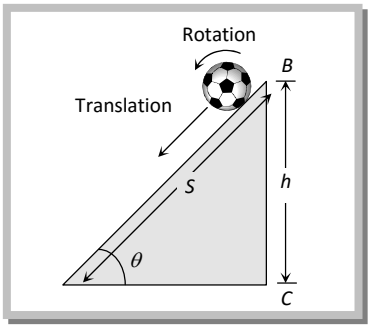
Rolling on an Inclined Plane.

When a body of mass m and radius R rolls down on inclined plane of height 'h' and angle of inclination θ, it loses potential energy. However it acquires both linear and angular speeds and hence, gain kinetic energy of translation and that of rotation.



By conservation of mechanical energy $mgh = \frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right)$

(1) **Velocity at the lowest point:** $v = \sqrt{\frac{2gh}{1+\frac{k^2}{R^2}}}$

(2) **Acceleration in motion:** From equation $v^2 = u^2 + 2aS$

By substituting $u = 0,\ S = \frac{h}{\sin\theta}$ and $v = \sqrt{\frac{2gh}{1+\frac{k^2}{R^2}}}$ we get

$$a = \frac{g\sin\theta}{1+\frac{k^2}{R^2}}$$

(3) **Time of descent:** From equation $v = u + at$

By substituting u = 0 and value of v and a from above expressions

$$t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left[1+\frac{k^2}{R^2}\right]}$$

From the above expressions it is clear that, $v \propto \frac{1}{\sqrt{1+\frac{k^2}{R^2}}};\ a \propto \frac{1}{1+\frac{k^2}{R^2}};\ t \propto \sqrt{1+\frac{k^2}{R^2}}$

Note: Here factor $\left(\frac{k^2}{R^2}\right)$ is a measure of moment of inertia of a body and its value is constant for given shape of the body and it does not depend on the mass and radius of a body.

❑ Velocity, acceleration and time of descent (for a given inclined plane) all depends on $\frac{k^2}{R^2}$. Lesser the moment of inertia of the rolling body lesser will be the value of $\frac{k^2}{R^2}$. So greater will be its velocity and acceleration and lesser will be the time of descent.

❑ If a solid and hollow body of same shape are allowed to roll down on inclined plane then as $\left(\frac{k^2}{R^2}\right)_S < \left(\frac{k^2}{R^2}\right)_H$, solid body will reach the bottom first with greater velocity.

❑ If a ring, cylinder, disc and sphere runs a race by rolling on an inclined plane then as $\left(\frac{k^2}{R^2}\right)_{\text{sphere}} = \text{minimum}$ while $\left(\frac{k^2}{R^2}\right)_{\text{Ring}} = \text{maximum}$, the sphere will reach the bottom first with greatest velocity while ring at last with least velocity.

❑ Angle of inclination has no effect on velocity, but time of descent and acceleration depends on it.

Velocity $\propto \theta^{\circ}$, time of decent $\propto \theta^{-1}$ and acceleration $\propto \theta$.