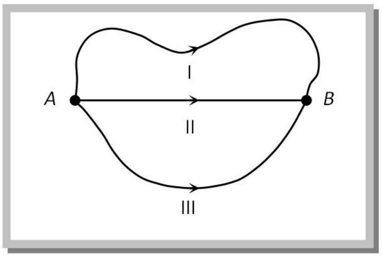
Work Done in Conservative and Non-Conservative Field.

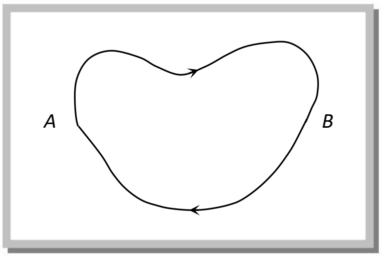
(1) In conservative field work done by the force (line integral of the force i.e. $\int \vec{F}.\vec{dl}$) is independent of the path followed between any two points.



$$\underset{\text{Path I}}{W_{A\to B}} = \underset{\text{Path II}}{W_{A\to B}} = \underset{\text{Path III}}{W_{A\to B}}$$

or $$\underset{\text{Path I}}{\int \vec{F}.\vec{dl}} = \underset{\text{Path II}}{\int \vec{F}.\vec{dl}} = \underset{\text{Path III}}{\int \vec{F}.\vec{dl}}$$

(2) In conservative field work done by the force (line integral of the force i.e. $\int \vec{F}.\vec{dl}$) over a closed path/loop is zero.



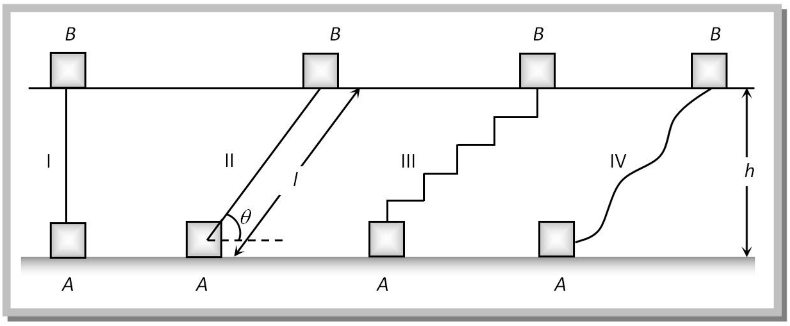
$$W_{A\to B} + W_{B\to A} = 0$$

or $$\oint \bar{F}.\bar{dl} = 0$$

Conservative force: The forces of these type of fields are known as conservative forces.

Example: Electrostatic forces, gravitational forces, elastic forces, magnetic forces etc and all the central forces are conservative in nature.

If a body of man m lifted to height h from the ground level by different path as shown in the figure



Work done through different paths

$$W_I = F.s = mg \times h = mgh$$

$$W_{II} = F.s = mg\sin\theta \times l = mg\sin\theta \times \frac{h}{\sin\theta} = mgh$$

$$W_{III} = mgh_1 + 0 + mgh_2 + 0 + mgh_3 + 0 + mgh_4 = mg(h_1 + h_2 + h_3 + h_4) = mgh$$

$$W_{IV} = \int \vec{F}.\vec{ds} = mgh$$

It is clear that $W_I = W_{II} = W_{III} = W_{IV} = mgh$.

Further if the body is brought back to its initial position A, similar amount of work (energy) is released from the system it means $W_{AB} = mgh$

and $W_{BA} = -mgh$.

Hence the net work done against gravity over a round strip is zero.

$$W_{Net} = W_{AB} + W_{BA}$$
$$= mgh + (-mgh) = 0$$

i.e. the gravitational force is conservative in nature.

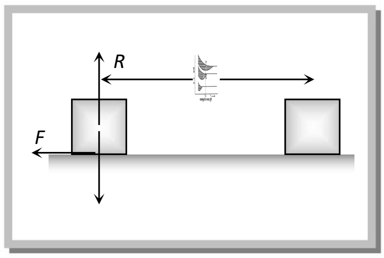
Non-conservative forces: A force is said to be non-conservative if work done by or against the force in moving a body from one position to another, depends on the path followed between these two positions and for complete cycle this work done can never be a zero.

Example: Frictional force, Viscous force, Airdrag etc.

If a body is moved from position A to another position B on a rough table, work done against frictional force shall depends on the length of the path between A and B and not only on the position A and B.

$$W_{AB} = \mu mgs$$

Further if the body is brought back to its initial position A, work has to be done against the frictional force, which always opposes the motion. Hence the net work done against the friction over a round trip is not zero.



$$W_{BA} = \mu mgs.$$

$$\therefore W_{Net} = W_{AB} + W_{BA} = \mu mgs + \mu mgs = 2\mu mgs \neq 0.$$

i.e. the friction is a non-conservative force.