \qquad e = \frac{V_2 - V_1}{U_1 - U_2} = \frac{V_2 - V_1}{U - 0}$$

 \Rightarrow $V_2 - V_1 = eu$ (i)

By conservation of momentum:

Momentum before collision = Momentum after collision

 $mu = mv_1 + mv_2$

$$\Rightarrow \qquad v_1 + v_2 = u \qquad \dots \dots (ii)$$

Solving equation (i) and (ii) we get $v_1 = \frac{u}{2}(1-e)$ and $v_2 = \frac{u}{2}(1+e)$

$$\therefore \qquad \frac{v_1}{v_2} = \frac{1-e}{1+e}$$

(3) Loss in kinetic energy

Loss (ΔK) = Total initial kinetic energy – Total final kinetic energy

$$= \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)$$



Substituting the value of v_1 and v_2 from the above expression

Loss (
$$\Delta K$$
) = $\frac{1}{2} \left(\frac{m_1 m_2}{m_1 + m_2} \right) (1 - e^2) (u_1 - u_2)^2$

By substituting e = 1 we get $\Delta K = 0$ i.e. for perfectly elastic collision loss of kinetic energy will be zero or kinetic energy remains constant before and after the collision.