Inequalities

14

DIAGNOSTIC TEST

Directions: Work out each problem. Circle the letter that appears before your answer.

Answers are at the end of the chapter.

1. If $4x < 6$, then

   (A) $x = 1.5$

   (B) $x < \frac{2}{3}$

   (C) $x > \frac{2}{3}$

   (D) $x < \frac{3}{2}$

   (E) $x > \frac{3}{2}$

2. a and b are positive numbers. If $a = b$ and $c > d$, then

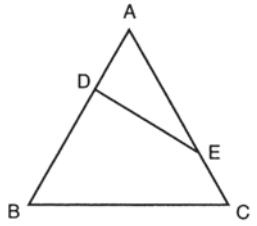
   (A) $a + c < b + d$
   (B) $a + c > b + d$
   (C) $a - c > b - d$
   (D) $ac < bd$
   (E) $a + c < b - d$

3. Which value of x will make the following expression true?

$$\frac{3}{5} < \frac{x}{10} < \frac{4}{5}$$

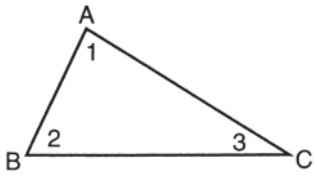
   (A) 5
   (B) 6
   (C) 7
   (D) 8
   (E) 9

4. In triangle ABC, $AB = AC$ and $EC < DB$. Then



   (A) $DB < AE$
   (B) $DB < AD$
   (C) $AD > AE$
   (D) $AD < AE$
   (E) $AD > EC$

5. In triangle ABC, $\angle 1 > \angle 2$ and $\angle 2 > \angle 3$. Then



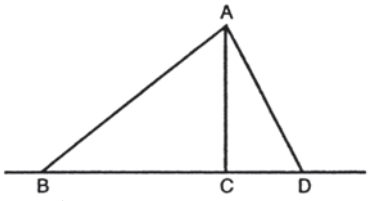
   (A) $AC < AB$
   (B) $AC > BC$
   (C) $BC > AC$
   (D) $BC < AB$
   (E) $\angle 3 > \angle 1$

6. If point C lies between A and B on line segment AB, which of the following is always true?

   (A) $AC = CB$
   (B) $AC > CB$
   (C) $CB > AC$
   (D) $AB < AC + CB$
   (E) $AB = CB + AC$

7. If AC is perpendicular to BD, which of the following is always true?

   I. $AC = BC$
   II. $AC < AB$
   III. $AB > AD$



   (A) I only
   (B) II and III only
   (C) II only
   (D) III only
   (E) I and II only

8. If $x < 0$ and $y > 0$, which of the following is always true?

   (A) $x + y > 0$
   (B) $x + y < 0$
   (C) $y - x < 0$
   (D) $x - y < 0$
   (E) $2x > y$

9. In triangle ABC, BC is extended to D. If $\angle A = 50°$ and $\angle ACD = 120°$, then

   (A) $BC > AB$
   (B) $AC > AB$
   (C) $BC > AC$
   (D) $AB > AC$
   (E) $\angle B < \angle A$

10. In right triangle ABC, $\angle A < \angle B$ and $\angle B < \angle C$. Then

   (A) $\angle A > 45°$
   (B) $\angle B = 90°$
   (C) $\angle B > 90°$
   (D) $\angle C = 90°$
   (E) $\angle C > 90°$

1. ALGEBRAIC INEQUALITIES

Algebraic inequality statements are solved in the same manner as equations. However, do not forget that whenever you multiply or divide by a negative number, the order of the inequality, that is, the inequality symbol must be reversed. In reading the inequality symbol, remember that it points to the smaller quantity. $a < b$ is read a is less than b. $a > b$ is read a is greater than b.

Example:

Solve for x: $12 - 4x < 8$

Solution:

Add –12 to each side.
$-4x < -4$
Divide by –4, remembering to reverse the inequality sign.
$x > 1$

Example:

$6x + 5 > 7x + 10$

Solution:

Collect all the terms containing x on the left side of the equation and all numerical terms on the right. As with equations, remember that if a term comes from one side of the inequality to the other, that term changes sign.
$-x > 5$
Divide (or multiply) by –1.
$x < -5$

Exercise 1

Work out each problem. Circle the letter that appears before your answer.

1. Solve for x: $8x < 5(2x + 4)$
   (A) $x > -10$
   (B) $x < -10$
   (C) $x > 10$
   (D) $x < 10$
   (E) $x < 18$

2. Solve for x: $6x + 2 - 8x < 14$
   (A) $x = 6$
   (B) $x = -6$
   (C) $x > -6$
   (D) $x < -6$
   (E) $x > 6$

3. A number increased by 10 is greater than 50. What numbers satisfy this condition?
   (A) $x > 60$
   (B) $x < 60$
   (C) $x > -40$
   (D) $x < 40$
   (E) $x > 40$

4. Solve for x: $-.4x < 4$
   (A) $x > -10$
   (B) $x > 10$
   (C) $x < 8$
   (D) $x < -10$
   (E) $x < 36$

5. Solve for x: $.03n > -.18$
   (A) $n < -.6$
   (B) $n > .6$
   (C) $n > 6$
   (D) $n > -6$
   (E) $n < -6$

6. Solve for b: $15b < 10$
   (A) $b < \frac{3}{2}$
   (B) $b > \frac{3}{2}$
   (C) $b < -\frac{3}{2}$
   (D) $b < \frac{2}{3}$
   (E) $b > \frac{2}{3}$

7. If $x^2 < 4$, then
   (A) $x > 2$
   (B) $x < 2$
   (C) $x > -2$
   (D) $-2 < x < 2$
   (E) $-2 \le x \le 2$

8. Solve for n: $n + 4.3 < 2.7$
   (A) $n > 1.6$
   (B) $n > -1.6$
   (C) $n < 1.6$
   (D) $n < -1.6$
   (E) $n = 1.6$

9. If $x < 0$ and $y < 0$, which of the following is always true?
   (A) $x + y > 0$
   (B) $xy < 0$
   (C) $x - y > 0$
   (D) $x + y < 0$
   (E) $x = y$

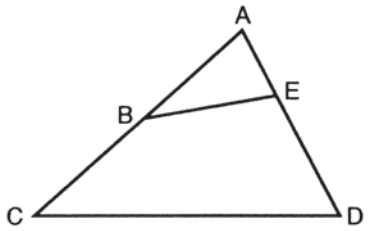
10. If $x < 0$ and $y > 0$, which of the following will always be greater than 0?
    (A) $x + y$
    (B) $x - y$
    (C) $\frac{x}{y}$
    (D) xy
    (E) $-2x$

2. GEOMETRIC INEQUALITIES

In working with geometric inequalities, certain postulates and theorems should be reviewed.

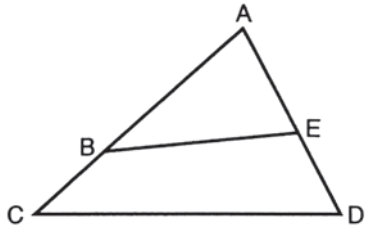
A. If unequal quantities are added to unequal quantities of the same order, the sums are unequal in the same order.

If $AB > AE$ and
(+) $BC > ED$ then
$AC > AD$



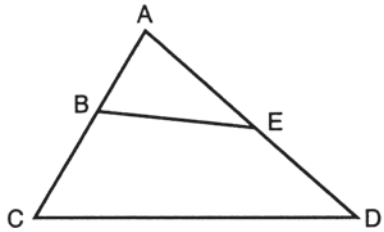
B. If equal quantities are added to unequal quantities, the sums are unequal in the same order.

$AB > AE$ and
(+) $BC = ED$ then
$AC > AD$



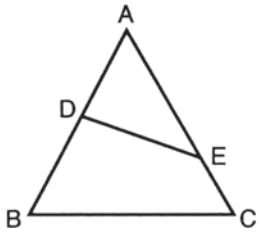
C. If equal quantities are subtracted from unequal quantities, the differences are unequal in the same order.

If $AC > AD$ and
(−) $BC = ED$ then
$AB > AE$



D. If unequal quantities are subtracted from equal quantities, the results are unequal in the *opposite* order.

$AB = AC$
(−) $AD < AE$
$DB > EC$



E. Doubles of unequals are unequal in the same order.

M is the midpoint of AB

N is the midpoint of CD

$AM > CN$

Therefore, $AB > CD$

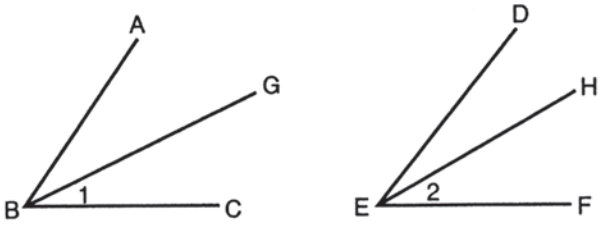
F. Halves of unequals are unequal in the same order.

$\angle ABC > \angle DEF$

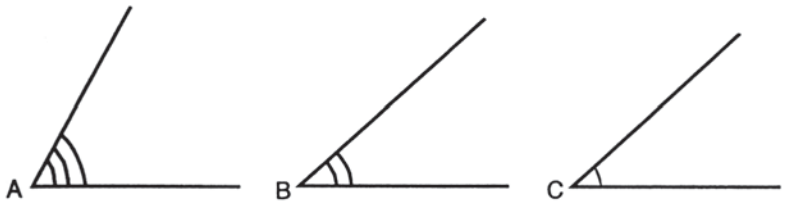
$\overline{BG}$ bisects $\angle ABC$

$\overline{EH}$ bisects $\angle DEF$

Therefore, $\angle 1 > \angle 2$



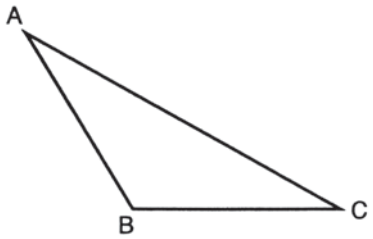
G. If the first of three quantities is greater than the second, and the second is greater than the third, then the first is greater than the third.



If $\angle A > \angle B$ and $\angle B > \angle C$, then $\angle A > \angle C$.

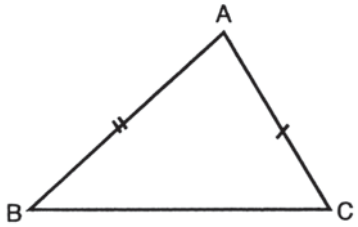
H. The sum of two sides of a triangle must be greater than the third side.

AB + BC > AC



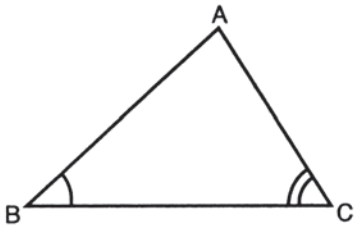
I. If two sides of a triangle are unequal, the angles opposite are unequal, with the larger angle opposite the larger side.

If $AB > AC$, then $\angle C > \angle B$.



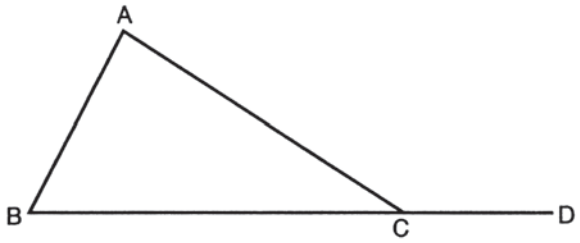
J. If two angles of a triangle are unequal, the sides opposite these angles are unequal, with the larger side opposite the larger angle.

If $\angle C > \angle B$, then AB > AC.



K. An exterior angle of a triangle is greater than either remote interior angle.

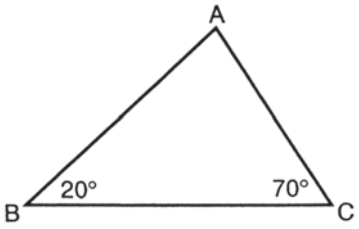
$\angle ACD > \angle B$ and $\angle ACD > \angle A$



Exercise 2

Work out each problem. Circle the letter that appears before your answer.

1. Which of the following statements is true regarding triangle *ABC?*

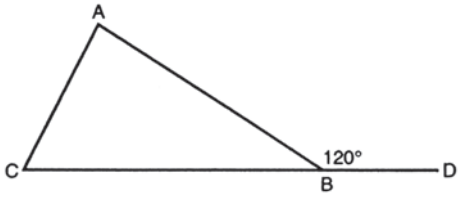


(A) $AC > AB$
(B) $AB > BC$
(C) $AC > BC$
(D) $BC > AB$
(E) $BC > AB + AC$

2. In triangle RST, $RS = ST$. If P is any point on RS, which of the following statements is always true?

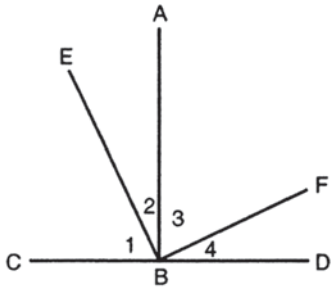
(A) $PT < PR$
(B) $PT > PR$
(C) $PT = PR$
(D) $PT = \frac{1}{2} PR$
(E) $PT \leq PR$

3. If $\angle A > \angle C$ and $\angle ABD = 120°$, then



(A) $AC < AB$
(B) $BC < AB$
(C) $\angle C > \angle ABC$
(D) $BC > AC$
(E) $\angle ABC > \angle A$

4. If $AB \perp CD$ and $\angle 1 > \angle 4$, then



(A) $\angle 1 > \angle 2$
(B) $\angle 4 > \angle 3$
(C) $\angle 2 > \angle 3$
(D) $\angle 2 < \angle 3$
(E) $\angle 2 < \angle 4$

5. Which of the following sets of numbers could be the sides of a triangle?

(A) 1, 2, 3
(B) 2, 2, 4
(C) 3, 3, 6
(D) 1, 1.5, 2
(E) 5, 6, 12

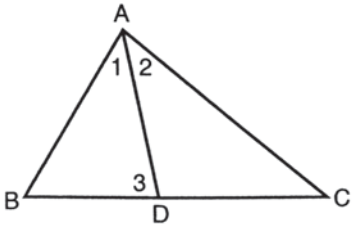
RETEST

Work out each problem. Circle the letter that appears before your answer.

1. If $2x > -5$, then
   (A) $x > \frac{5}{2}$
   (B) $x > -\frac{5}{2}$
   (C) $x > -\frac{2}{5}$
   (D) $x < \frac{5}{2}$
   (E) $x < -\frac{5}{2}$

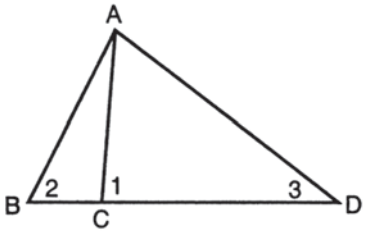
2. $m, n > 0$. If $m = n$ and $p < q$, then
   (A) $m - p < n - q$
   (B) $p - m > q - n$
   (C) $m - p > n - q$
   (D) $mp > nq$
   (E) $m + q < n + p$

3. If $\angle 3 > \angle 2$ and $\angle 1 = \angle 2$, then



   (A) $AB > BD$
   (B) $AB < BD$
   (C) $DC = BD$
   (D) $AD > BD$
   (E) $AB < AC$

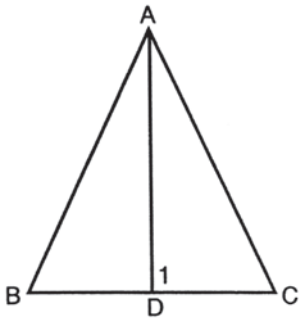
4. If $\angle 1 > \angle 2$ and $\angle 2 > \angle 3$, then



   (A) $AB > AD$
   (B) $AC > AD$
   (C) $AC < CD$
   (D) $AD > AC$
   (E) $AB > BC$

5. If $\frac{x}{2} > 6$, then
   (A) $x > 3$
   (B) $x < 3$
   (C) $x > 12$
   (D) $x < 12$
   (E) $x > -12$

6. If $AB = AC$ and $\angle 1 > \angle B$, then



   (A) $\angle B > \angle C$
   (B) $\angle 1 > \angle C$
   (C) $BD > AD$
   (D) $AB > AD$
   (E) $\angle ADC > \angle ADB$

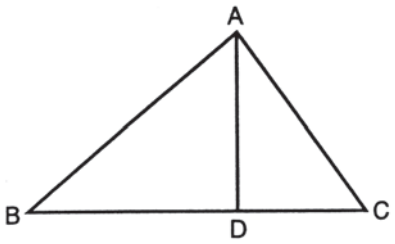
7. Which of the following sets of numbers may be used as the sides of a triangle?
   (A) 7, 8, 9
   (B) 3, 5, 8
   (C) 8, 5, 2
   (D) 3, 10, 6
   (E) 4, 5, 10

8. In isosceles triangle RST, $RS = ST$. If A is the midpoint of RS and B is the midpoint of ST, then
   (A) $SA > ST$
   (B) $BT > BS$
   (C) $BT = SA$
   (D) $SR > RT$
   (E) $RT > ST$

9. If $x > 0$ and $y < 0$, which of the following is always true?

   (A) $x - y > y - x$
   (B) $x + y > 0$
   (C) $xy > 0$
   (D) $y > x$
   (E) $x - y < 0$

10. In triangle ABC, AD is the altitude to BC. Then



   (A) $AD > DC$
   (B) $AD < BD$
   (C) $AD > AC$
   (D) $BD > DC$
   (E) $AB > BD$

SOLUTIONS TO PRACTICE EXERCISES

Diagnostic Test

1. (D) $4x < 6$

$$x < \frac{6}{4}$$

Simplify to $x < \frac{3}{2}$

2. (B) If equal quantities are added to unequal quantities, the sums are unequal in the same order.

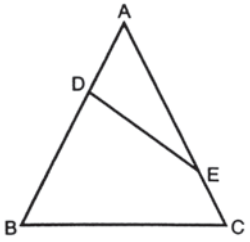
$$\begin{array}{r} c > d \\ (+)\, a = b \\ \hline a + c > b + d \end{array}$$

3. (C) $\frac{3}{5} < \frac{x}{10} < \frac{4}{5}$

Multiply through by 10.

$6 < x < 8$ or x must be between 6 and 8.

4. (D)



If unequal quantities are subtracted from equal quantities, the results are unequal in the opposite order.

$$\begin{array}{r} AC = AB \\ (-)\, EC < DB \\ \hline AE > AD \text{ or } AD < AE \end{array}$$

5. (C) If two angles of a triangle are unequal, the sides opposite these angles are unequal, with the larger side opposite the larger angle.

Since $\angle 1 > \angle 2$, $BC > AC$.

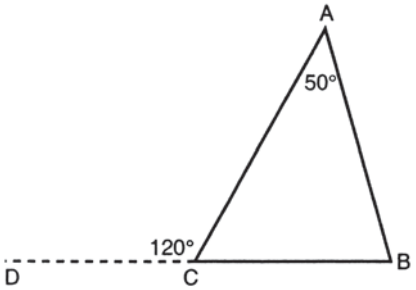
6. (E)

AB = CB + AC

7. (C) In right triangle ACB, the longest side is the hypotenuse AB. Therefore, side AC is less than AB.

8. (D) A positive subtracted from a negative is always negative.

9. (B)



$AB = CB + AC$

$\angle ACB$ is the supplement of $\angle ACD$. Therefore, $\angle ACB = 60°$. $\angle ABC$ must equal 70° because there are 180° in a triangle. Since $\angle ABC$ is the largest angle in the triangle, AC must be the longest side. Therefore, $AC > AB$.

10. (D) In a right triangle, the largest angle is the right angle. Since $\angle C$ is the largest angle, $\angle C = 90°$.

Exercise 1

1. (A) $8x < 10x + 20$
$-2x < 20$
$x > -10$

2. (C) $-2x < 12$
$x > -6$

3. (E) $x + 10 > 50$
$x > 40$

4. (A) $-.4x < 4$

Multiply by 10 to remove decimals.
$-4x < 40$
$x > -10$

5. (D) $.03n > -.18$

Multiply by 100
$3n > -18$
$n > -6$

6. (D) Divide by 15
$$b < \frac{10}{15}$$
Simplify to $b < \frac{2}{3}$

7. (D) x must be less than 2, but can go no lower than –2, as $(-3)^2$ would be greater than 4.

8. (D) $n + 4.3 < 2.7$
Subtract 4.3 from each side.
$n < -1.6$

9. (D) When two negative numbers are added, their sum will be negative.

10. (E) The product of two negative numbers is positive.

Exercise 2

1. (D) Angle A will contain 90°, which is the largest angle of the triangle. The sides from largest to smallest will be *BC*, *AB*, *AC*.

2. (B) Since $\angle SRT = \angle STR$, $\angle SRT$ will have to be greater than $\angle PTR$. Therefore, $PT > PR$ in triangle *PRT*.

3. (D) Angle *ABC* = 60°. Since there are 120° left for $\angle A$ and $\angle C$ together and, also $\angle A > \angle C$, then $\angle A$ must contain more than half of 120° and $\angle C$ must contain less than half of 120°. This makes $\angle A$ the largest angle of the triangle. The sides in order from largest to smallest are *BC*, *AC*, *AB*.

4. (D) $\angle ABC = \angle ABD$ as they are both right angles. If $\angle 1 > \angle 4$, then $\angle 2$ will be less than $\angle 3$ because we are subtracting unequal quantities ($\angle 1$ and $\angle 4$) from equal quantities ($\angle ABC$ and $\angle ABD$).

5. (D) The sum of any two sides (always try the shortest two) must be greater than the third side.

Retest

1. (B) $2x > -5$

$$x > -\frac{5}{2}$$

2. (C) If unequal quantities are subtracted from equal quantities, the differences are unequal in the opposite order.

$$m = n$$
$$(-)\,p < q$$
$$m - p > n - q$$

3. (A) Since $\angle 3 > \angle 2$ and $\angle 1 = \angle 2$, $\angle 3 > \angle 1$. If two angles of a triangle are unequal, the sides opposite these angles are unequal, with the larger side opposite the larger angle. Therefore, $AB > BD$.

4. (D) Since $\angle 1 > \angle 2$ and $\angle 2 > \angle 3$, $\angle 1 > \angle 3$. In triangle ACD side AD is larger than side AC, since AD is opposite the larger angle.

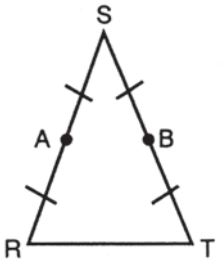
5. (C) $\frac{x}{2} > 6$

$$x > 12$$

6. (B) If two sides of a triangle are equal, the angles opposite them are equal. Therefore $\angle C = \angle B$. Since $\angle 1 > \angle B$, $\angle 1 > \angle C$.

7. (A) The sum of any two sides (always try the shortest two) must be greater than the third side.

8. (C)



$BT = \frac{1}{2}\ ST$ and $SA = \frac{1}{2}\ SR$. Since $ST = SR$, $BT = SA$.

9. (A) A positive minus a negative is always greater than a negative minus a positive.

10. (E) In right triangle ADB, the longest side is the hypotenuse AB. Therefore, $AB > BD$.