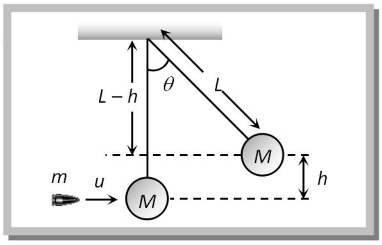
Collision between Bullet and Vertically Suspended Block.

A bullet of mass m is fired horizontally with velocity u in block of mass M suspended by vertical thread.

After the collision bullet gets embedded in block. Let the combined system raised upto height h and the string makes an angle θ with the vertical.

(1) **Velocity of system**



Let v be the velocity of the system (block + bullet) just after the collision.

Momentum $_{\text{bullet}}$ + Momentum $_{\text{block}}$ = Momentum $_{\text{bullet and block system}}$

$mu + 0 = (m + M)v$

$$\therefore \quad v = \frac{mu}{(m+M)} \quad ......(i)$$

(2) **Velocity of bullet:** Due to energy which remains in the bullet block system, just after the collision, the system (bullet + block) rises upto height h.

By the conservation of mechanical energy $\frac{1}{2}(m+M)v^2 = (m+M)gh \Rightarrow v = \sqrt{2gh}$

Now substituting this value in the equation (i) we get $\sqrt{2gh} = \frac{mu}{m+M}$

$$\therefore \quad u = \left[\frac{(m+M)\sqrt{2gh}}{m}\right]$$

(3) **Loss in kinetic energy:** We know the formula for loss of kinetic energy in perfectly inelastic collision

$$\Delta K = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(u_1 + u_2)^2$$

$$\therefore \quad \Delta K = \frac{1}{2}\frac{mM}{m+M}u^2 \quad [\text{As } u_1 = u,\ u_2 = 0,\ m_1 = m \text{ and } m_2 = M]$$

(4) **Angle of string from the vertical**

From the expression of velocity of bullet $u = \left[\frac{(m+M)\sqrt{2gh}}{m}\right]$ we can get $h = \frac{u^2}{2g}\left(\frac{m}{m+M}\right)^2$

From the figure $\cos\theta = \frac{L-h}{L} = 1 - \frac{h}{L} = 1 - \frac{u^2}{2gL}\left(\frac{m}{m+M}\right)^2$

or $$\theta = \cos^{-1}\left[1 - \frac{1}{2gL}\left(\frac{mu}{m+M}\right)^2\right]$$