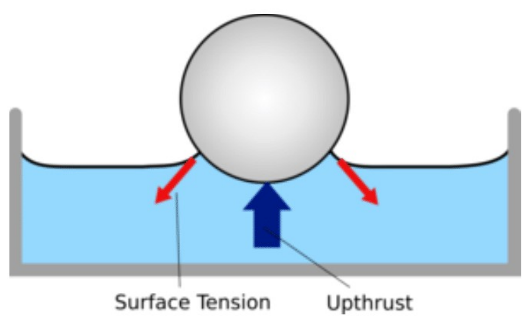
Properties Of Solid And Liquid

Liquids at Rest:-

- **Force of cohesion:-** It is force between two molecules of similar nature.
- **Force of adhesion:-** It is the force between two molecules of different nature.
- **Molecular range:-** The maximum distance between two molecules so that the force of attraction between them remains effective is called molecular range.
- **Sphere of influence:-** Sphere of influence of any molecule is the sphere with molecule as its center and having a radius equal to molecular range ($=10^{-7}$ cm).
- **Surface film:-** Surface film of a liquid is defined as the portion of liquid lying on the surface and caught between two parallel planes situated molecular range apart.
- **Surface tension:-**

Surface tension is the property of a liquid by virtue of which its free surface behaves like a stretched membrane and supports, comparatively heavier objects placed over it. It is measured in terms of force of surface tension.



- **Force of surface tension:-** It is defined as the amount of force acting per unit length on either side of an imaginary line drawn over the liquid surface.

(a) T = Force/length = F/l

(b) T = Surface energy/Surface area = W/A

Units:- S.I – Nm^{-1}

C.G.S- dyn cm^{-1}

- **Additional force:-**

(a) For a cylindrical rod:- $F = T\times 2\pi r$ (Here r is the radius of cylindrical rod)

(b) For a rectangular block:- $F = T\times 2(l+d)$ (Here l is the length and d is the thickness of the rectangular block)

(c) For a ring:- $F = T\times 2\times 2\pi r$ (Here r is the radius of cylindrical rod)

- **Surface energy:-**

Potential energy per unit area of the surface is called surface energy.

(a) Expansion under isothermal condition:-

To do work against forces of surface tension:-

W= $T\times A$ (Here A is the total increase in surface area)

To supply energy for maintaining the temperature of the film:-

$E = T+H$

(b) Expansion under adiabatic conditions:-

E = T

Force of surface tension is numerically equal to the surface energy under adiabatic conditions.

- **Drops and Bubbles:-**

(a) Drop:- Area of surface film of a spherical drop of radius R is given by, $A = 4\pi R^2$

(b) Bubble:- The surface area of the surface films of a bubble of radius R is, $A = 2\times 4\pi R^2$

- **Combination of *n* drops into one big drop:-**

(a) $R = n^{1/3} r$

(b) $E_i = n\ (4\pi r^2 T)$, $E_f = 4\pi R^2 T$

(c) $E_f / E_i = n^{-1/3}$

(d) $\Delta E/E_i = [1-(1/n^{1/3})]$

(e) $\Delta E = 4\pi R^2 T\ (n^{1/3}-1) = 4\pi R^3 T\ (1/r - 1/R)$

- **Angle of contact:-** Angle of contact, for a pair of solid and liquid, is defined as the angle between tangent to the liquid surface drawn at the point of contact and the solid surface inside the liquid.

(a) When $\theta < 90^o$ (acute):-

$F_a > F_c/\sqrt{2}$

(i) Force of cohesion between two molecules of liquid is less than the force of adhesion between molecules of solid and liquid.

(ii) Liquid molecules will stick with the solid, thus making solid wet.

(iii) Such liquid is put in the solid tube; it will have meniscus concave upwards.

(b) When $\theta > 90^o$ (obtuse):-$F_a < F_c/\sqrt{2}$

(i) Force of cohesion between two molecules of liquid is less than the force of adhesion between molecules of solid and liquid.

(ii) In this case, liquids do not wet the solids.

(iii) Such liquids when put in the solid tube will have a meniscus convex upwards.

(c) When $\theta = 90^o$:-?

$F_a = F_c/\sqrt{2}$

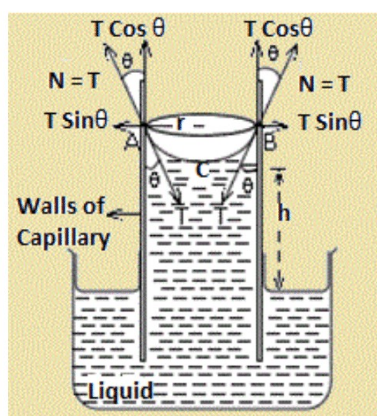
The surface of liquid at the point of contact is plane. In this case force of cohesion and adhesion are comparable to each other.

(d) $\cos\theta_c = T_{sa} - T_{sl}/T_{la}$

Here, T_{sa},T_{sl} and T_{la} represent solid-air, solid-liquid and liquid-air surface tension respectively). Here θ_c is acute if $T_{sl} < T_{sa}$ while θ_c is obtuse if $T_{sl} > T_{sa}$.

- **Capillarity:-**

??Capillarity is the phenomenon, by virtue of which the level of liquid in a capillary tube is different from that outside it, is called capillarity.



Weight of liquid, $W = V\rho g = \pi r^2[h + (r/3)]\rho g$ (Here r is the radius meniscus)

If weight of meniscus is taken into account, the force of surface tension will be,

$T = [r(h+(r/3))\,\rho g]/2\cos\theta$

For fine capillary, force of surface tension, $T = rh\rho g/2\cos\theta$

So height, $h = 2T\cos\theta/\,r\rho g$

This signifies, height of liquid risen (or depressed) in a capillary tube varies inversely as the radius of tube. Smaller the diameter of capillary tube, greater is the rise of liquid in it.

- **Tube of insufficient length:-**

$Rh = 2T/\rho g$

As, T, ρ and g are all constant, Rh = Constant

Smaller the value of h, greater will be the value of R. But liquid will never flow.

- **Effect of temperature affecting surface tension of liquids:-**

Surface tension of a liquid decreases with an increase in its temperature.

$T_\theta = K\ (\theta_c - \theta)$

Here T_θ is the surface tension at a particular temperature θ while θ_c is the critical temperature of the liquid and K is constant.

- **Effect of density:-** Density of liquid also affects its surface tension. Surface tension of a liquid is given by,

$T = A\ (\rho - \rho')^n$

Here, ρ is the density of liquid, ρ' is the density of saturated vapors of liquid and A is the constant depending on the nature of liquid.

- **Pressure difference across a liquid surface:-**

(a) Plane surface:- There is no difference of pressure on the two sides of the film.

(b) Convex surface:-Pressure below the surface film must be greater than that just above it.

(c) Concave surface:- Pressure on the upper side is greater than that just below it.

- **General formula for excess pressure:-**

$P_{excess} = T[1/R_1 + 1/R_2]$

- **Excess pressure in liquid drop:-**

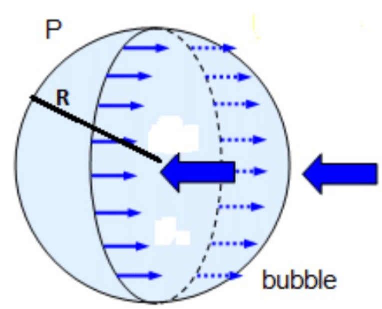
$P_{excess} = 2T/R$, Here R is the radius of liquid drop.

- **Excess pressure for an air bubble in liquid drop:-**

$P_{excess} = 2T/R$

- **Excess pressure in soap bubble:-**

$P_{excess} = 4T/R$, Here R is the radius of soap bubble.



- **Pressure inside an air bubble at a depth h in a liquid:-** $P_{in} = P_{atm} + hdg + (2T/R)$

- **Forces between two plates with thin water film separating them:-**

(a) $\Delta P = T\ (1/r - 1/R)$

(b) $F = AT\ (1/r - 1/R)$

(c) If separation between plate is d, then $\Delta P = 2T/d$ and $F = 2AT/d$

- **Radius of curvature of common film:-** $R_{comon} = rR/R\text{-}r$
- **Capillary depression,** $h = 2T \cos\ (\pi\text{-}\theta)/rdg$
- **Shape of liquid surface:-**

(a) Plane surface (as for water – silver) if $F_{adhesive} > F_{cohesive}/\sqrt{2}$

(b) Concave surface (as for water – glass) if $F_{adhesive} > F_{cohesive}/\sqrt{2}$

(c) Convex surface (as for mercury-glass) if $F_{adhesive} < F_{cohesive}/\sqrt{2}$

- **Increase in temperature:-**

$\Delta\theta = 3T/\rho s\ (1/r - 1/R)$ or $\Delta\theta = 3T/\rho sJ\ (1/r - 1/R)$

Flow of Liquids and Viscosity

(Mechanical Properties of fluids):-

- **Characteristic of Ideal fluid:-**

(a) It is incompressible

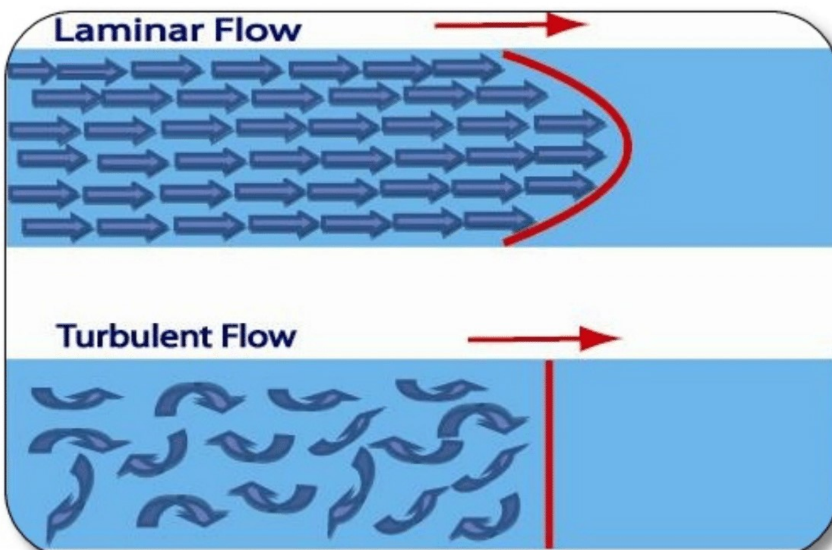
(b) It is non-viscous

(c) Flow of ideal fluid is irrational

(d) It is capable of exhibiting steady flow

- **Stream line flow:-** Flow of a liquid fluid is said to be streamlined if the velocity of a molecule, at any point, coincides with that of the preceding one.

- **Tube of flow:-** A bundle of streamlines having same velocity of fluid elements, over any cross-section perpendicular to the direction of flow, is called a tube of flow.

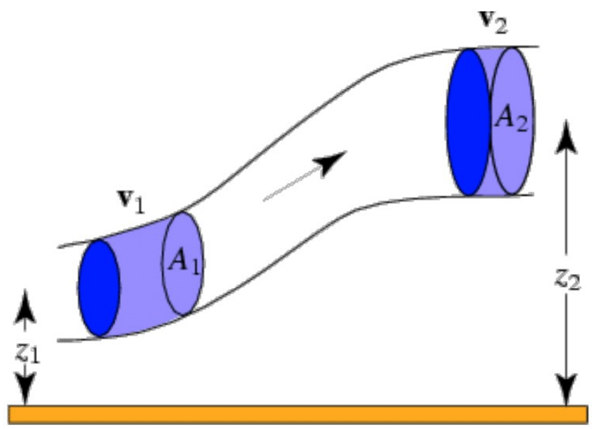


- **Laminar flow:-** It is a special case of streamline flow in which velocities of all the molecules on one streamline is same throughout its motion.

- **Turbulent flow:-** Whenever the velocity of a fluid is very high or it rushes past an obstacle so that there is a sudden change in its direction of motion, the motion of fluid becomes irregular, forming eddies or whirlpools. This type of motion of fluid is called turbulent flow.

- **Rate of flow (Equation of continuity):-**

?av= Constant ($a_1v_1=a_2v_2$)



Equation of continuity can be considered to be a statement of conservation of mass.

So, $v \propto 1/a$

Velocity of flow of liquid varies inversely as the area of cross-section of the opening from where the liquid comes out.

- **Total energy of a liquid:-**

(a) Kinetic energy:- It is the energy possessed by a liquid by virtue of its velocity.

K.E = $\frac{1}{2} mv^2$

K.E per unit mass = $\frac{1}{2} v^2$

K.E per unit volume = $\frac{1}{2} [mv^2/V] = \frac{1}{2} \rho v^2$

Here, ρ is the density of liquid.

(b) Potential energy:- It is the energy possessed by a liquid by virtue of which of its position.

Potential energy = mgh

P.E per unit mass = mgh/m = gh

P.E per unit volume = mgh/V = ρgh

(c) **Pressure energy:-** It is the energy possessed by a liquid by virtue of its pressure.

Pressure energy = $p \times V = m\,(p/\rho)$

Pressure energy per unit mass = p/ρ

Pressure energy per unit volume = $p \times V / V = p$

- **Total energy:-** Total energy of a liquid is the sum total of kinetic energy, potential energy and pressure energy.

$E = \frac{1}{2} mv^2 + mgh + mp/\rho$

Total energy per unit mass = $\frac{1}{2} v^2 + gh + p/\rho$

Total energy per unit volume = $\frac{1}{2} \rho v^2 + \rho gh + p$

- **Bernoulli's equation:-** It states that the total energy of a small amount of an incompressible non-viscous liquid flowing without friction from one point to another, in a streamlined flow, remains constant throughout the displacement.

(a) $\frac{1}{2} mv^2 + mgh + mp/\rho$ = Constant

(b) $\frac{1}{2} v^2 + gh + p/\rho$ = Constant

(c) $\frac{1}{2} \rho v^2 + \rho gh + p$ = Constant or $v^2/2g + h + p/\rho g$ = Constant

The term $v^2/2g$ is called velocity head, h is called gravitational head and $p/\rho g$ is called pressure head.

Therefore Bernoulli's theorem states that in case of an incompressible, non-viscous fluid, flowing from one point to another in a streamlined flow, the sum total of velocity head, gravitational head and the pressure head is a constant quantity.

- **Limitation of Bernoulli's equation:-**

(a) Force of viscosity, which comes into play in case of fluids in motion has not been accounted for.

(b) Loss of energy due to heat is not accounted for.

(c) When a fluid flows in a curved path, the energy due to centripetal force is also not accounted for.

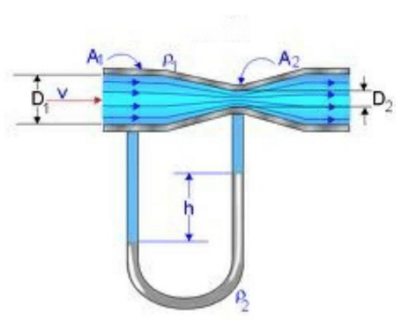
- If v is the relative velocity of top layer w.r.t. any other deeper layer (may be the lowest), then v is lesser for greater depth.

$v = K/bd$

or $v \propto 1/d$

- **Venturimeter:-**

?It is a device used for measuring the rate of flow of liquids, generally water, through pipes.



The rate of flow of water, $Q = a_1a_2\sqrt{2hg/[a_1^2-a_2^2]}$

- **Torricelli's theorem (velocity of efflux):-**

It states that the velocity of efflux of a liquid (V), from an orifice, is equal to the velocity acquired by a body, falling freely (v), from the surface of liquid to the orifice.

So, $V = v = \sqrt{2gh}$

- **Viscosity:-** Viscosity is the property of fluids by virtue of which they tend to destroy any relative motion between their layers.

- **Velocity gradient:-** Velocity gradient is defined as the rate of change of velocity with respect to distance.

(a) Velocity gradient = dv/dr

(b) Dimension of velocity gradient = $[dv/dr] = [T^{-1}]$

(c) Direction of velocity gradient is perpendicular to the direction of flow, directed in the direction of increasing velocity.

(d) Average velocity gradient:- Average velocity gradient is the difference between velocities of two layers separated a unit distance apart.

Average velocity gradient = $\Delta v/\Delta r$

- **Newton's law of viscosity:-**

In accordance to Newton's law of viscosity, the viscous drag force depends upon the nature of fluid along with following factors:-

(a) $F \propto A$ (common area of two layers)

(b) $F \propto dv/dr$ (velocity gradient)

(c) So, $F = \eta A\ (dv/dr)$

Here η is called coefficient of viscosity of fluid.

- **Coefficient of viscosity of fluid (η_v) or fugitive elasticity:-**

η_v = shear stress/velocity gradient = (F/A)/(dv/dr)

- **Modulus of rigidity(η_r):-**

η_r = shear stress/shear strain = (F/A)/(θ) = (F/A)/(dx/dr)

Here, θ = dx/dr = displacement gradient

- **Coefficient of viscosity (Absolute viscosity or Dynamic viscosity):-**

F= ηA (dv/dr)   if A = 1, dv = 1, dr =1, F = η

Co-efficient of viscosity of a fluid is defined as the tangential force per unit area which is required to maintain (or resist) a unit relative velocity between two layers a unit distance apart.

Or

Co-efficient of viscosity of a fluid is defined as the tangential force per unit area which is required to maintain a unit velocity gradient between its layers.

Unit of η:-

S.I:- η = 1 deca poise = 1 N sec/m^2

Co-efficient of viscosity of a fluid is said to be one deca-poise if a tangential force of 1 N per meter square is required to maintain a relative velocity of 1 ms^{-1} between its layer 1 m apart.

C.G.S:- η = 1 poise = 1 dyn sec/cm^2

Coefficient of viscosity of a fluid is said to be one poise if a tangential force of 1 dyn per square cm is required to maintain a relative velocity of 1

cms^{-1} between its layers 1 cm apart.

- **Relation between deca-poise and poise:-**

1 deca-poise = 10 poise

- **Dimension formula for η:-**

$\eta = Fdr/Adv = [M^1L^{-1}T^{-1}]$

- **Fluidity:-** Reciprocal of coefficient of viscosity of a fluid is called its fluidity.

Fluidity = $1/\eta$

Unit of fluidity: $poise^{-1}$

Dimension of fluidity: $[M^{-1}L^1T^1]$

- **Kinematic viscosity:-** Kinematic viscosity of a fluid is defined as the ration between its coefficient of viscosity to the density of fluid.

Kinematic viscosity = η/ρ

Units of kinematic viscosity:- C.G.S – 1 stoke = $cm^2\ s^{-1}$

Kinetic viscosity of a fluid having its dynamic viscosity one poise and density one g cm^{-3} is said to be 1 stoke.

Dimensional formula of kinematic viscosity = $\eta/\rho = [M^0L^2T^{-1}]$

- **Critical velocity (Reynold's Number):-** Critical velocity (v_c) is the maximum velocity of the flow of liquid flowing in a streamlined flow.

$v_c = N_R\ \eta/\rho D$

Here η is the coefficient of viscosity of liquid, ρ is the density of liquid and

D is the diameter of the tube.

Reynold's Number, $N_R = \rho v_c D/ \eta$

- **Stokes law:-** In accordance to Stoke's law, force of viscosity F depend upon,

(a) Co-efficient of viscosity of fluid η

(b) Radius of the moving body r

(c) Velocity of body v

So, force of viscosity, $F = 6\pi \eta r v$

- **Terminal velocity:-** $v = 2/9 [r^2 (\rho-\sigma)/\eta]$

- $\eta = 2/9 [r^2 (\rho-\sigma)g/v]$

- **Variation of viscosity with a change in temperature and pressure:-**

(a) Effect of temperature:-

$\eta = A /(1+Bt)^c$

Here A, B and C are constants.

Again, $\eta v^{1/2} = Ae^{c/vt}$

Here, A and C are constants and v is the relative velocity.

(b) Effect of pressure:- Co-efficient of viscosity of liquids increases due to an increase in pressure but there is no relation, so far, to explain the effect.

- **Change in viscosity of gases:-**

(a) Effect of temperature:- Co-efficient of viscosity of a gas at a given temperature is given by,

$\eta = \eta_0 A T^{1/2}$

Here T is the absolute temperature of gas.

Modified formula, $\eta = [\eta_0 A T^{1/2}]/[1+(S/T)]$

(b) Effect of pressure:- At low pressure, co-efficient of viscosity of a gas varies directly with pressure.

- **Rate of flow of liquid through a liquid through a capillary tube of radius r and length l,**

$V = \pi p r^4/8\eta l = p/(8\eta l/\pi r^4) = p/R$

Here p is the pressure difference between two ends of the capillary and R is the fluid resistance.

- **Accelerated fluid containers:-** $\tan\theta = a_x/g$

- If W be the weight of a body and U be the up thrust force of the liquid on the body then,

(a) The body sinks in the liquid of W>U

(b) The body floats just completely immersed if W=U

- **Pressure exerted by a column of liquid of height h:-** $P = h\rho g$

Here, ρ is the density of liquid.

- **Pressure at a point within the liquid**:-

$P = P_0 + h\rho g$

Here, P_0 is the atmospheric pressure and h is the depth of point with respect to free surface of liquid.