Geometry 13

DIAGNOSTIC TEST

Directions: Work out each problem. Circle the letter that appears before your answer.

Answers are at the end of the chapter.

1. If the angles of a triangle are in the ratio 5 : 6 : 7, the triangle is

   (A) acute
   (B) isosceles
   (C) obtuse
   (D) right
   (E) equilateral

2. A circle whose area is 4 has a radius of x. Find the area of a circle whose radius is $3x$.

   (A) 12
   (B) 36
   (C) $4\sqrt{3}$
   (D) 48
   (E) 144

3. A spotlight is attached to the ceiling 2 feet from one wall of a room and 3 feet from the wall adjacent. How many feet is it from the intersection of the two walls?

   (A) 4
   (B) 5
   (C) $3\sqrt{2}$
   (D) $\sqrt{13}$
   (E) $2\sqrt{3}$

4. In parallelogram *ABCD*, angle *B* is 5 times as large as angle *C*. What is the measure in degrees of angle *B*?

   (A) 30
   (B) 60
   (C) 100
   (D) 120
   (E) 150

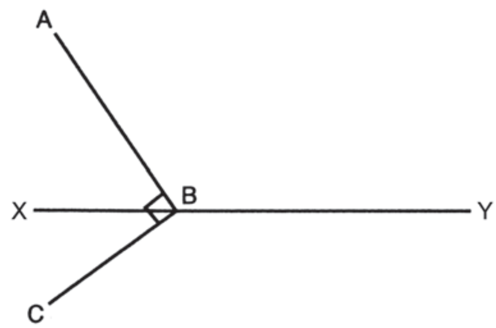
5. A rectangular box with a square base contains 24 cubic feet. If the height of the box is 18 inches, how many feet are there in each side of the base?

   (A) 4
   (B) 2
   (C) $\frac{2\sqrt{3}}{3}$
   (D) $\frac{\sqrt{3}}{2}$
   (E) $\sqrt{3}$

6. In triangle *ABC*, *AB* = *BC*. If angle *B* contains x degrees, find the number of degrees in angle *A*.

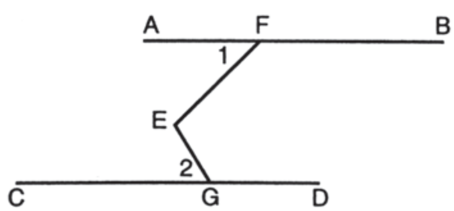
   (A) x
   (B) $180 - x$
   (C) $180 - \frac{x}{2}$
   (D) $90 - \frac{x}{2}$
   (E) $90 - x$

7. In the diagram below, AB is perpendicular to BC. If angle XBY is a straight angle and angle XBC contains 37°, find the number of degrees in angle ABY.



(A) 37
(B) 53
(C) 63
(D) 127
(E) 143

8. If $\overline{AB}$ is parallel to $\overline{CD}$, angle 1 contains 40°, and angle 2 contains 30°, find the number of degrees in angle FEG.



(A) 110
(B) 140
(C) 70
(D) 40
(E) 30

9. In a circle whose center is O, arc AB contains 100°. Find the number of degrees in angle ABO.

(A) 50
(B) 100
(C) 40
(D) 65
(E) 60

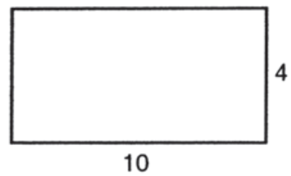
10. Find the length of the line segment joining the points whose coordinates are (–3, 1) and (5, –5).

(A) 10
(B) $2\sqrt{5}$
(C) $2\sqrt{10}$
(D) 100
(E) $\sqrt{10}$

The questions in the following area will expect you to recall some of the numerical relationships learned in geometry. If you are thoroughly familiar with these relationships, you should not find these questions difficult. As mentioned earlier, be particularly careful with units. For example, you cannot multiply a dimension given in feet by another given in inches when you are finding area. Read each question very carefully for the units given. In the following sections, all the needed formulas with illustrations and practice exercises are to help you prepare for the geometry questions on your test.

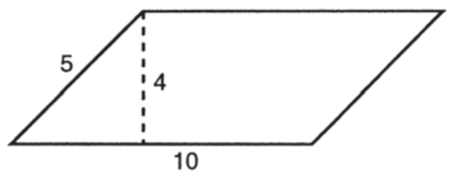
1. AREAS

A. Rectangle = base · altitude = *bh*



Area = 40

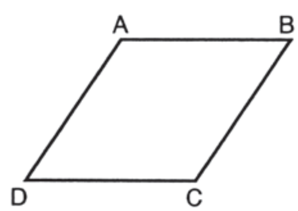
B. Parallelogram = base · altitude = *bh*



Area = 40

Notice that the altitude is different from the side. It is always shorter than the second side of the parallelogram, as a perpendicular is the shortest distance from a point to a line.

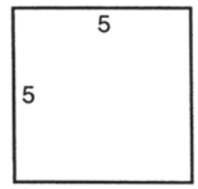
C. Rhombus = $\frac{1}{2}$ · product of the diagonals = $\frac{1}{2}d_1d_2$



If $AC = 20$ and $BD = 30$, the area of $ABCD = \frac{1}{2}(20)(30) = 300$

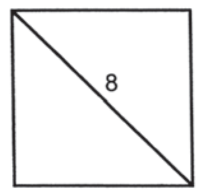
D. Square = side · side = s^2

Area = 25



Remember that every square is a rhombus, so that the rhombus formula may be used for a square if the diagonal is given. The diagonals of a square are equal.

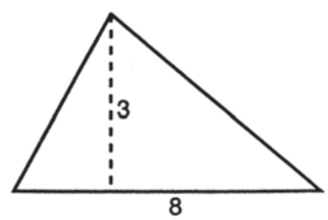
Area = $\frac{1}{2}(8)(8) = 32$



Remember also that a rhombus is *not* a square. Therefore do not use the s^2 formula for a rhombus. A rhombus, however, is a parallelogram, so you may use bh if you do not know the diagonals.

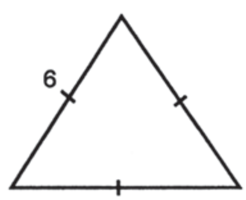
E. Triangle = $\frac{1}{2}$ · base · altitude = $\frac{1}{2}bh$

$A = \frac{1}{2}(8)(3) = 12$



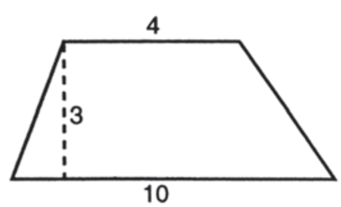
F. Equilateral Triangle = $\frac{1}{4}$ · side squared · $\sqrt{3} = \frac{s^2}{4}\sqrt{3}$

$A = \frac{36}{4}\sqrt{3} = 9\sqrt{3}$



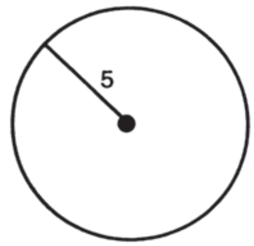
G. Trapezoid = $\frac{1}{2}$ · altitude · sum of bases = $\frac{1}{2}h(b_1 + b_2)$

$A = \frac{1}{2}(3)(14) = 21$



H. Circle = $\pi \cdot$ radius squared = $\pi \cdot r^2$

$A = \pi \cdot (5)^2 = 25\pi$

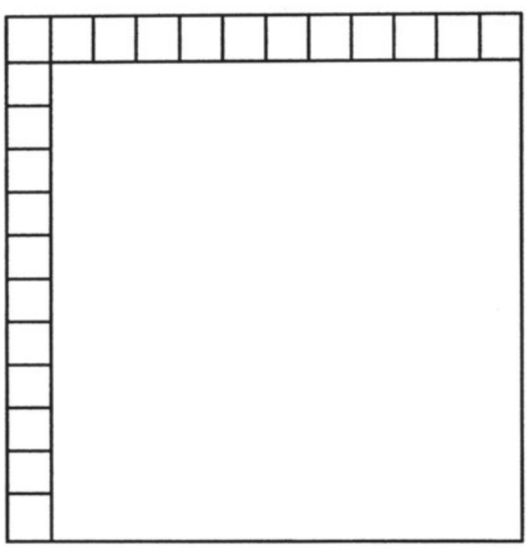


Remember that π is the ratio of the circumference of any circle and its diameter. $\pi = \frac{c}{d}$. The approximations you have used for π in the past (3.14 or $\frac{22}{7}$) are just that—approximations. π is an irrational number and cannot be expressed as a fraction or terminating decimal. Therefore all answers involving π should be left in terms of π unless you are given a specific value to substitute for π.

A word about units—Area is measured in square units. That is, we wish to compute how many squares one inch on each side (a square inch) or one foot on each side (a square foot), etc., can be used to cover a given surface. To change from square inches to square feet or square yards, remember that

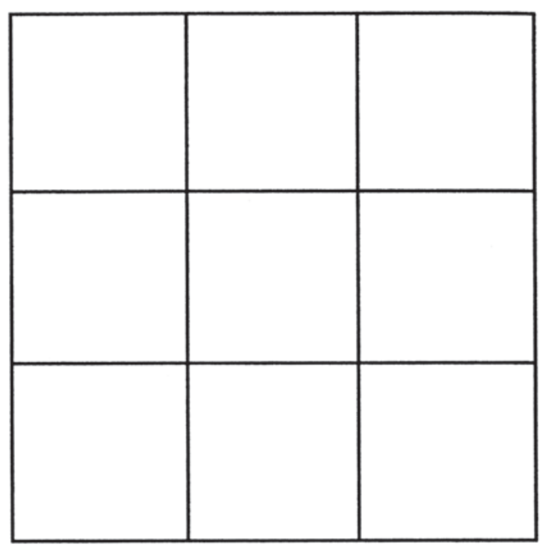
144 square inches = 1 square foot
9 square feet = 1 square yard

1 square foot



12" = 1'
12 one inch squares in a row
12 rows
144 square inches in 1 sq. ft.

1 square yard



3' = 1 yd.
3 one foot squares in a row
3 rows
9 square feet in 1 sq. yd.

Exercise 1

Work out each problem. Circle the letter that appears before your answer.

1. The dimensions of a rectangular living room are 18 feet by 20 feet. How many square yards of carpeting are needed to cover the floor?
   (A) 360
   (B) 42
   (C) 40
   (D) 240
   (E) 90

2. In a parallelogram whose area is 15, the base is represented by $x + 7$ and the altitude is $x - 7$. Find the base of the parallelogram.
   (A) 8
   (B) 15
   (C) 1
   (D) 34
   (E) 5

3. The sides of a right triangle are 6, 8, and 10. Find the altitude drawn to the hypotenuse.
   (A) 2.4
   (B) 4.8
   (C) 3.4
   (D) 3.5
   (E) 4.2

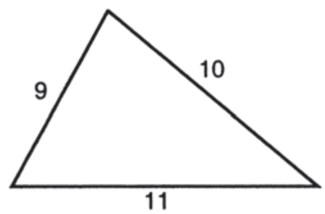
4. If the diagonals of a rhombus are represented by $4x$ and $6x$, the area may be represented by
   (A) $6x$
   (B) $24x$
   (C) $12x$
   (D) $6x^2$
   (E) $12x^2$

5. A circle is inscribed in a square whose side is 6. Express the area of the circle in terms of π.
   (A) 6π
   (B) 3π
   (C) 9π
   (D) 36π
   (E) 12π

2. PERIMETER

The perimeter of a figure is the distance around the outside. If you were fencing in an area, the number of feet of fencing you would need is the perimeter. Perimeter is measured in linear units, that is, centimeters, inches, feet, meters, yards, etc.

A. Any polygon = sum of all sides



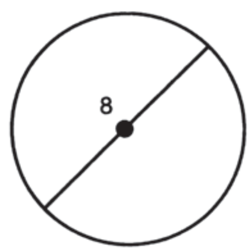
$P = 9 + 10 + 11 = 30$

B. Circle = π · diameter = πd

or

$2 \cdot \pi \cdot \text{radius} = 2\pi r$

Since $2r = d$, these formulas are the same. The perimeter of a circle is called its circumference.

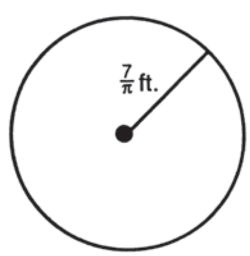


$C = \pi \cdot 8 = 8\pi$

or

$C = 2 \cdot \pi \cdot 4 = 8\pi$

The distance covered by a wheel in one revolution is equal to the circumference of the wheel. In making one revolution, every point on the rim comes in contact with the ground. The distance covered is then the same as stretching the rim out into a straight line.



The distance covered by this wheel in one revolution is $2 \cdot \pi \cdot \frac{7}{\pi} = 14$ feet.

Exercise 2

Work out each problem. Circle the letter that appear before your answer.

1. The area of an equilateral triangle is $16\sqrt{3}$. Find its perimeter.
   (A) 24
   (B) 16
   (C) 48
   (D) $24\sqrt{3}$
   (E) $48\sqrt{3}$

2. The hour hand of a clock is 3 feet long. How many feet does the tip of this hand move between 9:30 P.M. and 1:30 A.M. the following day?
   (A) π
   (B) 2π
   (C) 3π
   (D) 4π
   (E) 24π

3. If the radius of a circle is increased by 3, the circumference is increased by
   (A) 3
   (B) 3π
   (C) 6
   (D) 6π
   (E) 4.5

4. The radius of a wheel is 18 inches. Find the number of feet covered by this wheel in 20 revolutions.
   (A) 360π
   (B) 360
   (C) 720π
   (D) 720
   (E) 60π

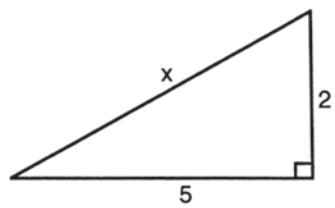
5. A square is equal in area to a rectangle whose base is 9 and whose altitude is 4. Find the perimeter of the square.
   (A) 36
   (B) 26
   (C) 13
   (D) 24
   (E) none of these

3. RIGHT TRIANGLES

A. Pythagorean theorem

$(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2$

$$(5)^2 + (2)^2 = x^2$$
$$25 + 4 = x^2$$
$$29 = x^2$$
$$x = \sqrt{29}$$



B. Pythagorean triples

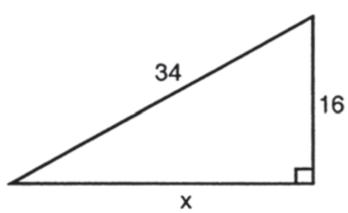
These are sets of numbers that satisfy the Pythagorean Theorem. When a given set of numbers such as 3, 4, 5 forms a Pythagorean triple ($3^2 + 4^2 = 5^2$), any multiples of this set such as 6, 8, 10 or 30, 40, 50 also form a Pythagorean triple. Memorizing the sets of Pythagorean triples that follow will save you valuable time in solving problems, for, if you recognize given numbers as multiples of Pythagorean triples, you do not have to do any arithmetic at all. The most common Pythagorean triples that should be memorized are

3, 4, 5

5, 12, 13

8, 15, 17

7, 24, 25



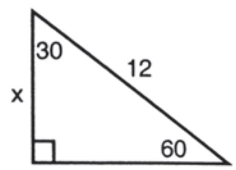
Squaring 34 and 16 to apply the Pythagorean theorem would take too much time. Instead, recognize the hypotenuse as 2(17). Suspect an 8, 15, 17 triangle. Since the given leg is 2(8), the missing leg will be 2(15) or 30, without any computation at all.

C. 30°–60°–90° triangle

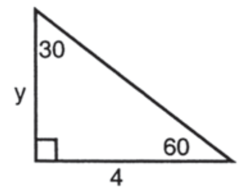
a) The leg opposite the 30° angle is one-half the hypotenuse.

b) The leg opposite the 60° angle is one-half the hypotenuse • $\sqrt{3}$.

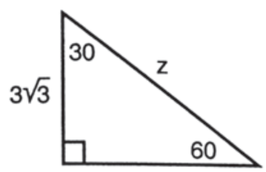
c) An altitude in an equilateral triangle forms a 30°–60°–90° triangle and is therefore equal to one-half the side • $\sqrt{3}$.



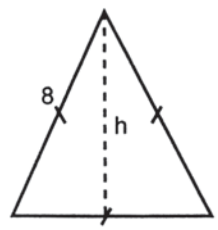
$x = 6\sqrt{3}$



$y = 4\sqrt{3}$



$z = 6$



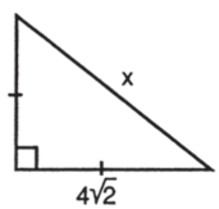
$h = 4\sqrt{3}$

D. 45°–45°–90° triangle (isosceles right triangle)

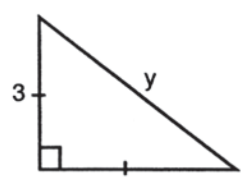
a) Each leg is one-half the hypotenuse times $\sqrt{2}$.

b) Hypotenuse is leg times $\sqrt{2}$.

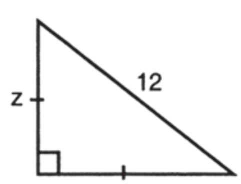
c) The diagonal of a square forms a 45°–45°–90° triangle and is therefore equal to a side times $\sqrt{2}$.



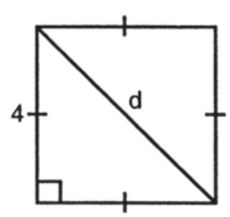
$x = 8$



$y = 3\sqrt{2}$



$z = 6\sqrt{2}$



$d = 4\sqrt{2}$

Exercise 3

Work out each problem. Circle the letter that appears before your answer.

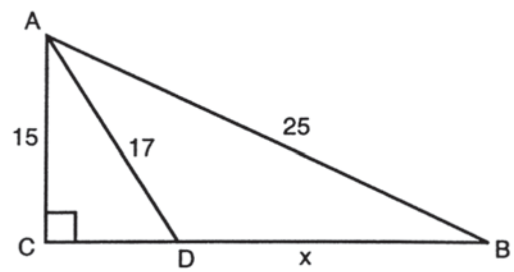
1. A farmer uses 140 feet of fencing to enclose a rectangular field. If the ratio of length to width is 3 : 4, find the diagonal, in feet, of the field.
   (A) 50
   (B) 100
   (C) 20
   (D) 10
   (E) cannot be determined

2. Find the altitude of an equilateral triangle whose side is 20.
   (A) 10
   (B) $20\sqrt{3}$
   (C) $10\sqrt{3}$
   (D) $20\sqrt{2}$
   (E) $10\sqrt{2}$

3. Two boats leave the same dock at the same time, one traveling due west at 8 miles per hour and the other due north at 15 miles per hour. How many miles apart are the boats after three hours?
   (A) 17
   (B) 69
   (C) 75
   (D) 51
   (E) 39

4. Find the perimeter of a square whose diagonal is $6\sqrt{2}$.
   (A) 24
   (B) $12\sqrt{2}$
   (C) 12
   (D) 20
   (E) $24\sqrt{2}$

5. Find the length of *DB*.



   (A) 8
   (B) 10
   (C) 12
   (D) 15
   (E) 20

4. COORDINATE GEOMETRY

A. Distance between two points =

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The distance between (–3, 2) and (5, –1) is

$$\sqrt{[-3-5]^2 + [2-(-1)]^2} = \sqrt{(-8)^2 + (3)^2} = \sqrt{64+9} = \sqrt{73}$$

B. The midpoint of a line segment =

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Since a midpoint is in the middle, its coordinates are found by averaging the *x* coordinates and averaging the *y* coordinates. Remember that to find the average of two numbers, you add them and divide by two. Be very careful of signs in adding signed numbers. Review the rules given earlier if necessary.

The midpoint of the segment joining (–4, 1) to (–2, –9) is

$$\left(\frac{-4+(-2)}{2}, \frac{1+(-9)}{2}\right) = \left(\frac{-6}{2}, \frac{-8}{2}\right) = (-3, -4)$$

Exercise 4

Work out each problem. Circle the letter that appears before your answer.

1. *AB* is the diameter of a circle whose center is *O*. If the coordinates of *A* are (2, 6) and the coordinates of *B* are (6, 2), find the coordinates of *O*.
   (A) (4, 4)
   (B) (4, –4)
   (C) (2, –2)
   (D) (0, 0)
   (E) (2, 2)

2. *AB* is the diameter of a circle whose center is *O*. If the coordinates of *O* are (2, 1) and the coordinates of *B* are (4, 6), find the coordinates of *A*.
   (A) $\left(3, 3\frac{1}{2}\right)$
   (B) $\left(1, 2\frac{1}{2}\right)$
   (C) (0, –4)
   (D) $\left(2\frac{1}{2}, 1\right)$
   (E) $\left(-1, -2\frac{1}{2}\right)$

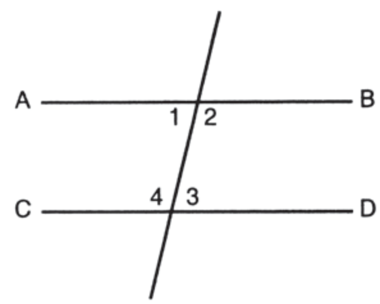
3. Find the distance from the point whose coordinates are (4, 3) to the point whose coordinates are (8, 6).
   (A) 5
   (B) 25
   (C) $\sqrt{7}$
   (D) $\sqrt{67}$
   (E) 15

4. The vertices of a triangle are (2, 1), (2, 5), and (5, 1). The area of the triangle is
   (A) 12
   (B) 10
   (C) 8
   (D) 6
   (E) 5

5. The area of a circle whose center is at (0,0) is 16π. The circle passes through each of the following points *except*
   (A) (4, 4)
   (B) (0, 4)
   (C) (4, 0)
   (D) (–4, 0)
   (E) (0, –4)

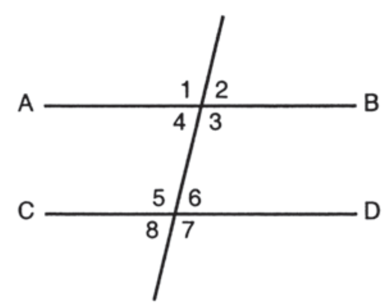
5. PARALLEL LINES

A. If two lines are parallel and cut by a transversal, the alternate interior angles are congruent.



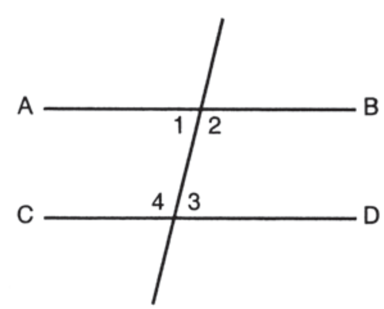
If $\overline{AB}$ is parallel to $\overline{CD}$, then angle 1 ≅ angle 3 and
angle 2 ≅ angle 4.

B. If two parallel lines are cut by a transversal, the corresponding angles are congruent.



If $\overline{AB}$ is parallel to $\overline{CD}$, then angle 1 ≅ angle 5
angle 2 ≅ angle 6
angle 3 ≅ angle 7
angle 4 ≅ angle 8

C. If two parallel lines are cut by a transversal, interior angles on the same side of the transversal are supplementary.

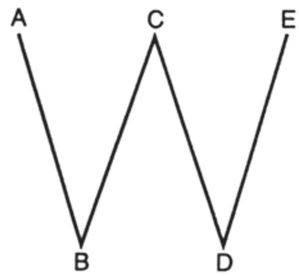


If $\overline{AB}$ is parallel to $\overline{CD}$, angle 1 + angle 4 = 180°
angle 2 + angle 3 = 180°

Exercise 5

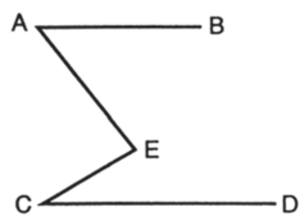
Work out each problem. Circle the letter that appears before your answer.

1. If $\overline{AB}$ is parallel to $\overline{CD}$, $\overline{BC}$ is parallel to $\overline{ED}$, and angle $B = 30°$, find the number of degrees in angle D.



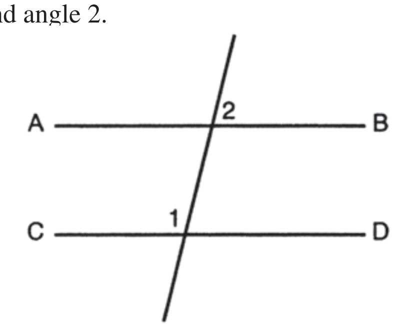
(A) 30
(B) 60
(C) 150
(D) 120
(E) none of these

2. If $\overline{AB}$ is parallel to $\overline{CD}$, angle $A = 35°$, and angle $C = 45°$, find the number of degrees in angle AEC.



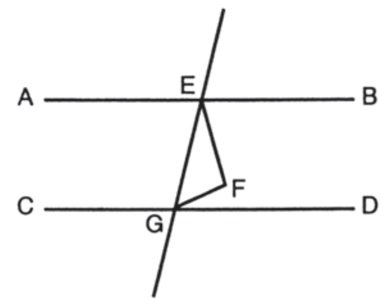
(A) 35
(B) 45
(C) 70
(D) 80
(E) 100

3. If $\overline{AB}$ is parallel to $\overline{CD}$ and angle $1 = 130°$, find angle 2.



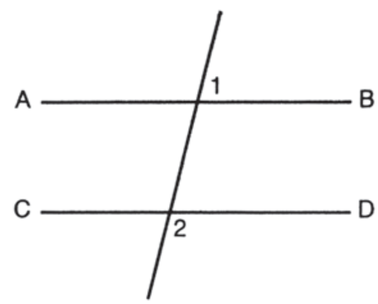
(A) 130°
(B) 100°
(C) 40°
(D) 60°
(E) 50°

4. If $\overline{AB}$ is parallel to $\overline{CD}$, $\overline{EF}$ bisects angle BEG, and $\overline{GF}$ bisects angle EGD, find the number of degrees in angle EFG.



(A) 40
(B) 60
(C) 90
(D) 120
(E) cannot be determined

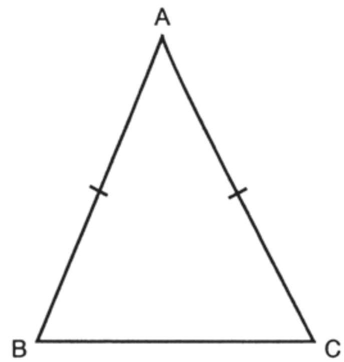
5. If $\overline{AB}$ is parallel to $\overline{CD}$ and angle $1 = x°$, then the sum of angle 1 and angle 2 is



(A) $2x°$
(B) $(180 - x)°$
(C) 180°
(D) $(180 + x)°$
(E) none of these

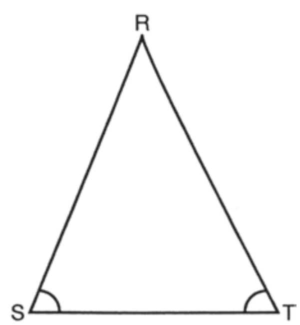
6. TRIANGLES

A. If two sides of a triangle are congruent, the angles opposite these sides are congruent.



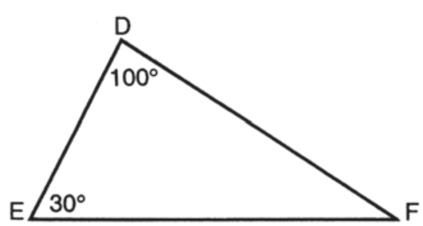
If $\overline{AB} \cong \overline{AC}$, then angle $B \cong$ angle C.

B. If two angles of a triangle are congruent, the sides opposite these angles are congruent.



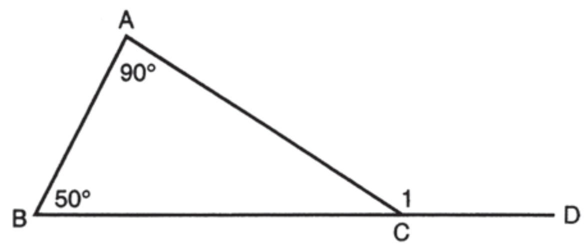
If angle $S \cong$ angle T, then $\overline{RS} \cong \overline{RT}$.

C. The sum of the measures of the angles of a triangle is 180°.



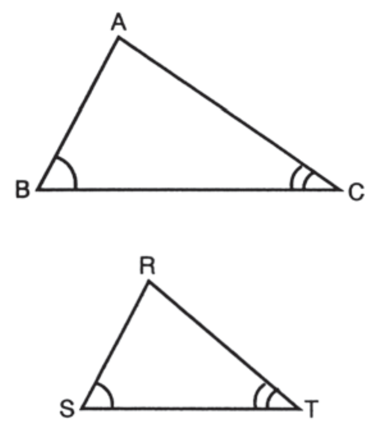
Angle $F = 180° - 100° - 30° = 50°$.

D. The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.



Angle 1 = 140°

E. If two angles of one triangle are congruent to two angles of a second triangle, the third angles are congruent.



Angle *A* will be congruent to angle *R*.

Exercise 6

Work out each problem. Circle the letter that appears before your answer.

1. The angles of a triangle are in the ratio 1 : 5 : 6. This triangle is

   (A) acute
   (B) obtuse
   (C) isosceles
   (D) right
   (E) equilateral

2. If the vertex angle of an isosceles triangle is 50°, find the number of degrees in one of the base angles.

   (A) 50
   (B) 130
   (C) 60
   (D) 65
   (E) 55

3. In triangle *ABC*, angle *A* is three times as large as angle *B*. The exterior angle at *C* is 100°. Find the number of degrees in angle *A*.

   (A) 60
   (B) 80
   (C) 20
   (D) 25
   (E) 75

4. If a base angle of an isosceles triangle is represented by $x°$, represent the number of degrees in the vertex angle.

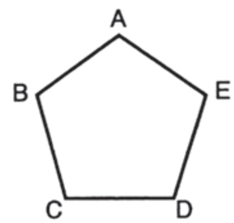
   (A) $180 - x$
   (B) $x - 180$
   (C) $2x - 180$
   (D) $180 - 2x$
   (E) $90 - 2x$

5. In triangle *ABC*, $\overline{AB} = \overline{BC}$. If angle $A = (4x - 30)°$ and angle $C = (2x + 10)°$, find the number of degrees in angle *B*.

   (A) 20
   (B) 40
   (C) 50
   (D) 100
   (E) 80

7. POLYGONS

A. The sum of the measures of the angles of a polygon of n sides is $(n - 2)180°$.



Since $ABCDE$ has 5 sides, angle A + angle B + angle C + angle D + angle $E = (5 - 2)180° = 3(180)° = 540°$

B. Properties of a parallelogram

a) Opposite sides are parallel

b) Opposite sides are congruent

c) Opposite angles are congruent

d) Consecutive angles are supplementary

e) Diagonals bisect each other

C. Properties of a rectangle

a) All 5 properties of a parallelogram

b) All angles are right angles

c) Diagonals are congruent

D. Properties of a rhombus

a) All 5 properties of a parallelogram

b) All sides are congruent

c) Diagonals are perpendicular to each other

d) Diagonals bisect the angles

E. Properties of a square

a) All 5 parallelogram properties

b) Two additional rectangle properties

c) Three additional rhombus properties

Exercise 7

Work out each problem. Circle the letter that appears before your answer.

1. Find the number of degrees in the sum of the interior angles of a hexagon.
   (A) 360
   (B) 540
   (C) 720
   (D) 900
   (E) 1080

2. In parallelogram *ABCD*, $AB = x + 4$, $BC = x - 6$, and $CD = 2x - 16$. Find *AD*.
   (A) 20
   (B) 24
   (C) 28
   (D) 14
   (E) 10

3. In parallelogram *ABCD*, $AB = x + 8$, $BC = 3x$, and $CD = 4x - 4$. *ABCD* must be a
   (A) rectangle
   (B) rhombus
   (C) trapezoid
   (D) square
   (E) pentagon

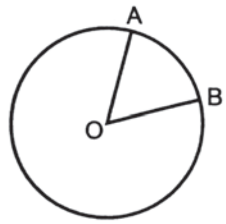
4. The sum of the angles in a rhombus is
   (A) 180°
   (B) 360°
   (C) 540°
   (D) 720°
   (E) 450°

5. Which of the following statements is *false?*
   (A) A square is a rhombus.
   (B) A rhombus is a parallelogram.
   (C) A rectangle is a rhombus.
   (D) A rectangle is a parallelogram.
   (E) A square is a rectangle.

8. CIRCLES

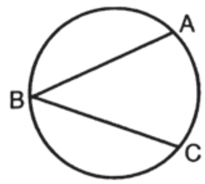
A. A central angle is equal in degrees to its intercepted arc.

If arc $AB = 50°$, then angle $AOB = 50°$.



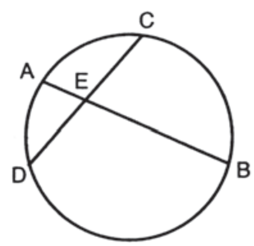
B. An inscribed angle is equal in degrees to one-half its intercepted arc.

If arc $AC = 100°$, then angle $ABC = 50°$.



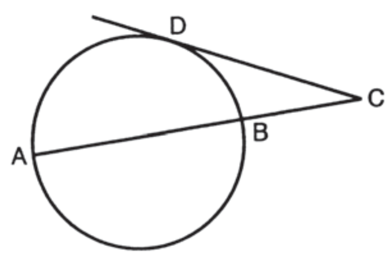
C. An angle formed by two chords intersecting in a circle is equal in degrees to one-half the sum of its intercepted arcs.

If arc $AD = 30°$ and arc $CB = 120°$, then angle $AED = 75°$.



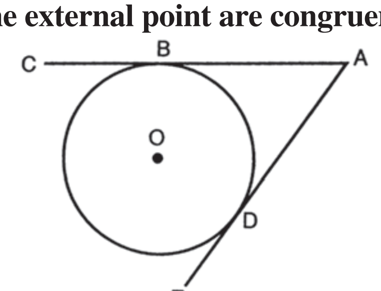
D. An angle outside the circle formed by two secants, a secant and a tangent, or two tangents is equal in degrees to one-half the difference of its intercepted arcs.

If arc $AD = 120°$ and arc $BD = 30°$, then angle $C = 45°$.



E. Two tangent segments drawn to a circle from the same external point are congruent.

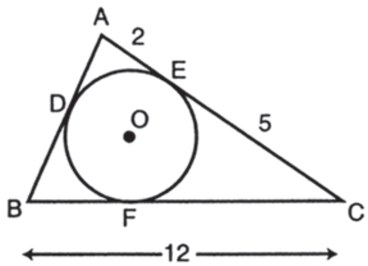
If $\overline{AC}$ and $\overline{AE}$ are tangent to circle O at B and D, then $AB \cong AD$.



Exercise 8

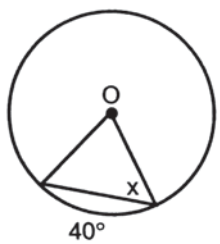
Work out each problem. Circle the letter that appears before your answer.

1. If circle O is inscribed in triangle ABC, find the length of side AB.



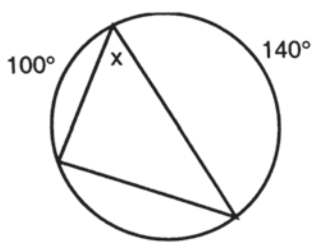
(A) 12
(B) 14
(C) 9
(D) 10
(E) 7

2. Find angle x.



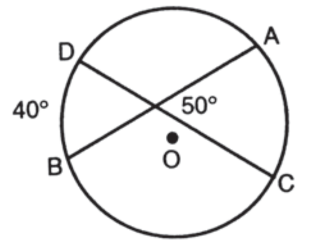
(A) 40°
(B) 20°
(C) 50°
(D) 70°
(E) 80°

3. Find angle x.



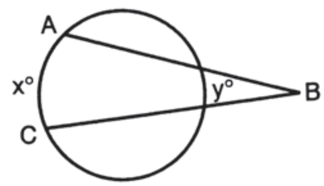
(A) 120°
(B) 50°
(C) 70°
(D) 40°
(E) 60°

4. Find the number of degrees in arc AC.



(A) 60
(B) 50
(C) 25
(D) 100
(E) 20

5. The number of degrees in angle ABC is



(A) $\frac{1}{2}y$

(B) y

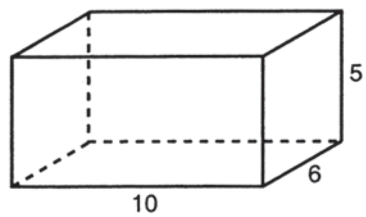
(C) $\frac{1}{2}x$

(D) $\frac{1}{2}(x-y)$

(E) $\frac{1}{2}(x+y)$

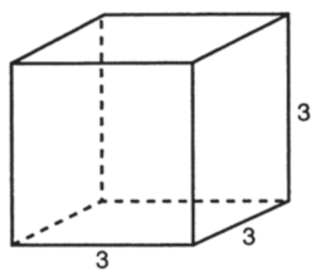
9. VOLUMES

A. The volume of a rectangular solid is equal to the product of its length, width, and height.



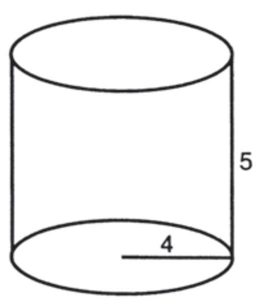
$V = (10)(6)(5) = 300$

B. The volume of a cube is equal to the cube of an edge, since the length, width, and height are all equal.



$V = (3)^3 = 27$

C. The volume of a cylinder is equal to π times the square of the radius of the base times the height.



$V = \pi\ (4)^2\ (5) = 80\pi$

Exercise 9

Work out each problem. Circle the letter that appears before your answer.

1. The surface area of a cube is 96 square feet. How many cubic feet are there in the volume of the cube?
   (A) 16
   (B) 4
   (C) 12
   (D) 64
   (E) 32

2. A cylindrical pail has a radius of 7 inches and a height of 10 inches. Approximately how many gallons will the pail hold if there are 231 cubic inches to a gallon? (Use $\pi = \frac{22}{7}$)
   (A) .9
   (B) 4.2
   (C) 6.7
   (D) 5.1
   (E) 4.8

3. Water is poured into a cylindrical tank at the rate of 9 cubic inches a minute. How many minutes will it take to fill the tank if its radius is 3 inches and its height is 14 inches? (Use $\pi = \frac{22}{7}$)
   (A) $14\frac{2}{3}$
   (B) 44
   (C) 30
   (D) $27\frac{2}{9}$
   (E) 35

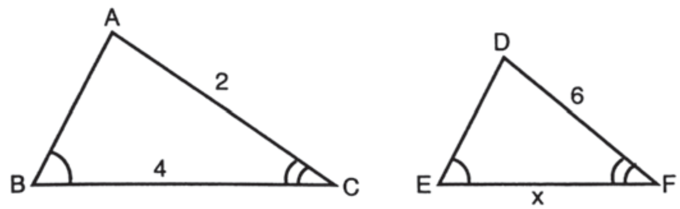
4. A rectangular tank 10 inches by 8 inches by 4 inches is filled with water. If the water is to be transferred to smaller tanks in the form of cubes 4 inches on a side, how many of these tanks are needed?
   (A) 4
   (B) 5
   (C) 6
   (D) 7
   (E) 8

5. The base of a rectangular tank is 6 feet by 5 feet and its height is 16 inches. Find the number of cubic feet of water in the tank when it is $\frac{5}{8}$ full.
   (A) 25
   (B) 40
   (C) 480
   (D) 768
   (E) 300

10. SIMILAR POLYGONS

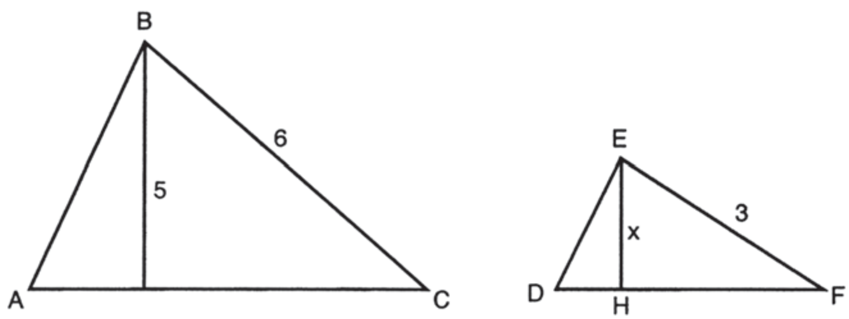
A. Corresponding angles of similar polygons are congruent.

B. Corresponding sides of similar polygons are in proportion.



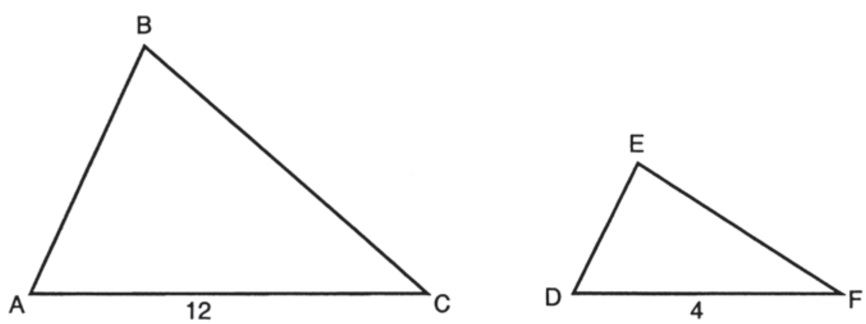
If triangle *ABC* is similar to triangle *DEF* and the sides and angles are given as marked, then *EF* must be equal to 12 as the ratio of corresponding sides is 2 : 6 or 1 : 3.

C. When figures are similar, all ratios between corresponding lines are equal. This includes the ratios of corresponding sides, medians, altitudes, angle bisectors, radii, diameters, perimeters, and circumferences. The ratio is referred to as the linear ratio or ratio of similitude.



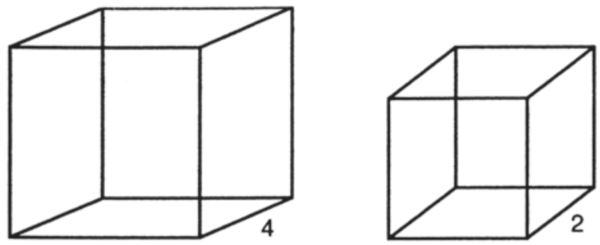
If triangle *ABC* is similar to triangle *DEF* and the segments are given as marked, then *EH* is equal to 2.5 because the linear ratio is 6 : 3 or 2 : 1.

D. When figures are similar, the ratio of their areas is equal to the square of the linear ratio.



If triangle *ABC* is similar to triangle *DEF*, the area of triangle *ABC* will be 9 times as great as the area of triangle *DEF*. The linear ratio is 12 : 4 or 3 : 1. The area ratio will be the square of this or 9 : 1. If the area of triangle *ABC* had been given as 27, the area of triangle *DEF* would be 3.

E. When figures are similar, the ratio of their volumes is equal to the cube of their linear ratio.



The volume of the larger cube is 8 times the volume of the smaller cube. The ratio of sides is 4 : 2 or 2 : 1. The ratio of areas would be 4 : 1. The ratio of volumes would be 8 : 1.

Exercise 10

Work out each problem. Circle the letter that appears before your answer.

1. If the area of a circle of radius x is 5π, find the area of a circle of radius $3x$.
   (A) 10π
   (B) 15π
   (C) 20π
   (D) 30π
   (E) 45π

2. If the length and width of a rectangle are each doubled, the area is increased by
   (A) 50%
   (B) 100%
   (C) 200%
   (D) 300%
   (E) 400%

3. The area of one circle is 9 times as great as the area of another. If the radius of the smaller circle is 3, find the radius of the larger circle.
   (A) 9
   (B) 12
   (C) 18
   (D) 24
   (E) 27

4. If the radius of a circle is doubled, then
   (A) the circumference and area are both doubled
   (B) the circumference is doubled and the area is multiplied by 4
   (C) the circumference is multiplied by 4 and the area is doubled
   (D) the circumference and area are each multiplied by 4
   (E) the circumference stays the same and the area is doubled

5. The volumes of two similar solids are 250 and 128. If a dimension of the larger solid is 25, find the corresponding side of the smaller solid.
   (A) 12.8
   (B) 15
   (C) 20
   (D) 40
   (E) cannot be determined

RETEST

Work out each problem. Circle the letter that appears before your answer.

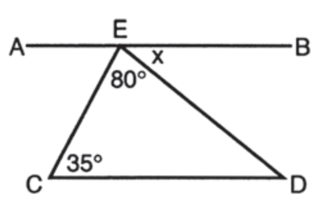
1. The area of a trapezoid whose bases are 10 and 12 and whose altitude is 3 is
   (A) 66
   (B) 11
   (C) 33
   (D) 25
   (E) $16\frac{1}{2}$

2. The circumference of a circle whose area is 16π is
   (A) 8π
   (B) 4π
   (C) 16π
   (D) 8
   (E) 16

3. Find the perimeter of a square whose diagonal is 8.
   (A) 32
   (B) 16
   (C) $32\sqrt{2}$
   (D) $16\sqrt{2}$
   (E) $32\sqrt{3}$

4. The length of the line segment joining the point $A(4, -3)$ to $B(7, -7)$ is
   (A) $\sqrt{221}$
   (B) $\sqrt{185}$
   (C) 7
   (D) $6\frac{1}{2}$
   (E) 5

5. Find angle x if $\overline{AB}$ is parallel to $\overline{CD}$.

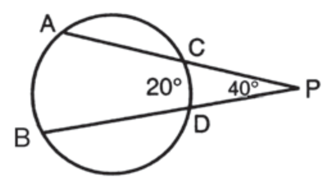


   (A) 35°
   (B) 80°
   (C) 245°
   (D) 65°
   (E) 55°

6. In triangle ABC, the angles are in a ratio of 1 : 1 : 2. The largest angle of the triangle is
   (A) 45°
   (B) 60°
   (C) 90°
   (D) 120°
   (E) 100°

7. Find the number of degrees in each angle of a regular pentagon.
   (A) 72
   (B) 108
   (C) 60
   (D) 180
   (E) 120

8. Find the number of degrees in arc AB.



   (A) 80
   (B) 20
   (C) 60
   (D) 100
   (E) 90

9. Find the edge, in inches, of a cube whose volume is equal to the volume of a rectangular solid 2 in. by 6 in. by 18 in.
   (A) 4
   (B) 8
   (C) 5
   (D) 6
   (E) 7

10. If the volume of one cube is 8 times as great as another, then the ratio of the area of a face of the larger cube to the area of a face of the smaller cube is
   (A) 2 : 1
   (B) 4 : 1
   (C) $\sqrt{2}:1$
   (D) 8 : 1
   (E) $2\sqrt{2}:1$

SOLUTIONS TO PRACTICE EXERCISES

Diagnostic Test

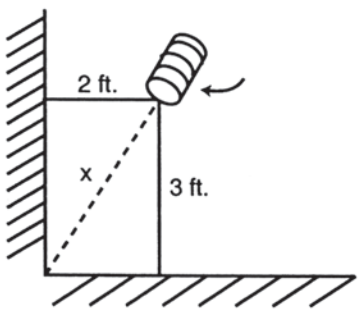
1. (A) Represent the angles as $5x$, $6x$, and $7x$. They must add up to 180°.

$$18x = 180$$
$$x = 10$$

The angles are 50°, 60°, and 70°, an acute triangle.

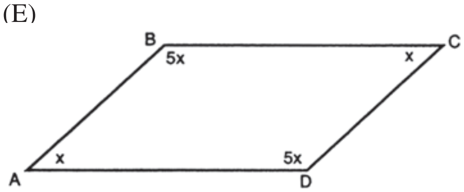
2. (B) The area of a circle is πr^2. The area of a circle with radius x is πx^2, which equals 4. The area of a circle with radius $3x$ is $\pi\,(3x)^2 = 9\pi x^2 = 9 \cdot 4 = 36$.

3. (D)



$$2^2 + 3^2 = x^2$$
$$4 + 9 = x^2$$
$$13 = x^2$$
$$\sqrt{13} = x$$

4. (E)



The sum of the angles in a parallelogram is 360°.

$$12x = 360°$$
$$x = 30°$$

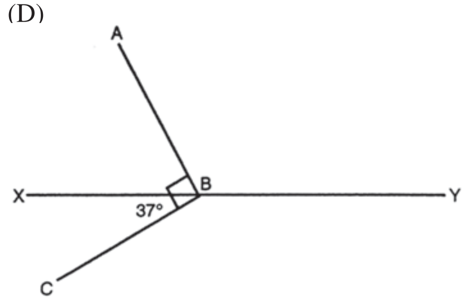
Angle $B = 5x = 5 \cdot 30° = 150°$

5. (A) The volume of a rectangular box is the product of its length, width, and height. Since the height is 18 inches, or $1\frac{1}{2}$ feet, and the length and width of the square base are the same, we have

$$x \cdot x \cdot 1\frac{1}{2} = 24$$
$$x^2 = 16$$
$$x = 4$$

6. (D) The remaining degrees of the triangle are $180 - x$. Since the triangle is isosceles, the remaining angles are equal, each $\frac{180 - x}{2} = 90 - \frac{x}{2}$.

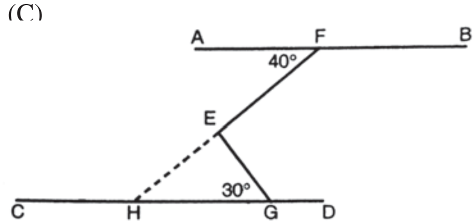
7. (D)



Angle $ABX = 90° - 37° = 53°$

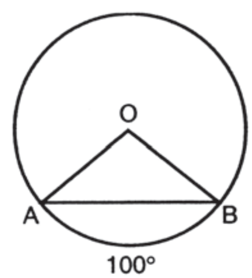
Angle $ABY = 180° - 53° = 127°$

8. (C)



Extend $\overline{FE}$ to H. $\angle EHG = \angle AFE = 40°$. $\angle HEG$ must equal 110° because there are 180° in a triangle. Since $\angle FEG$ is the supplement of $\angle HEG$, $\angle FEG = 70°$.

9. (C)



Angle O is a central angle equal to its arc, 100°. This leaves 80° for the other two angles. Since the triangle is isosceles (because the legs are both radii and therefore equal), angle ABO is 40°.

10. (A) $d = \sqrt{\left(5-(3)\right)^2 + \left(-5-1\right)^2}$

$$= \sqrt{(8)^2 + (-6)^2} = \sqrt{64 + 36}$$
$$= \sqrt{100} = 10$$

Exercise 1

1. (C) Find the area in square feet and then convert to square yards by dividing by 9. Remember there are 9 square feet in one square yard.

   $(18 \cdot 20) \div 9 = 360 \div 9 = 40$ square yards

2. (B) Area of parallelogram = $b \cdot h$

   $$\begin{aligned}(x+7)(x-7) &= 15\\ x^2-49 &= 15\\ x^2 &= 64\\ x &= 8\end{aligned}$$

   Base $= x+7=15$

3. (B) Area of triangle $= \frac{1}{2} \cdot b \cdot h$

   Using one leg as base and the other as altitude, the area is $\frac{1}{2} \cdot 6 \cdot 8 = 24$. Using the hypotenuse as base and the altitude to the hypotenuse will give the same area.

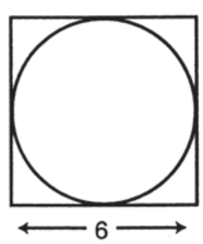
   $$\begin{aligned}\frac{1}{2}\cdot 10\cdot h &= 24\\ 5h &= 24\end{aligned}$$

   $h = 4.8 \therefore \frac{1}{2}\cdot 10\cdot 4.8 = 24$

4. (E) Area of rhombus $= \frac{1}{2}$ · product of diagonals

   Area $= \frac{1}{2}(4x)(6x) = \frac{1}{2}(24x^2) = 12x^2$

5. (C)



   radius of circle = 3

   Area $= \pi r^2 = 9\pi$

Exercise 2

1. (A) Area of equilateral triangle $= \frac{s^2}{4}\sqrt{3}$

   Therefore, $\frac{s^2}{4}$ must equal 16

   $s^2 = 64$

   $s = 8$

   Perimeter is $8 + 8 + 8 = 24$

2. (B) In 4 hours the hour hand moves through one-third of the circumference of the clock.

   $C = 2\pi r = 2\pi(3) = 6\pi$

   $\frac{1}{3}\cdot 6\pi = 2\pi$

3. (D) Compare $2\pi r$ with $2\pi(r + 3)$.

   $2\pi(r + 3) = 2\pi r + 6\pi$

   Circumference was increased by 6π. Trying this with a numerical value for r will give the same result.

4. (E) In one revolution, the distance covered is equal to the circumference.

   $C = 2\pi r = 2\pi(18) = 36\pi$ inches

   To change this to feet, divide by 12.

   $\frac{36\pi}{12} = 3\pi$ feet

   In 20 revolutions, the wheel will cover $20(3\pi)$ or 60π feet.

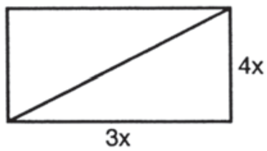
5. (D) Area of rectangle $= b \cdot h = 36$

   Area of square $= s^2 = 36$

   Therefore, $s = 6$ and perimeter $= 24$

Exercise 3

1. (A)

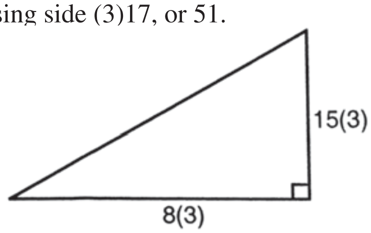


$$14x = 140$$
$$x = 10$$

The rectangle is 30′ by 40′. This is a 3, 4, 5 right triangle, so the diagonal is 50′.

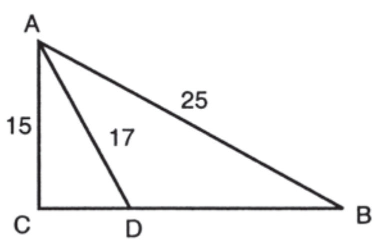
2. (C) The altitude in an equilateral triangle is always $\frac{1}{2}$ side $\cdot \sqrt{3}$.

3. (D) This is an 8, 15, 17 triangle, making the missing side (3)17, or 51.



4. (A) The diagonal in a square is equal to the side times $\sqrt{2}$. Therefore, the side is 6 and the perimeter is 24.

5. (C)



Triangle *ABC* is a 3, 4, 5 triangle with all sides multiplied by 5. Therefore *CB* = 20. Triangle *ACD* is an 8, 15, 17 triangle. Therefore *CD*= 8. *CB* – *CD* = *DB* = 12.

Exercise 4

1. (A) Find the midpoint of *AB* by averaging the *x* coordinates and averaging the *y* coordinates. $\left(\frac{6+2}{2}, \frac{2+6}{2}\right) = (4,4)$

2. (C) *O* is the midpoint of *AB*.

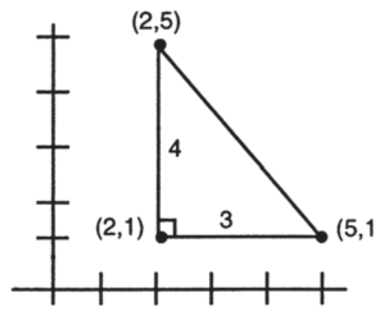
$\frac{x+4}{2} = 2$ $\quad$ $x + 4 = 4,\ x = 0$

$\frac{y+6}{2} = 1$ $\quad$ $y + 6 = 2,\ y = -4$

A is the point (0, –4).

3. (A) $d = \sqrt{(8-4)^2 + (6-3)^2} = \sqrt{4^2 + 3^2}$
$= \sqrt{16+9} = \sqrt{25} = 5$

4. (D) Sketch the triangle and you will see it is a right triangle with legs of 4 and 3.



$$\text{Area} = \frac{1}{2} \cdot b \cdot h = \frac{1}{2} \cdot 4 \cdot 3 = 6$$

5. (A) Area of a circle = πr^2

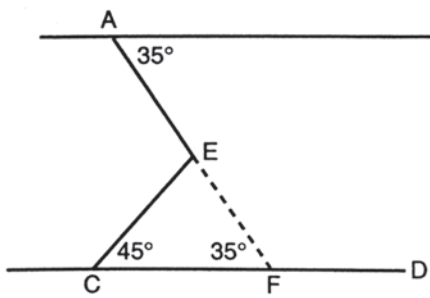
$\pi r^2 = 16\pi \qquad r = 4$

The point (4, 4) lies at a distance of $\sqrt{(4-0)^2 + (4-0)^2} = \sqrt{32}$ units from (0, 0). All the other points lie 4 units from (0, 0).

Exercise 5

1. (A) Angle B = Angle C because of alternate interior angles. Then Angle C = Angle D for the same reason. Therefore, Angle $D = 30°$.

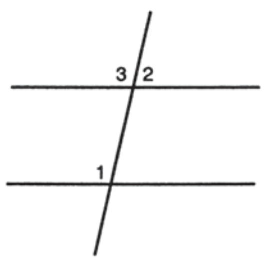
2. (D)



Extend $\overline{AE}$ to F. $\angle A = \angle EFC$

$\angle CEF$ must equal 100° because there are 180° in a triangle. $\angle AEC$ is supplementary to $\angle CEF$. $\angle AEC = 80°$

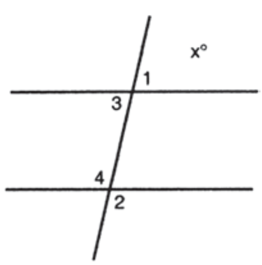
3. (E)



$$\angle 1 = \angle 3$$
$$\angle 2 + \angle 3 = 180°$$
$$\angle 2 = 50°$$

4. (C) Since $\angle BEG$ and $\angle EGD$ add to 180°, halves of these angles must add to 90°. Triangle EFG contains 180°, leaving 90° for $\angle EFG$.

5. (C)



$$\angle 1 = \angle 3$$
$$\angle 2 = \angle 4$$
$$\angle 1 + \angle 2 = \angle 3 + \angle 4$$

But $\angle 3 + \angle 4 = 180°$. Therefore, $\angle 1 + \angle 2 = 180°$

Exercise 6

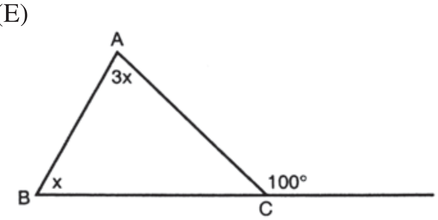
1. (D) Represent the angles as x, $5x$, and $6x$. They must add to 180°.

$$12x = 180$$
$$x = 15$$

The angles are 15°, 75°, and 90°. Thus, it is a right triangle.

2. (D) There are 130° left to be split evenly between the base angles (the base angles must be equal). Each one must be 65°.

3. (E)

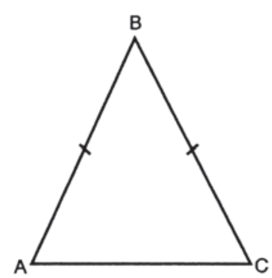


The exterior angle is equal to the sum of the two remote interior angles.

$$4x = 100$$
$$x = 25$$
$$\text{Angle } A = 3x = 75°$$

4. (D) The other base angle is also x. These two base angles add to $2x$. The remaining degrees of the triangle, or $180 - 2x$, are in the vertex angle.

5. (E)



$$\angle A = \angle C$$
$$4x - 30 = 2x + 10$$
$$2x = 40$$
$$x = 20$$

$\angle A$ and $\angle C$ are each 50°, leaving 80° for $\angle B$.

Exercise 7

1. (C) A hexagon has 6 sides. Sum = $(n - 2)\ 180 = 4(180) = 720$

2. (D) Opposite sides of a parallelogram are congruent, so $AB = CD$.

$$\begin{aligned} x+4 &= 2x-16 \\ 20 &= x \\ AD = BC = x-6 &= 14 \end{aligned}$$

3. (B) $AB = CD$

$$\begin{aligned} x+8 &= 4x-4 \\ 12 &= 3x \\ x &= 4 \end{aligned}$$

$$AB = 12 \qquad BC = 12 \qquad CD = 12$$

If all sides are congruent, it must be a rhombus. Additional properties would be needed to make it a square.

4. (B) A rhombus has 4 sides. Sum = $(n - 2)\ 180 = 2(180) = 360$

5. (C) Rectangles and rhombuses are both types of parallelograms but do not share the same special properties. A square is both a rectangle and a rhombus with *added* properties.

Exercise 8

1. (C) Tangent segments drawn to a circle from the same external point are congruent. If $CE = 5$, then $CF = 5$, leaving 7 for BF. Therefore BD is also 7. If $AE = 2$, then $AD = 2$.

$BD + DA = BA = 9$

2. (D) Angle O is a central angle equal to its arc, 40°. This leaves 140° for the other two angles. Since the triangle is isosceles, because the legs are equal radii, each angle is 70°.

3. (E) The remaining arc is 120°. The inscribed angle x is $\frac{1}{2}$ its intercepted arc.

4. (A)

$$\begin{aligned} 50° &= \frac{1}{2}\left(40° + AC\right) \\ 100° &= 40° + AC \\ 60° &= AC \end{aligned}$$

5. (D) An angle outside the circle is $\frac{1}{2}$ the difference of its intercepted arcs.

Exercise 9

1. (D) There are 6 equal squares in the surface area of a cube. Each square will have an area of $\frac{96}{6}$ or 16. Each edge is 4.
   $V = e^3 = 4^3 = 64$
2. (C) $V = \pi r^2 h = \frac{22}{7} \cdot 49 \cdot 10 = 1540$ cubic inches

   Divide by 231 to find gallons.
3. (B) $V = \pi r^2 h = \frac{22}{7} \cdot 9 \cdot 14 = 396$ cubic inches

   Divide by 9 to find minutes.
4. (B) $V = l \cdot w \cdot h = 10 \cdot 8 \cdot 4 = 320$ cubic inches

   Each small cube $= 4^3 = 64$ cubic inches. Therefore it will require 5 cubes.
5. (A) Change 16 inches to $1\frac{1}{3}$ feet.
   $V = 6 \cdot 5 \cdot 1\frac{1}{3} = 40$ cubic feet when full.
   $\frac{5}{8} \cdot 40 = 25$

Exercise 10

1. (E) If the radius is multiplied by 3, the area is multiplied by 3^2 or 9.
2. (D) If the dimensions are all doubled, the area is multiplied by 2^2 or 4. If the new area is 4 times as great as the original area, is has been *increased* by 300%.
3. (A) If the area ratio is 9 : 1, the linear ratio is 3 : 1. Therefore, the larger radius is 3 times the smaller radius.
4. (B) Ratio of circumferences is the same as ratio of radii, but the area ratio is the square of this.
5. (C) We must take the cube root of the volume ratio to find the linear ratio. This becomes much easier if you simplify the ratio first.
   $\frac{250}{128} = \frac{125}{64}$
   The linear ratio is then 5 : 4.
   $$\frac{5}{4} = \frac{25}{x}$$
   $$5x = 100$$
   $$x = 20$$

Retest

1. (C) Area of trapezoid = $\frac{1}{2}h(b_1 + b_2)$

$\text{Area} = \frac{1}{2} \cdot 3(10 + 12) = 33$

2. (A) Area of circle = $\pi r^2 = 16\pi$

Therefore, $r^2 = 16$ or $r = 4$

Circumference of circle = $2\pi r = 2\pi\ (4) = 8\pi$

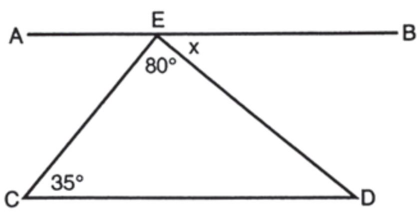
3. (D) The side of a square is equal to the diagonal times $\frac{\sqrt{2}}{2}$. Therefore, the side is $4\sqrt{2}$ and the perimeter is $16\sqrt{2}$.

4. (E) $d = \sqrt{(7-4)^2 + (-7-(-3))^2}$

$= \sqrt{(3)^2 + (-4)^2} = \sqrt{9+16}$

$= \sqrt{25} = 5$

5. (D)



$\angle CDE$ must equal 65° because there are 180° in a triangle. Since $\overline{AB}$ is parallel to $\overline{CD}$, $\angle x = \angle CDE = 65°$.

6. (C) Represent the angles as x, x, and $2x$. They must add to 180°.

$4x = 180$

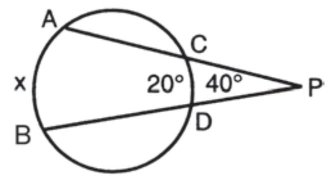
$x = 45$

Therefore, the largest angle is $2x = 2(45°) = 90°$.

7. (B) A pentagon has 5 sides. Sum $(n - 2)180 = 3(180) = 540°$

In a regular pentagon, all the angles are equal. Therefore, each angle = $\frac{540}{5} = 108°$.

8. (D)



An angle outside the circle is $\frac{1}{2}$ the difference of its intercepted arcs.

$40 = \frac{1}{2}(x - 20)$

$80 = x - 20$

$100 = x$

9. (D) $V = l \cdot w \cdot h = 2 \cdot 6 \cdot 18 = 216$

The volume of a cube is equal to the cube of an edge.

$V = e^3$

$216 = e^3$

$6 = e$

10. (B) If the volume ratio is 8 : 1, the linear ratio is 2 : 1, and the area ratio is the square of this, or 4:1.