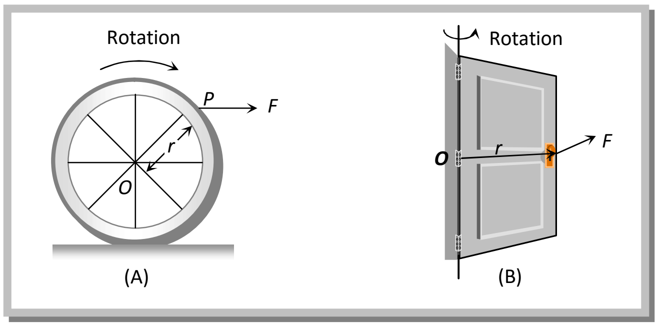
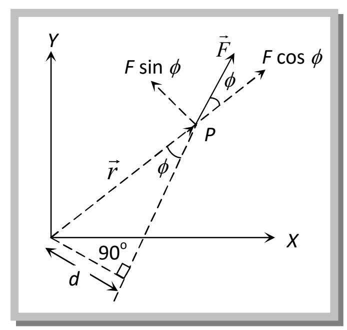
Torque.

If a pivoted, hinged or suspended body tends to rotate under the action of a force, it is said to be acted upon by a torque. orthe turning effect of a force about the axis of rotation is called moment of force or torque due to the force.



If the particle rotating in xy plane about the origin under the effect of force $\vec{F}$ and at any instant the position vector of the particle is $\vec{r}$ then,



Torque $\vec{\tau} = \vec{r} \times \vec{F}$

$$\tau = r\,F \sin\phi$$

[Where ϕ is the angle between the direction of $\vec{r}$ and $\vec{F}$]

(1) Torque is an axial vector. i.e., its direction is always perpendicular to the plane containing vector $\vec{r}$ and $\vec{F}$ in accordance with right hand screw rule. For a given figure the sense of rotation is anti-clockwise so the direction of torque is perpendicular to the plane, outward through the axis of rotation.

(2) Rectangular components of force

$$\vec{F}_r = F\cos\phi = \text{radial component of force},$$

$$\vec{F}_\phi = F\sin\phi = \text{transverse component of force}$$

As $\tau = r\,F\,\sin\phi$

or $\tau = r\,F_\phi$ = (position vector) · (transverse component of force)

Thus the magnitude of torque is given by the product of transverse component of force and its perpendicular distance from the axis of rotation i.e., Torque is due to transverse component of force only.

(3) As $\tau = rF\ \sin\phi$

or $\cdot\ \tau = F(r\sin\phi) = Fd$ [As $d = r\sin\phi$ from the figure]

i.e. Torque = Force $\cdot$ Perpendicular distance of line of action of force from the axis of rotation.

Torque is also called as moment of force and d is called moment or lever arm.

(4) Maximum and minimum torque: As $\vec{\tau} = \vec{r} \times \vec{F}$ or $\tau = rF\sin\phi$

$\tau_{maximum} = rF$	When $\lvert\sin\phi\rvert = \max = 1$ i.e., $\phi = 90°$	$\vec{F}$ is perpendicular to $\vec{r}$
$\tau_{minimum} = 0$	When $\lvert\sin\phi\rvert = \min = 0$ $i.e.\ \phi = 0°$ or $180°$	$\vec{F}$ is collinear to $\vec{r}$

(5) For a given force and angle, magnitude of torque depends on r. The more is the value of r, the more will be the torque and easier to rotate the body.

Example: (i) Handles are provided near the free edge of the Planck of the door.

(ii) The handle of screw driver is taken thick.

(iii) In villages handle of flourmill is placed near the circumference.

(iv) The handle of hand-pump is kept long.

(v) The arm of wrench used for opening the tap, is kept long.

(6) Unit: Newton-meter (M.K.S.) and Dyne-cm (C.G.S.)

(7) Dimension: $[ML^2T^{-2}]$.

(8) If a body is acted upon by more than one force, the total torque is the vector sum of each torque.

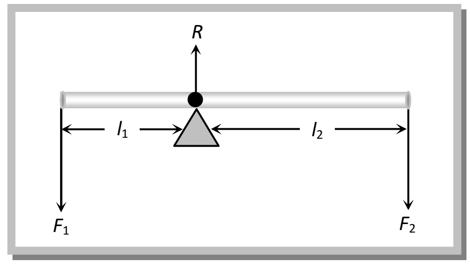
$$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \ldots\ldots$$

(9) A body is said to be in rotational equilibrium if resultant torque acting on it is zero i.e. $\Sigma \vec{\tau} = 0$.

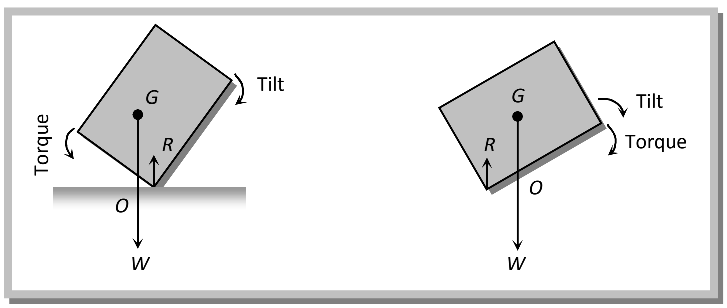
(10) In case of beam balance or see-saw the system will be in rotational equilibrium if,

$\vec{\tau}_1 + \vec{\tau}_2 = 0$ or $F_1 l_1 - F_2 l_2 = 0 \therefore F_1 l_1 = F_2 l_2$

However if, $\vec{\tau}_1 > \vec{\tau}_2$ L.H.S. will move downwards and if $\vec{\tau}_1 < \vec{\tau}_2$. R.H.S. will move downward. and the system will not be in rotational equilibrium.



(11) On tilting, a body will restore its initial position due to torque of weight about the point O till the line of action of weight passes through its base on tilting, a body will topple due to torque of weight about O, if the line of action of weight does not pass through the base.



(12) Torque is the cause of rotatory motion and in rotational motion it plays same role as force plays in translatory motion i.e., torque is rotational analogue of force. This all is evident from the following correspondences between rotatory and translatory motion.

Rotatory Motion	Translatory Motion
$\vec{\tau} = I \vec{\alpha}$	$\vec{F} = m \vec{a}$
$W = \int \vec{\tau} \cdot d\vec{\theta}$	$W = \int \vec{F} \cdot d\vec{s}$
$P = \vec{\tau} \cdot \vec{\omega}$	$P = \vec{F} \cdot \vec{v}$
$\vec{\tau} = \frac{d\vec{L}}{dt}$	$\vec{F} = \frac{d\vec{P}}{dt}$