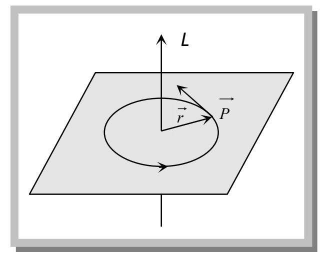
Angular Momentum.

The turning momentum of particle about the axis of rotation is called the angular momentum of the particle.

Or

The moment of linear momentum of a body with respect to any axis of rotation is known as angular momentum. If $\vec{P}$ is the linear momentum of particle and $\vec{r}$ its position vector from the point of rotation then angular momentum.



$$\vec{L} = \vec{r} \times \vec{P}$$

$$\vec{L} = r P \sin\phi \, \hat{n}$$

Angular momentum is an axial vector i.e. always directed perpendicular to the plane of rotation and along the axis of rotation.

(1) S.I. Unit: kg-m^2-s^{-1} or J-sec.

(2) Dimension: $[ML^2T^{-1}]$ and it is similar to Planck's constant (h).

(3) In Cartesian co-ordinates if $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{P} = P_x\hat{i} + P_y\hat{j} + P_z\hat{k}$

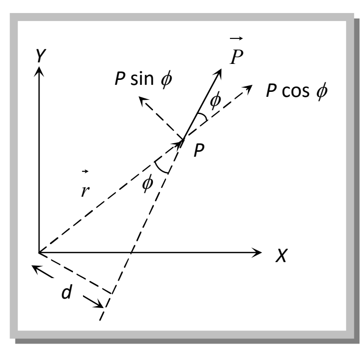
Then $\vec{L} = \vec{r} \times \vec{P} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ P_x & P_y & P_z \end{vmatrix} = (yP_z - zP_y)\hat{i} - (xP_z - zP_x)\hat{j} + (xP_y - yP_x)\hat{k}$

(4) As it is clear from the figure radial component of momentum $\vec{P}_r = P\cos\phi$

Transverse component of momentum $\vec{P}_\phi = P\sin\phi$

So magnitude of angular momentum $L = r P \sin\phi$

$$L = r P_\phi$$



- Angular momentum = Position vector × Transverse component of angular momentum i.e., The radial component of linear momentum has no role to play in angular momentum.

(5) Magnitude of angular momentum $L = P \ (r\sin\phi) = L = Pd$ [As $d = r\sin\phi$ from the figure.]

$\therefore$ Angular momentum = (Linear momentum) × · (Perpendicular distance of line of action of force from the axis of rotation)

(6) Maximum and minimum angular momentum: We know $\vec{L} = \vec{r} \times \vec{P}$

$\therefore \qquad \vec{L} = m[\vec{r} \times \vec{v}] = m\,v\,r\sin\phi = P\,r\sin\phi \qquad$ [As $\vec{P} = m\,\vec{v}$]

$L_{maximum} = mvr$	When $\lvert\sin\phi\rvert = \max = 1$ i.e., $\phi = 90°$	$\vec{v}$ is perpendicular to $\vec{r}$
$L_{minimum} = 0$	When $\lvert\sin\phi\rvert = \min = 0$ $i.e. \phi = 0°$ or $180°$	$\vec{v}$ is parallel or anti-parallel to $\vec{r}$

(7) A particle in translatory motion always have an angular momentum unless it is a point on the line of motion because $L = mvr\,\sin\phi$ and $L > 1$ if $\phi \neq 0^o$ or 180^o

(8) In case of circular motion, $\vec{L} = \vec{r} \times \vec{P} = m(\vec{r} \times \vec{v}) = mvr\,\sin\phi$

$\therefore \qquad L = mvr = mr^2\omega \qquad$ [As $\vec{r} \perp \vec{v}$ and $v = r\omega$]

or $\qquad L = I\omega \qquad$ [As $mr^2 = I$]

In vector form $\vec{L} = I\vec{\omega}$

(9) From $\vec{L} = I\vec{\omega}$ $\therefore \dfrac{d\vec{L}}{dt} = I\dfrac{d\vec{\omega}}{dt} = I\vec{\alpha} = \vec{\tau}$ [As $\dfrac{d\vec{\omega}}{dt} = \vec{\alpha}$ and $\vec{\tau} = I\vec{\alpha}$]

I.e. the rate of change of angular momentum is equal to the net torque acting on the particle. [Rotational analogue of Newton's second law]

(10) If a large torque acts on a particle for a small time then 'angular impulse' of torque is given by

$$\vec{J} = \int \vec{\tau}\, dt = \vec{\tau}_{av} \int_{t_1}^{t_2} dt$$

or Angular impulse $\vec{J} = \vec{\tau}_{av}\, \Delta t = \Delta \vec{L}$

$\therefore$ Angular impulse = Change in angular momentum

(11) The angular momentum of a system of particles is equal to the vector sum of angular momentum of each particle i.e., $\vec{L} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + ... + \vec{L}_n$.

(12) According to Bohr Theory angular momentum of an electron in nth orbit of atom can be taken as,

$L = n\dfrac{h}{2\pi}$ [Where n is an integer used for number of orbit]