Problem Solving in Algebra 12

DIAGNOSTIC TEST

Directions: Work out each problem. Circle the letter that appears before your answer.

Answers are at the end of the chapter.

1. Find three consecutive odd integers such that the sum of the first two is four times the third.
   (A) 3, 5, 7
   (B) –3, –1, 1
   (C) –11, –9, –7
   (D) –7, –5, –3
   (E) 9, 11, 13

2. Find the shortest side of a triangle whose perimeter is 64, if the ratio of two of its sides is 4 : 3 and the third side is 20 less than the sum of the other two.
   (A) 6
   (B) 18
   (C) 20
   (D) 22
   (E) 24

3. A purse contains 16 coins in dimes and quarters. If the value of the coins is $2.50, how many dimes are there?
   (A) 6
   (B) 8
   (C) 9
   (D) 10
   (E) 12

4. How many quarts of water must be added to 18 quarts of a 32% alcohol solution to dilute it to a solution that is only 12% alcohol?
   (A) 10
   (B) 14
   (C) 20
   (D) 30
   (E) 34

5. Danny drove to Yosemite Park from his home at 60 miles per hour. On his trip home, his rate was 10 miles per hour less and the trip took one hour longer. How far is his home from the park?
   (A) 65 mi.
   (B) 100 mi.
   (C) 200 mi.
   (D) 280 mi.
   (E) 300 mi.

6. Two cars leave a restaurant at the same time and travel along a straight highway in opposite directions. At the end of three hours they are 300 miles apart. Find the rate of the slower car, if one car travels at a rate 20 miles per hour faster than the other.
   (A) 30
   (B) 40
   (C) 50
   (D) 55
   (E) 60

7. The numerator of a fraction is one half the denominator. If the numerator is increased by 2 and the denominator is decreased by 2, the value of the fraction is $\frac{2}{3}$. Find the numerator of the original fraction.
   (A) 4
   (B) 8
   (C) 10
   (D) 12
   (E) 20

8. Darren can mow the lawn in 20 minutes, while Valerie needs 30 minutes to do the same job. How many minutes will it take them to mow the lawn if they work together?
   (A) 10
   (B) 8
   (C) 16
   (D) $6\frac{1}{2}$
   (E) 12

9. Meredith is 3 times as old as Adam. Six years from now, she will be twice as old as Adam will be then. How old is Adam now?
   (A) 6
   (B) 12
   (C) 18
   (D) 20
   (E) 24

10. Mr. Barry invested some money at 5% and an amount half as great at 4%. His total annual income from both investments was $210. Find the amount invested at 4%.
   (A) $1000
   (B) $1500
   (C) $2000
   (D) $2500
   (E) $3000

In the following sections, we will review some of the major types of algebraic problems. Although not every problem you come across will fall into one of these categories, it will help you to be thoroughly familiar with these types of problems. By practicing with the problems that follow, you will learn to translate words into mathematical equations. You should then be able to handle other types of problems confidently.

In solving verbal problems, it is most important that you read carefully and know what it is that you are trying to find. Once this is done, represent your unknown algebraically. Write the equation that translates the words of the problem into the symbols of mathematics. Solve that equation by the techniques previously reviewed.

1. COIN PROBLEMS

In solving coin problems, it is best to change the value of all monies to cents before writing an equation. Thus, the number of nickels must be multiplied by 5 to give the value in cents, dimes by 10, quarters by 25, half dollars by 50, and dollars by 100.

Example:

Sue has $1.35, consisting of nickels and dimes. If she has 9 more nickels than dimes, how many nickels does she have?

Solution:

Let x = the number of dimes
$x + 9$ = the number of nickels
$10x$ = the value of dimes in cents
$5x + 45$ = the value of nickels in cents
135 = the value of money she has in cents
$10x + 5x + 45 = 135$
$15x = 90$
$x = 6$

She has 6 dimes and 15 nickles.

In a problem such as this, you can be sure that 6 would be among the multiple choice answers given. You must be sure to read carefully what you are asked to find and then continue until you have found the quantity sought.

Exercise 1

Work out each problem. Circle the letter that appears before your answer.

1. Marie has $2.20 in dimes and quarters. If the number of dimes is $\frac{1}{4}$ the number of quarters, how many dimes does she have?
   (A) 2
   (B) 4
   (C) 6
   (D) 8
   (E) 10
2. Lisa has 45 coins that are worth a total of $3.50. If the coins are all nickels and dimes, how many more dimes than nickels does she have?
   (A) 5
   (B) 10
   (C) 15
   (D) 20
   (E) 25
3. A postal clerk sold 40 stamps for $5.40. Some were 10-cent stamps and some were 15-cent stamps. How many 10-cent stamps were there?
   (A) 10
   (B) 12
   (C) 20
   (D) 24
   (E) 28
4. Each of the 30 students in Homeroom 704 contributed either a nickel or a quarter to the Cancer Fund. If the total amount collected was $4.70, how many students contributed a nickel?
   (A) 10
   (B) 12
   (C) 14
   (D) 16
   (E) 18
5. In a purse containing nickels and dimes, the ratio of nickels to dimes is 3 : 4. If there are 28 coins in all, what is the value of the dimes?
   (A) 60¢
   (B) $1.12
   (C) $1.60
   (D) 12¢
   (E) $1.00

2. CONSECUTIVE INTEGER PROBLEMS

Consecutive integers are one apart and can be represented algebraically as x, $x + 1$, $x + 2$, and so on. Consecutive even and odd integers are both two apart and can be represented by x, $x + 2$, $x + 4$, and so on. *Never* try to represent consecutive odd integers by x, $x + 1$, $x + 3$, etc., for if x is odd, $x + 1$ would be even.

Example:

Find three consecutive odd integers whose sum is 219.

Solution:

Represent the integers as x, $x + 2$, and $x + 4$. Write an equation stating that their sum is 219.

$3x + 6 = 219$

$3x = 213$

$x = 71$, making the integers 71, 73, and 75.

Exercise 2

Work out each problem. Circle the letter that appears before your answer.

1. If $n + 1$ is the largest of four consecutive integers, represent the sum of the four integers.
   (A) $4n + 10$
   (B) $4n - 2$
   (C) $4n - 4$
   (D) $4n - 5$
   (E) $4n - 8$

2. If n is the first of two consecutive odd integers, which equation could be used to find these integers if the difference of their squares is 120?
   (A) $(n + 1)^2 - n^2 = 120$
   (B) $n^2 - (n + 1)^2 = 120$
   (C) $n^2 - (n + 2)^2 = 120$
   (D) $(n + 2)^2 - n^2 = 120$
   (E) $[(n + 2) - n]^2 = 120$

3. Find the average of four consecutive odd integers whose sum is 112.
   (A) 25
   (B) 29
   (C) 31
   (D) 28
   (E) 30

4. Find the second of three consecutive integers if the sum of the first and third is 26.
   (A) 11
   (B) 12
   (C) 13
   (D) 14
   (E) 15

5. If $2x - 3$ is an odd integer, find the next even integer.
   (A) $2x - 5$
   (B) $2x - 4$
   (C) $2x - 2$
   (D) $2x - 1$
   (E) $2x + 1$

3. AGE PROBLEMS

In solving age problems, you are usually called upon to represent a person's age at the present time, several years from now, or several years ago. A person's age x years from now is found by adding x to his present age. A person's age x years ago is found by subtracting x from his present age.

Example:

Michelle was 15 years old y years ago. Represent her age x years from now.

Solution:

Her present age is $15 + y$. In x years, her age will be her present age plus x, or $15 + y + x$.

Example:

Jody is now 20 years old and her brother, Glenn, is 14. How many years ago was Jody three times as old as Glenn was then?

Solution:

We are comparing their ages x years ago. At that time, Jody's age $(20 - x)$ was three times Glenn's age $(14 - x)$. This can be stated as the equation

$$20 - x = 3(14 - x)$$
$$20 - x = 42 - 3x$$
$$2x = 22$$
$$x = 11$$

To check, find their ages 11 years ago. Jody was 9 while Glenn was 3. Therefore, Jody was three times as old as Glenn was then.

Exercise 3

Work out each problem. Circle the letter that appears before your answer.

1. Mark is now 4 times as old as his brother Stephen. In 1 year Mark will be 3 times as old as Stephen will be then. How old was Mark two years ago?

   (A) 2
   (B) 3
   (C) 6
   (D) 8
   (E) 9

2. Mr. Burke is 24 years older than his son Jack. In 8 years, Mr. Burke will be twice as old as Jack will be then. How old is Mr. Burke now?

   (A) 16
   (B) 24
   (C) 32
   (D) 40
   (E) 48

3. Lili is 23 years old and Melanie is 15 years old. How many years ago was Lili twice as old as Melanie?

   (A) 7
   (B) 16
   (C) 9
   (D) 5
   (E) 8

4. Two years from now, Karen's age will be $2x + 1$. Represent her age two years ago.

   (A) $2x - 4$
   (B) $2x - 1$
   (C) $2x + 3$
   (D) $2x - 3$
   (E) $2x - 2$

5. Alice is now 5 years younger than her brother Robert, whose age is $4x + 3$. Represent her age 3 years from now.

   (A) $4x - 5$
   (B) $4x - 2$
   (C) $4x$
   (D) $4x + 1$
   (E) $4x - 1$

4. INVESTMENT PROBLEMS

All interest referred to is simple interest. The annual amount of interest paid on an investment is found by multiplying the amount invested, called the principal, by the percent of interest, called the rate.

PRINCIPAL · RATE = INTEREST INCOME

Example:

Mrs. Friedman invested some money in a bank paying 4% interest annually and a second amount, $500 less than the first, in a bank paying 6% interest. If her annual income from both investments was $50, how much money did she invest at 6%?

Solution:

Represent the two investments algebraically.
x = amount invested at 4%
$x - 500$ = amount invested at 6%
$.04x$ = annual interest from 4% investment
$.06(x - 500)$ = annual interest from 6% investment
$.04x + .06(x - 500) = 50$
Multiply by 100 to remove decimals.

$$\begin{aligned} 4x + 6(x - 500) &= 5000 \\ 4x + 6x - 3000 &= 5000 \\ 10x &= 8000 \\ x &= 800 \\ x - 500 &= 300 \end{aligned}$$

She invested $300 at 6%.

Exercise 4

Work out each problem. Circle the letter that appears before your answer.

1. Barbara invested x dollars at 3% and $400 more than this amount at 5%. Represent the annual income from the 5% investment.

   (A) $.05x$
   (B) $.05\ (x + 400)$
   (C) $.05x + 400$
   (D) $5x + 40000$
   (E) none of these

2. Mr. Blum invested $10,000, part at 6% and the rest at 5%. If x represents the amount invested at 6%, represent the annual income from the 5% investment.

   (A) $5(x - 10{,}000)$
   (B) $5(10{,}000 - x)$
   (C) $.05(x + 10{,}000)$
   (D) $.05(x - 10{,}000)$
   (E) $.05(10{,}000 - x)$

3. Dr. Kramer invested $2000 in an account paying 6% interest annually. How many more dollars must she invest at 3% so that her total annual income is 4% of her entire investment?

   (A) $120
   (B) $1000
   (C) $2000
   (D) $4000
   (E) $6000

4. Marion invested $7200, part at 4% and the rest at 5%. If the annual income from both investments was the same, find her total annual income from these investments.

   (A) $160
   (B) $320
   (C) $4000
   (D) $3200
   (E) $1200

5. Mr. Maxwell inherited some money from his father. He invested $\frac{1}{2}$ of this amount at 5%, $\frac{1}{3}$ of this amount at 6%, and the rest at 3%. If the total annual income from these investments was $300, what was the amount he inherited?

   (A) $600
   (B) $60
   (C) $2000
   (D) $3000
   (E) $6000

5. FRACTION PROBLEMS

A fraction is a ratio between two numbers. If the value of a fraction is $\frac{3}{4}$, it does not mean that the numerator is 3 and the denominator 4. The numerator and denominator could be 9 and 12, respectively, or 1.5 and 2, or 45 and 60, or an infinite number of other combinations. All we know is that the ratio of numerator to denominator will be 3 : 4. Therefore, the numerator may be represented by $3x$ and the denominator by $4x$. The fraction is then represented by $\frac{3x}{4x}$.

Example:

The value of a fraction is $\frac{2}{3}$. If one is subtracted from the numerator and added to the denominator, the value of the fraction is $\frac{1}{2}$. Find the original fraction.

Solution:

Represent the original fraction as $\frac{2x}{3x}$. If one is subtracted from the numerator and added to the denominator, the new fraction is $\frac{2x-1}{3x+1}$. The value of this new fraction is $\frac{1}{2}$.

$$\frac{2x-1}{3x+1}=\frac{1}{2}$$

Cross multiply to eliminate fractions.

$$4x-2=3x+1$$
$$x=3$$

The original fraction is $\frac{2x}{3x}$, which is $\frac{6}{9}$.

Exercise 5

Work out each problem. Circle the letter that appears before your answer.

1. A fraction is equivalent to $\frac{4}{5}$. If the numerator is increased by 4 and the denominator is increased by 10, the value of the resulting fraction is $\frac{2}{3}$. Find the numerator of the original fraction.

   (A) 4
   (B) 5
   (C) 12
   (D) 16
   (E) 20

2. What number must be added to both the numerator and the denominator of the fraction $\frac{5}{21}$ to give a fraction equal to $\frac{3}{7}$?

   (A) 3
   (B) 4
   (C) 5
   (D) 6
   (E) 7

3. The value of a certain fraction is $\frac{3}{5}$. If both the numerator and denominator are increased by 5, the new fraction is equivalent to $\frac{7}{10}$. Find the original fraction.

   (A) $\frac{3}{5}$
   (B) $\frac{6}{10}$
   (C) $\frac{9}{15}$
   (D) $\frac{12}{20}$
   (E) $\frac{15}{25}$

4. The denominator of a certain fraction is 5 more than the numerator. If 3 is added to both numerator and denominator, the value of the new fraction is $\frac{2}{3}$. Find the original fraction.

   (A) $\frac{3}{8}$
   (B) $\frac{4}{9}$
   (C) $\frac{11}{16}$
   (D) $\frac{12}{17}$
   (E) $\frac{7}{12}$

5. The denominator of a fraction is twice as large as the numerator. If 4 is added to both the numerator and denominator, the value of the fraction is $\frac{5}{8}$. Find the denominator of the original fraction.

   (A) 6
   (B) 10
   (C) 12
   (D) 14
   (E) 16

6. MIXTURE PROBLEMS

There are two kinds of mixture problems with which you should be familiar. The first is sometimes referred to as dry mixture, in which we mix dry ingredients of different values, such as nuts or coffee. Also solved by the same method are problems dealing with tickets at different prices, and similar problems. In solving this type of problem it is best to organize the data in a chart with three rows and columns, labeled as illustrated in the following example.

Example:

Mr. Sweet wishes to mix candy worth 36 cents a pound with candy worth 52 cents a pound to make 300 pounds of a mixture worth 40 cents a pound. How many pounds of the more expensive candy should he use?

Solution:

	No. of pounds	·	Price per pound	=	Total value
More expensive	x		52		$52x$
Less expensive	$300 - x$		36		$36(300 - x)$
Mixture	300		40		12000

The value of the more expensive candy plus the value of the less expensive candy must be equal to the value of the mixture. Almost all mixture problems derive their equation from adding the final column in the chart.

$52x + 36(300 - x) = 12000$

Notice that all values were computed in cents to avoid decimals.

$$52x + 10{,}800 - 36x = 12{,}000$$
$$16x = 1200$$
$$x = 75$$

He should use 75 pounds of the more expensive candy.

In solving the second type of mixture problem, we are dealing with percents instead of prices and amounts of a certain ingredient instead of values. As we did with prices, we may omit the decimal point from the percents, as long as we do it in every line of the chart.

Example:

How many quarts of pure alcohol must be added to 15 quarts of a solution that is 40% alcohol to strengthen it to a solution that is 50% alcohol?

Solution:

	No. of quarts	·	Percent Alcohol	=	Amount of Alcohol
Diluted	15		40		600
Pure	x		100		$100x$
Mixture	$15 + x$		50		$50(15 + x)$

Notice that the percent of alcohol in pure alcohol is 100. If we had added pure water to weaken the solution, the percent of alcohol in pure water would have been 0. Again, the equation comes from adding the final column since the amount of alcohol in the original solution plus the amount of alcohol added must equal the amount of alcohol in the new solution.

$$600 + 100x = 50(15 + x)$$
$$600 + 100x = 750 + 50x$$
$$50x = 150$$
$$x = 3$$

3 quarts of alcohol should be added.

Exercise 6

Work out each problem. Circle the letter that appears before your answer.

1. Express, in terms of x, the value, in cents, of x pounds of 40–cent cookies and $(30 - x)$ pounds of 50-cent cookies.
   (A) $150 + 10x$
   (B) $150 - 50x$
   (C) $1500 - 10x$
   (D) $1500 - 50x$
   (E) $1500 + 10x$

2. How many pounds of nuts selling for 70 cents a pound must be mixed with 30 pounds of nuts selling at 90 cents a pound to make a mixture that will sell for 85 cents a pound?
   (A) 7.5
   (B) 10
   (C) 22.5
   (D) 40
   (E) 12

3. A container holds 10 pints of a solution which is 20% acid. If 3 quarts of pure acid are added to the container, what percent of the resulting mixture is acid?
   (A) 5
   (B) 10
   (C) 20
   (D) 50
   (E) $33\frac{1}{3}$

4. A solution of 60 quarts of sugar and water is 20% sugar. How much water must be added to make a solution that is 5% sugar?
   (A) 180 qts.
   (B) 120 qts.
   (C) 100 qts.
   (D) 80 qts.
   (E) 20 qts.

5. How much water must be evaporated from 240 pounds of a solution that is 3% alcohol to strengthen it to a solution that is 5% alcohol?
   (A) 120 lbs.
   (B) 96 lbs.
   (C) 100 lbs.
   (D) 84 lbs.
   (E) 140 lbs.

7. MOTION PROBLEMS

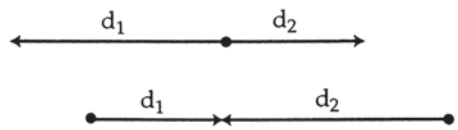
The fundamental relationship in all motion problems is that rate times time is equal to distance.

RATE · TIME = DISTANCE

The problems at the level of this examination usually deal with a relationship between distances. Most motion problems fall into one of three categories.

A. Motion in opposite directions

This can occur when objects start at the same point and move apart, or when they start at a given distance apart and move toward each other. In either case, the distance covered by the first object plus the distance covered by the second is equal to the total distance covered. This can be shown in the following diagram.



In either case, $d_1 + d_2$ = total distance covered.

B. Motion in the same direction

This type of problem is sometimes referred to as a "catch up" problem. Usually two objects leave the same place at different times and at different rates, but the one that leaves later "catches up" to the one that leaves earlier. In such cases the two distances must be equal. If one is still ahead of the other, then an equation must be written expressing this fact.

C. Round trip

In this type of problem, the rate going is different from the rate returning. The times are also different. But if we go somewhere and then return to the starting point, the distances must be equal.

To solve any type of motion problem, it is helpful to organize the information in a chart with columns for rate, time, and distance. A separate line should be used for each moving object. Be very careful of units used. If the rate is given in *miles per hour*, the time must be in *hours* and the distance will be in *miles*.

Example:

A passenger train and a freight train leave at 10:30 A.M. from stations that are 405 miles apart and travel toward each other. The rate of the passenger train is 45 miles per hour faster than that of the freight train. If they pass each other at 1:30 P.M., how fast was the passenger train traveling?

Solution:

Notice that each train traveled exactly 3 hours.

	Rate ·	Time =	Distance
Passenger	$x + 45$	3	$3x + 135$
Freight	x	3	$3x$

$$3x + 135 + 3x = 405$$
$$6x = 270$$
$$x = 45$$

The rate of the passenger train was 90 m.p.h.

Example:

Susie left her home at 11 A.M., traveling along Route 1 at 30 miles per hour. At 1 P.M., her brother Richard left home and started after her on the same road at 45 miles per hour. At what time did Richard catch up to Susie?

Solution:

	Rate ·	Time =	Distance
Susie	30	x	$30x$
Richard	45	$x - 2$	$45x - 90$

Since Richard left 2 hours later than Susie, he traveled for $x - 2$ hours, while Susie traveled for x hours. Notice that we do not fill in 11 and 1 in the time column, as these are times on the clock and not actual hours traveled. Since Richard caught up to Susie, the distances must be equal.

$$30x = 45x - 90$$
$$90 = 15x$$
$$x = 6$$

Susie traveled for 6 hours, which means it was 6 hours past 11 A.M., or 5 P.M. when Richard caught up to her.

Example:

How far can Scott drive into the country if he drives out at 40 miles per hour and returns over the same road at 30 miles per hour and spends 8 hours away from home including a one-hour stop for lunch?

Solution:

His actual driving time is 7 hours, which must be divided into two parts. If one part is x, the other is what is left, or $7 - x$.

	Rate ·	Time =	Distance
Going	40	x	$40x$
Return	30	$7 - x$	$210x - 30x$

The distances are equal.

$$40x = 210 - 30x$$
$$70x = 210$$
$$x = 3$$

If he traveled 40 miles per hour for 3 hours, he went 120 miles.

Exercise 7

Work out each problem. Circle the letter that appears before your answer.

1. At 10 A.M. two cars started traveling toward each other from towns 287 miles apart. They passed each other at 1:30 P.M. If the rate of the faster car exceeded the rate of the slower car by 6 miles per hour, find the rate, in miles per hour, of the faster car.

   (A) 38
   (B) 40
   (C) 44
   (D) 48
   (E) 50

2. A motorist covers 350 miles in 8 hours. Before noon he averages 50 miles per hour, but after noon he averages only 40 miles per hour. At what time did he leave?

   (A) 7 A.M.
   (B) 8 A.M.
   (C) 9 A.M.
   (D) 10 A.M.
   (E) 11 A.M.

3. At 3 P.M. a plane left Kennedy Airport for Los Angeles traveling at 600 m.p.h. At 3:30 P.M. another plane left the same airport on the same route traveling at 650 m.p.h. At what time did the second plane overtake the first?

   (A) 5:15 P.M.
   (B) 6:45 P.M.
   (C) 6:50 P.M.
   (D) 7:15 P.M.
   (E) 9:30 P.M.

4. Joe left home at 10 A.M. and walked out into the country at 4 miles per hour. He returned on the same road at 2 miles per hour. If he arrived home at 4 P.M., how many miles into the country did he walk?

   (A) 6
   (B) 8
   (C) 10
   (D) 11
   (E) 12

5. Two cars leave a restaurant at the same time and proceed in the same direction along the same route. One car averages 36 miles per hour and the other 31 miles per hour. In how many hours will the faster car be 30 miles ahead of the slower car?

   (A) 3
   (B) $3\frac{1}{2}$
   (C) 4
   (D) 6
   (E) $6\frac{1}{4}$

8. WORK PROBLEMS

In most work problems, a job is broken up into several parts, each representing a fractional portion of the entire job. For each part represented, the numerator should represent the time actually spent working, while the denominator should represent the total time needed to do the job alone. The sum of all the individual fractions must be 1 if the job is completed.

Example:

John can complete a paper route in 20 minutes. Steve can complete the same route in 30 minutes. How long will it take them to complete the route if they work together?

Solution:

	John		**Steve**		
$\frac{\text{Time actually spent}}{\text{Time needed to do entire job alone}}$	$\frac{x}{20}$	+	$\frac{x}{30}$	=	1

Multiply by 60 to clear fractions.

$$3x + 2x = 60$$
$$5x = 60$$
$$x = 12$$

Example:

Mr. Powell can mow his lawn twice as fast as his son Mike. Together they do the job in 20 minutes. How many minutes would it take Mr. Powell to do the job alone?

Solution:

If it takes Mr. Powell x hours to mow the lawn, Mike will take twice as long, or $2x$ hours, to mow the lawn.

Mr. Powell		**Mike**		
$\frac{20}{x}$	+	$\frac{20}{2x}$	=	1

Multiply by $2x$ to clear fractions.

$$40 + 20 = 2x$$
$$60 = 2x$$
$$x = 30 \text{ minutes}$$

Exercise 8

Work out each problem. Circle the letter that appears before your answer.

1. Mr. White can paint his barn in 5 days. What part of the barn is still unpainted after he has worked for x days?

   (A) $\frac{x}{5}$

   (B) $\frac{5}{x}$

   (C) $\frac{x-5}{x}$

   (D) $\frac{5-x}{x}$

   (E) $\frac{5-x}{5}$

2. Mary can clean the house in 6 hours. Her younger sister Ruth can do the same job in 9 hours. In how many hours can they do the job if they work together?

   (A) $3\frac{1}{2}$

   (B) $3\frac{3}{5}$

   (C) 4

   (D) $4\frac{1}{4}$

   (E) $4\frac{1}{2}$

3. A swimming pool can be filled by an inlet pipe in 3 hours. It can be drained by a drainpipe in 6 hours. By mistake, both pipes are opened at the same time. If the pool is empty, in how many hours will it be filled?

   (A) 4

   (B) $4\frac{1}{2}$

   (C) 5

   (D) $5\frac{1}{2}$

   (E) 6

4. Mr. Jones can plow his field with his tractor in 4 hours. If he uses his manual plow, it takes three times as long to plow the same field. After working with the tractor for two hours, he ran out of gas and had to finish with the manual plow. How long did it take to complete the job after the tractor ran out of gas?

   (A) 4 hours

   (B) 6 hours

   (C) 7 hours

   (D) 8 hours

   (E) $8\frac{1}{2}$ hours

5. Michael and Barry can complete a job in 2 hours when working together. If Michael requires 6 hours to do the job alone, how many hours does Barry need to do the job alone?

   (A) 2

   (B) $2\frac{1}{2}$

   (C) 3

   (D) $3\frac{1}{2}$

   (E) 4

RETEST

Work out each problem. Circle the letter that appears before your answer.

1. Three times the first of three consecutive odd integers is 10 more than the third. Find the middle integer.

   (A) 7
   (B) 9
   (C) 11
   (D) 13
   (E) 15

2. The denominator of a fraction is three times the numerator. If 8 is added to the numerator and 6 is subtracted from the denominator, the resulting fraction is equivalent to $\frac{8}{9}$. Find the original fraction.

   (A) $\frac{16}{18}$
   (B) $\frac{1}{3}$
   (C) $\frac{8}{24}$
   (D) $\frac{5}{3}$
   (E) $\frac{8}{16}$

3. How many quarts of water must be added to 40 quarts of a 5% acid solution to dilute it to a 2% solution?

   (A) 80
   (B) 40
   (C) 60
   (D) 20
   (E) 50

4. Miriam is 11 years older than Charles. In three years she will be twice as old as Charles will be then. How old was Miriam 2 years ago?

   (A) 6
   (B) 8
   (C) 9
   (D) 17
   (E) 19

5. One printing press can print the school newspaper in 12 hours, while another press can print it in 18 hours. How long will the job take if both presses work simultaneously?

   (A) 7 hrs. 12 min.
   (B) 6 hrs. 36 min.
   (C) 6 hrs. 50 min.
   (D) 7 hrs. 20 min.
   (E) 7 hrs. 15 min.

6. Janet has $2.05 in dimes and quarters. If she has four fewer dimes than quarters, how much money does she have in dimes?

   (A) 30¢
   (B) 80¢
   (C) $1.20
   (D) 70¢
   (E) 90¢

7. Mr. Cooper invested a sum of money at 6%. He invested a second sum, $150 more than the first, at 3%. If his total annual income was $54, how much did he invest at 3%?

   (A) $700
   (B) $650
   (C) $500
   (D) $550
   (E) $600

8. Two buses are 515 miles apart. At 9:30 A.M. they start traveling toward each other at rates of 48 and 55 miles per hour. At what time will they pass each other?

   (A) 1:30 P.M.
   (B) 2:30 P.M.
   (C) 2 P.M.
   (D) 3 P.M.
   (E) 3:30 P.M.

9. Carol started from home on a trip averaging 30 miles per hour. How fast must her mother drive to catch up to her in 3 hours if she leaves 30 minutes after Carol?

   (A) 35 m.p.h.
   (B) 39 m.p.h.
   (C) 40 m.p.h.
   (D) 55 m.p.h.
   (E) 60 m.p.h.

10. Dan has twice as many pennies as Frank. If Frank wins 12 pennies from Dan, both boys will have the same number of pennies. How many pennies did Dan have originally?

    (A) 24
    (B) 12
    (C) 36
    (D) 48
    (E) 52

SOLUTIONS TO PRACTICE EXERCISES

Diagnostic Test

1. (D) Represent the integers as x, $x + 2$, and $x + 4$.

$$x + x + 2 = 4(x + 4)$$
$$2x + 2 = 4x + 16$$
$$-14 = 2x$$
$$x = -7, x + 2 = -5, x + 4 = -3$$

2. (B) Represent the first two sides as $4x$ and $3x$, then the third side is $7x - 20$.

$$4x + 3x + (7x - 20) = 64$$
$$14x - 20 = 64$$
$$14x = 84$$
$$x = 6$$

The shortest side is 3(6) = 18.

3. (D)

$$\text{Let } x = \text{the number of dimes}$$
$$16 - x = \text{the number of quarters}$$
$$10x = \text{value of dimes in cents}$$
$$400 - 25x = \text{value of quarters in cents}$$
$$10x + 400 - 25x = 250$$
$$-15x = -150$$
$$x = 10$$

4. (D)

	No of Quarts ·	Percent Alcohol =	Amount of Alcohol
Original	18	32	576
Added	x	0	0
New	$18 + x$	12	$216 + 12x$

$$576 = 216 + 12x$$
$$360 = 12x$$
$$x = 30$$

5. (E)

	R ·	T =	D
Going	60	x	$60x$
Return	50	$x + 1$	$50x + 50$

$$60x = 50x + 50$$
$$10x = 50$$
$$x = 5$$

If he drove for 5 hours at 60 miles per hour, he drove 300 miles.

6. (B)

	R ·	T =	D
Slow	x	3	$3x$
Fast	$x + 20$	3	$3x + 60$

$$3x + 3x + 60 = 300$$
$$6x = 240$$
$$x = 40$$

7. (C) Represent the original fraction by $\frac{x}{2x}$.

$$\frac{x+2}{2x-2} = \frac{2}{3}$$

Cross multiply.

$$3x + 6 = 4x - 4$$
$$x = 10$$

8. (E)

$$\underset{\text{Darren}}{\frac{x}{20}} + \underset{\text{Valerie}}{\frac{x}{30}} = 1$$

Multiply by 60.

$$3x + 2x = 60$$
$$5x = 60$$
$$x = 12$$

9. (A)

$$\text{Let } x = \text{Adam's age now}$$
$$3x = \text{Meredith's age now}$$
$$x + 6 = \text{Adam's age in 6 years}$$
$$3x + 6 = \text{Meredith's age in 6 years}$$
$$3x + 6 = 2(x + 6)$$
$$3x + 6 = 2x + 12$$
$$x = 6$$

10. (B)

$$\text{Let } x = \text{amount invested at 4\%}$$
$$2x = \text{amount invested at 5\%}$$
$$.04x + .05(2x) = 210$$

Multiply by 100 to eliminate decimals.

$$4x + 5(2x) = 21{,}000$$
$$14x = 21{,}000$$
$$x = \$1500$$

Exercise 1

1. (A) Let x = number of dimes

$$\begin{aligned} 4x &= \text{number of quarters} \\ 10x &= \text{value of dimes in cents} \\ 100x &= \text{value of quarters in cents} \end{aligned}$$

$$\begin{aligned} 10x + 100x &= 220 \\ 100x &= 220 \\ x &= 2 \end{aligned}$$

2. (A) Let x = number of nickels

$$\begin{aligned} 45 - x &= \text{number of dimes} \\ 5x &= \text{value of nickels in cents} \\ 450 - 10x &= \text{value of dimes in cents} \end{aligned}$$

$$\begin{aligned} 5x + 450 - 10x &= 350 \\ -5x &= -100 \\ x &= 20 \end{aligned}$$

20 nickels and 25 dimes

3. (B) Let x = number of 10-cent stamps

$$\begin{aligned} 40 - x &= \text{number of 15-cent stamps} \\ 10x &= \text{value of 10-cent stamps} \\ 600 - 15x &= \text{value of 15-cent stamps} \end{aligned}$$

$$\begin{aligned} 10x + 600 - 15x &= 540 \\ -5x &= -60 \\ x &= 12 \end{aligned}$$

4. (C) Let x = number of nickels

$$\begin{aligned} 30 - x &= \text{number of quarters} \\ 5x &= \text{value of nickels in cents} \\ 750 - 25x &= \text{value of quarters in cents} \end{aligned}$$

$$\begin{aligned} 5x + 750 - 25x &= 470 \\ -20x &= -280 \\ x &= 14 \end{aligned}$$

5. (C) Let $3x$ = number of nickels

$$4x = \text{number of dimes}$$

$$\begin{aligned} 3x + 4x &= 28 \\ 7x &= 28 \\ x &= 4 \end{aligned}$$

There are 16 dimes, worth $1.60.

Exercise 2

1. (B) Consecutive integers are 1 apart. If the fourth is $n + 1$, the third is n, the second is $n - 1$, and the first is $n - 2$. The sum of these is $4n - 2$.

2. (D) The other integer is $n + 2$. If a difference is positive, the larger quantity must come first.

3. (D) To find the average of any 4 numbers, divide their sum by 4.

4. (C) Represent the integers as x, $x + 1$, and $x + 2$.

$$\begin{aligned} x + x + 2 &= 26 \\ 2x &= 24 \\ x &= 12 \\ x + 1 &= 13 \end{aligned}$$

5. (C) An even integer follows an odd integer, so simply add 1.

Exercise 3

1. (C) Let x = Stephen's age now

$$4x = \text{Mark's age now}$$
$$x + 1 = \text{Stephen's age in 1 year}$$
$$4x + 1 = \text{Mark's age in 1 year}$$
$$4x + 1 = 3(x + 1)$$
$$4x + 1 = 3x + 3$$
$$x = 2$$

Mark is now 8, so 2 years ago he was 6.

2. (D) Let x = Jack's age now

$$x + 24 = \text{Mr. Burke's age now}$$
$$x + 8 = \text{Jack's age in 8 years}$$
$$x + 32 = \text{Mr. Burke's age in 8 years}$$
$$x + 32 = 2(x + 8)$$
$$x + 32 = 2x + 16$$
$$16 = x$$

Jack is now 16, Mr. Burke is 40.

3. (A) The fastest reasoning here is from the answers. Subtract each number from both ages, to see which results in Lili being twice as old as Melanie. 7 years ago, Lili was 16 and Melanie was 8.

Let x = number of years ago

Then $23 - x = 2(15 - x)$

$$23 - x = 30 - 2x$$
$$7 = x$$

4. (D) Karen's age now can be found by subtracting 2 from her age 2 years from now. Her present age is $2x - 1$. To find her age 2 years ago, subtract another 2.

5. (D) Alice's present age is $4x - 2$. In 3 years her age will be $4x + 1$.

Exercise 4

1. (B) She invested $x + 400$ dollars at 5%. The income is $.05(x + 400)$.

2. (E) He invested $10{,}000 - x$ dollars at 5%. The income is $.05(10{,}000 - x)$.

3. (D) Let x = amount invested at 3%

$$2000 + x = \text{her total investment}$$
$$.06(2000) + .03x = .04(2000 + x)$$

Multiply by 100 to eliminate decimals.

$$6(2000) + 3x = 4(2000 + x)$$
$$12{,}000 + 3x = 8000 + 4x$$
$$4000 = x$$

4. (B) Let x = amount invested at 4%

$$7200 - x = \text{amount invested at 5\%}$$
$$.04x = .05(7200 - x)$$

Multiply by 100 to eliminate decimals.

$$4x = 5(7200 - x)$$
$$4x = 36{,}000 - 5x$$
$$9x = 36{,}000$$
$$x = 4000$$

Her income is .04(4000) + .05(3200). This is \$160 + \$160, or \$320.

5. (E) In order to avoid fractions, represent his inheritance as $6x$. Then $\frac{1}{2}$ his inheritance is $3x$ and $\frac{1}{3}$ his inheritance is $2x$.

Let $3x$ = amount invested at 5%

$$2x = \text{amount invested at 6\%}$$
$$x = \text{amount invested at 3\%}$$

$.05(3x) + .06(2x) + .03(x) = 300$

Multiply by 100 to eliminate decimals.

$$5(3x) + 6(2x) + 3(x) = 30{,}000$$
$$15x + 12x + 3x = 30{,}000$$
$$30x = 30{,}000$$
$$x = 1000$$

His inheritance was $6x$, or \$6000.

Exercise 5

1. (D) Represent the original fraction as $\frac{4x}{5x}$.

   $\frac{4x+4}{5x+10}=\frac{2}{3}$

   Cross multiply.

   $12x+12=10x+20$

   $2x=8$

   $x=4$

   The original numerator was 4*x*, or 16.

2. (E) While this can be solved using the equation $\frac{5+x}{21+x}=\frac{3}{7}$, it is probably easier to work from the answers. Try adding each choice to the numerator and denominator of $\frac{5}{21}$ to see which gives a result equal to $\frac{3}{7}$.

   $\frac{5+7}{21+7}=\frac{12}{28}=\frac{3}{7}$

3. (C) Here again, it is fastest to reason from the answers. Add 5 to each numerator and denominator to see which will result in a new fraction equal to $\frac{7}{10}$.

   $\frac{9+5}{15+5}=\frac{14}{20}=\frac{7}{10}$

4. (E) Here again, add 3 to each numerator and denominator of the given choices to see which will result in a new fraction equal to $\frac{2}{3}$.

   $\frac{7+3}{12+3}=\frac{10}{15}=\frac{2}{3}$

5. (C) Represent the original fraction by $\frac{x}{2x}$.

   $\frac{x+4}{2x+4}=\frac{5}{8}$

   Cross multiply.

   $8x+32=10x+20$

   $12=2x$

   $x=6$

   The original denominator is 2*x*, or 12.

Exercise 6

1. (C) Multiply the number of pounds by the price per pound to get the total value.

   $40(x)+50(30-x)=$

   $40x+1500-50x=$

   $1500-10x$

2. (B)

No. of Pounds ·	Price per Pound =	Total Value
x	70	$70x$
30	90	2700
$x+30$	85	$85(x+30)$

   $70x+2700=85(x+30)$

   $70x+2700=85x+2550$

   $150=15x$

   $x=10$

3. (D)

	No. of Pints	% of Acid =	Amount of Acid
Original	10	.20	2
Added	6	1.00	6
New	16		8

   Remember that 3 quarts of acid are 6 pints. There are now 8 pints of acid in 16 pints of solution. Therefore, the new solution is $\frac{1}{2}$ or 50% acid.

4. (A)

No. of Quarts ·	% of Sugar =	Amount of Sugar
60	20	1200
x	0	0
$60+x$	5	$5(60+x)$

   $1200=5(60+x)$

   $1200=300+5x$

   $900=5x$

   $x=180$

5. (B)

No. of Pounds ·	% of Alcohol =	Amount of Sugar
240	3	720
x	0	0
$240-x$	5	$5(240-x)$

   Notice that when *x* quarts were evaporated, *x* was *subtracted* from 240 to represent the number of pounds in the mixture.

   $720=5(240-x)$

   $720=1200-5x$

   $5x=480$

   $x=96$

Exercise 7

1. (C) $R \cdot T = D$

Slow	x	3.5	$3.5x$
Fast	$x + 6$	3.5	$3.5(x + 6)$

The cars each traveled from 10 A.M. to 1:30 P.M., which is $3\frac{1}{2}$ hours.

$3.5x + 3.5(x + 6) = 287$

Multiply by 10 to eliminate decimals.

$$35x + 35(x+6) = 2870$$
$$35x + 35x + 210 = 2870$$
$$70x = 2660$$
$$x = 38$$

The rate of the faster car was $x + 6$ or 44 m.p.h.

2. (C) $R \cdot T = D$

Before noon	50	x	$50x$
After noon	40	$8 - x$	$40(8 - x)$

The 8 hours must be divided into 2 parts.

$$50x + 40(8-x) = 350$$
$$50x + 320 - 40x = 350$$
$$10x = 30$$
$$x = 3$$

If he traveled 3 hours before noon, he left at 9 A.M.

3. (E) $R \cdot T = D$

600	x	$600x$
650	$x - \frac{1}{2}$	$650(x - \frac{1}{2})$

The later plane traveled $\frac{1}{2}$ hour less.

$$600x = 650\left(x - \frac{1}{2}\right)$$
$$600x = 650x - 325$$
$$325 = 50x$$
$$6\tfrac{1}{2} = x$$

The plane that left at 3 P.M. traveled for $6\frac{1}{2}$ hours. The time is then 9:30 P.M.

4. (B)

$R \cdot T = D$

Going	4	x	$4x$
Return	2	$6 - x$	$2(6 - x)$

He was gone for 6 hours.

$$4x = 2(6-x)$$
$$4x = 12 - 2x$$
$$6x = 12$$
$$x = 2$$

If he walked for 2 hours at 4 miles per hour, he walked for 8 miles.

5. (D)

$R \cdot T = D$

36	x	$36x$
31	x	$31x$

They travel the same number of hours.

$$36x - 31x = 30$$
$$5x = 30$$
$$x = 6$$

This problem may be reasoned without an equation. If the faster car gains 5 miles per hour on the slower car, it will gain 30 miles in 6 hours.

Exercise 8

1. (E) In x days, he has painted $\frac{x}{5}$ of the barn. To find what part is still unpainted, subtract the part completed from 1. Think of 1 as $\frac{5}{5}$.

$$\frac{5}{5}-\frac{x}{5}=\frac{5-x}{5}$$

2. (B)

Mary		Ruth	
$\frac{x}{6}$	+	$\frac{x}{9}$	$=1$

Multiply by 18.

$$3x+2x=18$$
$$5x=18$$
$$x=3\frac{3}{5}$$

3. (E)

Inlet		Drain	
$\frac{x}{3}$	−	$\frac{x}{6}$	$=1$

Multiply by 6.

$$2x-x=6$$
$$x=6$$

Notice the two fractions are subtracted, as the drainpipe does not help the inlet pipe but works against it.

4. (B)

Tractor		Plow	
$\frac{2}{4}$	+	$\frac{x}{12}$	$=1$

This can be done without algebra, as half the job was completed by the tractor; therefore, the second fraction must also be equal to $\frac{1}{2}$. x is therefore 6.

5. (C)

Michael		Barry	
$\frac{2}{6}$	+	$\frac{2}{x}$	$=1$

Multiply by $6x$.

$$2x+12=6x$$
$$12=4x$$
$$x=3$$

Retest

1. (B) Represent the integers as x, $x+2$, and $x+4$.

$$3x=(x+4)+10$$
$$2x=14$$
$$x=7$$
$$x+2=9$$

2. (C) Represent the original fraction by $\frac{x}{3x}$.

$$\frac{x+8}{3x-6}=\frac{8}{9}$$

Cross multiply.

$$9x+72=24x-48$$
$$120=15x$$
$$x=8$$
$$3x=24$$

The original fraction is $\frac{8}{24}$.

3. (C)

	No. of Quarts	· Percent Alcohol =	Amount of Alcohol
Original	40	5	200
Added	x	0	0
New	$40+x$	2	$80+2x$

$$200=80+2x$$
$$120=2x$$
$$x=60$$

4. (D) Let x = Charles' age now

$x+11$ = Miriam's age now

$x+3$ = Charles' age in 3 years

$x+14$ = Miriam's age in 3 years

$$x+14=2(x+3)$$
$$x+14=2x+6$$
$$x=8$$

Therefore, Miriam is 19 now and 2 years ago was 17.

5. (A) Fast Press Slow Press

$$\frac{x}{12} + \frac{x}{18} = 1$$

Multiply by 36.

$$3x + 2x = 36$$
$$5x = 36$$
$$x = 7\frac{1}{5} \text{ hours}$$
$$= 7 \text{ hours } 12 \text{ minutes}$$

6. (A) Let x = the number of dimes
$x + 4$ = the number of quarters
$10x$ = the value of dimes in cents
$25x + 100$ = the value of quarters in cents

$$10x + 25x + 100 = 205$$
$$35x = 105$$
$$x = 3$$

She has 30¢ in dimes.

7. (A) Let x = amount invested at 6%
$x + 150$ = amount invested at 3%
$.06x + .03(x + 150) = 54$

Multiply by 100 to eliminate decimals

$$6x + 3(x + 150) = 5400$$
$$6x + 3x + 450 = 5400$$
$$9x = 4950$$
$$x = \$550$$
$$x + 150 = \$700$$

8. (B)

	R ·	T =	D
Slow	48	x	$48x$
Fast	55	x	$55x$

$$48x + 55x = 515$$
$$103x = 515$$
$$x = 5 \text{ hours}$$

Therefore, they will pass each other 5 hours after 9:30 A.M., 2:30 P.M.

9. (A)

	R ·	T =	D
Carol	30	3.5	105
Mother	x	3	$3x$

$$3x = 105$$
$$x = 35 \text{ m.p.h.}$$

10. (D) Let x = number of pennies Frank has
$2x$ = number of pennies Dan has

$$x + 12 = 2x - 12$$
$$x = 24$$

Therefore, Dan originally had 48 pennies.