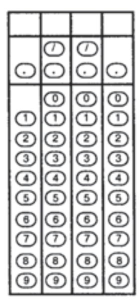
15 Numbers and Operations, Algebra, and Functions

DIAGNOSTIC TEST

Directions: Answer the following 10 questions, limiting your time to 15 minutes. Note that question 1 is a *grid-in* question, in which you provide the numerical solution. (All other questions are in multiple-choice format.)

Answers are at the end of the chapter.

1. The population of Urbanville has always doubled every five years. Urbanville's current population is 25,600. What was its population 20 years ago?



2. Which of the following describes the union of the factors of 15, the factors of 30, and the factors of 75?

   (A) The factors of 15
   (B) The factors of 30
   (C) The factors of 45
   (D) The factors of 75
   (E) None of the above

3. $|-1 - 2| - |5 - 6| - |-3 + 4| =$

   (A) –5
   (B) –3
   (C) 1
   (D) 3
   (E) 5

4. For all $x \neq 0$ and $y \neq 0$, $\dfrac{x^6y^3y}{y^6x^3x}$ is equivalent to:

   (A) $\dfrac{y^2}{x^2}$
   (B) xy
   (C) 1
   (D) $\dfrac{x^2}{y^2}$
   (E) $\dfrac{x^3}{y^3}$

5. If $f(x) = x + 1$, then $\dfrac{1}{f(x)} \times f\left(\dfrac{1}{x}\right) =$

   (A) 1
   (B) $\dfrac{1}{x}$
   (C) x
   (D) $\dfrac{x}{x+1}$
   (E) x^2

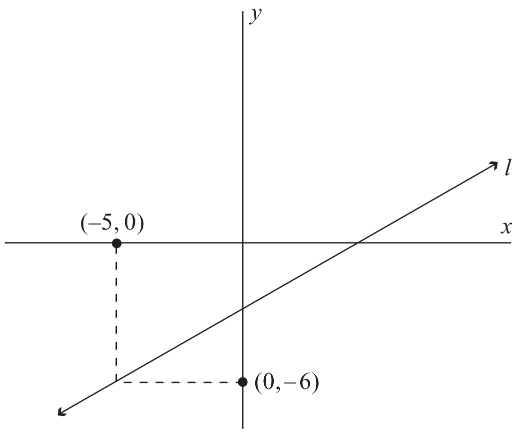
6. If the domain of $f(x) = \dfrac{x}{5^{-x}}$ is the set $\{-2, -1, 0, 2\}$, then $f(x)$ CANNOT equal

   (A) $-\dfrac{2}{25}$
   (B) $-\dfrac{1}{5}$
   (C) 0
   (D) 5
   (E) 50

7. Which of the following equations defines a function containing the (x,y) pairs $(-1,-1)$ and $(-\frac{1}{2},0)$?

   (A) $y = 3x + 2$
   (B) $y = 2x + 1$
   (C) $y = 6x + 5$
   (D) $y = -4x - 2$
   (E) $y = 4x + 3$

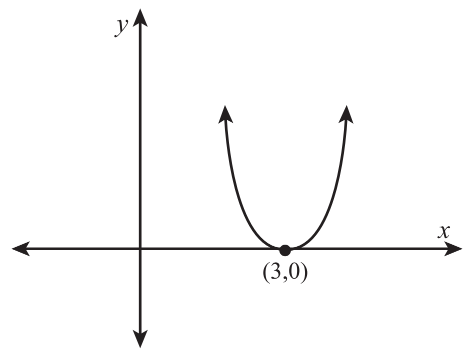
8. The figure below shows the graph of a linear function on the *xy*-plane.



If the *x*-intercept of line *l* is 4, what is the slope of *l* ?

(A) $\frac{2}{3}$
(B) $\frac{3}{4}$
(C) $\frac{5}{6}$
(D) $\frac{6}{5}$
(E) Not enough information to answer the question is given.

9. The figure below shows a parabola in the *xy*-plane.



Which of the following equations does the graph best represent?

(A) $y = -x^2 + 6x - 9$
(B) $y = x^2 - 2x + 6$
(C) $y = \frac{2}{3}x^2 - 4x + 6$
(D) $y = -x^2 + x - 3$
(E) $y = x^2 + 3x + 9$

10. Which of the following best describes the relationship between the graph of $y = \dfrac{2}{x^2}$ and the graph of $x = \dfrac{2}{y^2}$ in the *xy*-plane?

(A) Mirror images symmetrical about the *x*-axis
(B) Mirror images symmetrical about the *y*-axis
(C) Mirror images symmetrical about the line of the equation $x = y$
(D) Mirror images symmetrical about the line of the equation $x = -y$
(E) None of the above

1. SEQUENCES INVOLVING EXPONENTIAL GROWTH (GEOMETRIC SEQUENCES)

In a sequence of terms involving exponential growth, which the testing service also calls a *geometric sequence*, there is a constant ratio between consecutive terms. In other words, each successive term is the same multiple of the preceding one. For example, in the sequence 2, 4, 8, 16, 32, . . . , notice that you multipy each term by 2 to obtain the next term, and so the constant ratio (multiple) is 2.

To solve problems involving geometric sequence, you can apply the following standard equation:

$$a \cdot r^{(n-1)} = T$$

In this equation:

The variable a is the value of the first term in the sequence

The variable r is the constant ratio (multiple)

The variable n is the position number of any particular term in the sequence

The variable T is the value of term n

If you know the values of any three of the four variables in this standard equation, then you can solve for the fourth one. (On the SAT, geometric sequence problems generally ask for the value of either a or T.)

Example (solving for T when a and r are given):

The first term of a geometric sequence is 2, and the constant multiple is 3. Find the second, third, and fourth terms.

Solution:

2nd term (T) = $2 \cdot 3^{(2-1)} = 2 \cdot 3^1 = 6$

3rd term (T) = $2 \cdot 3^{(3-1)} = 2 \cdot 3^2 = 2 \cdot 9 = 18$

4th term (T) = $2 \cdot 3^{(4-1)} = 2 \cdot 3^3 = 2 \cdot 27 = 54$

To solve for T when a and r are given, as an alternative to applying the standard equation, you can multiply a by $r^{(n-1)}$ times. Given $a = 2$ and $r = 3$:

2nd term (T) = $2 \cdot 3 = 6$

3rd term (T) = $2 \cdot 3 = 6 \cdot 3 = 18$

4th term (T) = $2 \cdot 3 = 6 \cdot 3 = 18 \cdot 3 = 54$

NOTE: Using the alternative method, you may wish to use your calculator to find T if a and/or r are large numbers.

Example (solving for a when r and T are given):

The fifth term of a geometric sequence is 768, and the constant multiple is 4. Find the 1st term (a).

Solution:

$$\begin{aligned} a \times 4^{(5-1)} &= 768 \\ a \times 4^4 &= 768 \\ a \times 256 &= 768 \\ a &= \frac{768}{256} \\ a &= 3 \end{aligned}$$

Example (solving for T when a and another term in the sequence are given):

To find a particular term (T) in a geometric sequence when the first term and another term are given, first determine the constant ratio (r), and then solve for T. For example, assume that the first and sixth terms of a geometric sequence are 2 and 2048, respectively. To find the value of the fourth term, first apply the standard equation to determine r:

Solution:

$$2 \times r^{(6-1)} = 2048$$
$$2 \times r^5 = 2048$$
$$r^5 = \frac{2048}{2}$$
$$r^5 = 1024$$
$$r = \sqrt[5]{1024}$$
$$r = 4$$

The constant ratio is 4. Next, in the standard equation, let $a = 2$, $r = 4$, and $n = 4$, and then solve for T:

$$2 \times 4^{(4-1)} = T$$
$$2 \times 4^3 = T$$
$$2 \times 64 = T$$
$$128 = T$$

The fourth term in the sequence is 128.

Exercise 1

Work out each problem. For questions 1–3, circle the letter that appears before your answer. Questions 4 and 5 are grid-in questions.

1. On January 1, 1950, a farmer bought a certain parcel of land for $1,500. Since then, the land has doubled in value every 12 years. At this rate, what will the value of the land be on January 1, 2010?

   (A) $7,500
   (B) $9,000
   (C) $16,000
   (D) $24,000
   (E) $48,000

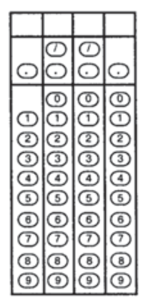
2. A certain type of cancer cell divides into two cells every four seconds. How many cells are observable 32 seconds after observing a total of four cells?

   (A) 1,024
   (B) 2,048
   (C) 4,096
   (D) 5,512
   (E) 8,192

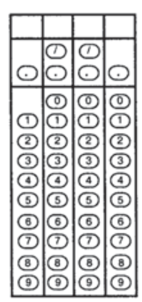
3. The seventh term of a geometric sequence with constant ratio 2 is 448. What is the first term of the sequence?

   (A) 6
   (B) 7
   (C) 8
   (D) 9
   (E) 11

4. Three years after an art collector purchases a certain painting, the value of the painting is $2,700. If the painting increased in value by an average of 50 percent per year over the three year period, how much did the collector pay for the painting, in dollars?



5. What is the second term in a geometric series with first term 3 and third term 147?



2. SETS (UNION, INTERSECTION, ELEMENTS)

A *set* is simply a collection of elements; elements in a set are also referred to as the "members" of the set. An SAT problem involving sets might ask you to recognize either the union or the intersection of two (or more) sets of numbers.

The *union* of two sets is the set of all members of either or both sets. For example, the union of the set of all negative integers and the set of all non-negative integers is the set of all integers. The *intersection* of two sets is the set of all common members – in other words, members of *both* sets. For example, the intersection of the set of integers less than 11 and the set of integers greater than 4 but less than 15 is the following set of six consecutive integers: {5,6,7,8,9,10}.

On the new SAT, a problem involving either the union or intersection of sets might apply any of the following concepts: the real number line, integers, multiples, factors (including prime factors), divisibility, or counting.

Example:

Set A is the set of all positive multiples of 3, and set B is the set of all positive multiples of 6. What is the union and intersection of the two sets?

Solution:

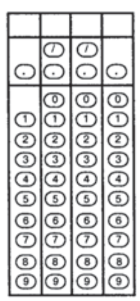
The union of sets A and B is the set of all postitive multiples of 3.
The intersection of sets A and B is the set of all postitive multiples of 6.

Exercise 2

Work out each problem. Note that question 2 is a grid-in question. For all other questions, circle the letter that appears before your answer.

1. Which of the following describes the union of the set of integers less than 20 and the set of integers greater than 10?
   (A) Integers 10 through 20
   (B) All integers greater than 10 but less than 20
   (C) All integers less than 10 and all integers greater than 20
   (D) No integers
   (E) All integers

2. Set A consists of the positive factors of 24, and set B consists of the positive factors of 18. The intersection of sets A and B is a set containing how many members?



3. The union of sets X and Y is a set that contains exactly two members. Which of the following pairs of sets could be sets X and Y ?
   (A) The prime factors of 15; the prime factors of 30
   (B) The prime factors of 14; the prime factors of 51
   (C) The prime factors of 19; the prime factors of 38
   (D) The prime factors of 22; the prime factors of 25
   (E) The prime factors of 39; the prime factors of 52

4. The set of all multiples of 10 could be the intersection of which of the following pairs of sets?
   (A) The set of all multiples of $\frac{5}{2}$; the set of all multiples of 2
   (B) The set of all multiples of $\frac{3}{5}$; the set of all multiples of 5
   (C) The set of all multiples of $\frac{3}{2}$; the set of all multiples of 10
   (D) The set of all multiples of $\frac{3}{4}$; the set of all multiples of 2
   (E) The set of all multiples of $\frac{5}{2}$; the set of all multiples of 4

5. For all real numbers x, sets P, Q, and R are defined as follows:

   P:$\{x \geq -10\}$

   Q:$\{x \geq 10\}$

   R:$\{|x| \leq 10\}$

   Which of the following indicates the intersection of sets P, Q, and R ?
   (A) x = any real number
   (B) $x \geq -10$
   (C) $x \geq 10$
   (D) $x = 10$
   (E) $-10 \leq x \leq 10$

3. ABSOLUTE VALUE

The *absolute value* of a real number refers to the number's distance from zero (the origin) on the real-number line. The absolute value of x is indicated as $|x|$. The absolute value of a negative number always has a positive value.

Example:

$|-2-3|-|2-3| =$
(A) -2
(B) -1
(C) 0
(D) 1
(E) 4

Solution:

The correct answer is (E). $|-2-3| = |-5| = 5$, and $|2-3| = |-1| = 1$. Performing subtraction: $5-1=4$.

The concept of absolute value can be incorporated into many different types of problems on the new SAT, including those involving algebraic expressions, equations, and inequalities, as well as problems involving functional notation and the graphs of functions.

Exercise 3

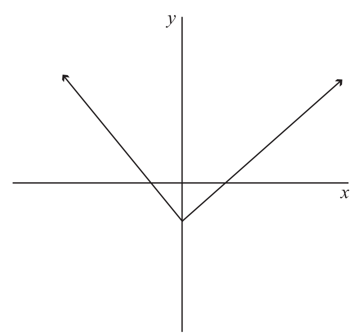
Work out each problem. Circle the letter that appears before your answer.

1. $|7-2|-|2-7| =$
(A) -14
(B) -9
(C) -5
(D) 0
(E) 10

2. For all integers a and b, where $b \neq 0$, subtracting b from a must result in a positive integer if:
(A) $|a-b|$ is a positive integer
(B) $\left(\frac{a}{b}\right)$ is a positive integer
(C) $(b-a)$ is a negative integer
(D) $(a+b)$ is a positive integer
(E) (ab) is a positive integer

3. What is the complete solution set for the inequality $|x-3| > 4$?
(A) $x > -1$
(B) $x > 7$
(C) $-1 < x < 7$
(D) $x < -7, x > 7$
(E) $x < -1, x > 7$

4. The figure below shows the graph of a certain equation in the xy-plane.



Which of the following could be the equation?
(A) $x = |y|-1$
(B) $y = |x|-1$
(C) $|y| = x-1$
(D) $y = x+1$
(E) $|x| = y-1$

5. If $f(x) = |\frac{1}{x} - 3| - x$, then $f\left(\frac{1}{2}\right) =$
(A) -1
(B) $-\frac{1}{2}$
(C) 0
(D) $\frac{1}{2}$
(E) 1

4. EXPONENTS (POWERS)

An *exponent*, or *power*, refers to the number of times that a number (referred to as the *base* number) is multiplied by itself, plus 1. In the number 2^3, the base number is 2 and the exponent is 3. To calculate the value of 2^3, you multiply 2 by itself twice: $2^3 = 2 \cdot 2 \cdot 2 = 8$. In the number $\left(\frac{2}{3}\right)^4$, the base number is $\frac{2}{3}$ and the exponent is 4. To calculate the value of $\left(\frac{2}{3}\right)^4$, you multiply $\frac{2}{3}$ by itself three times: $\left(\frac{2}{3}\right)^4 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{16}{81}$.

An SAT problem might require you to combine two or more terms that contain exponents. Whether you can you combine base numbers—using addition, subtraction, multiplication, or division—*before* applying exponents to the numbers depends on which operation you're performing. When you add or subtract terms, you cannot combine base numbers or exponents:

$$a^x + b^x \neq (a + b)^x$$
$$a^x - b^x \neq (a - b)^x$$

Example:

If $x = -2$, then $x^5 - x^2 - x =$
(A) 26
(B) 4
(C) –34
(D) –58
(E) –70

Solution:

The correct answer is (C). You cannot combine exponents here, even though the base number is the same in all three terms. Instead, you need to apply each exponent, in turn, to the base number, then subtract:

$x^5 - x^2 - x = (-2)^5 - (-2)^2 - (-2) = -32 - 4 + 2 = -34$

There are two rules you need to know for combining exponents by multiplication or division. First, you can combine base numbers first, but only if the exponents are the same:

$$a^x \cdot b^x = (ab)^x$$

$$\frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$$

Second, you can combine exponents first, but only if the base numbers are the same. When multiplying these terms, add the exponents. When dividing them, subtract the denominator exponent from the numerator exponent:

$$a^x \cdot a^y = a^{(x + y)}$$

$$\frac{a^x}{a^y} = a^{(x-y)}$$

When the same base number (or term) appears in both the numerator and denominator of a fraction, you can factor out, or cancel, the number of powers common to both.

Example:

Which of the following is a simplified version of $\frac{x^2y^3}{x^3y^2}$?

(A) $\frac{y}{x}$
(B) $\frac{x}{y}$
(C) $\frac{1}{xy}$
(D) 1
(E) x^5y^5

Solution:

The correct answer is (A). The simplest approach to this problem is to cancel, or factor out, x^2 and y^2 from numerator and denominator. This leaves you with x^1 in the denominator and y^1 in the denominator.

You should also know how to raise exponential numbers to powers, as well as how to raise base numbers to negative and fractional exponents. To raise an exponential number to a power, multiply exponents together:

$$\left(a^x\right)^y = a^{xy}$$

Raising a base number to a negative exponent is equivalent to 1 divided by the base number raised to the exponent's absolute value:

$$a^{-x} = \frac{1}{a^x}$$

To raise a base number to a fractional exponent, follow this formula:

$$a^{\frac{x}{y}} = \sqrt[y]{a^x}$$

Also keep in mind that any number other than 0 (zero) raised to the power of 0 (zero) equals 1:

$$a^0 = 1 \; [a \neq 0]$$

Example:

$(2^3)^2 \cdot 4^{-3} =$
(A) 16
(B) 1
(C) $\frac{2}{3}$
(D) $\frac{1}{2}$
(E) $\frac{1}{8}$

Solution:

The correct answer is (B). $(2^3)^2 \cdot 4^{-3} = 2^{(2)(3)} \cdot \frac{1}{4^3} = \frac{2^6}{4^3} = \frac{2^6}{2^6} = 1$

Exercise 4

Work out each problem. For questions 1–4, circle the letter that appears before your answer. Question 5 is a grid-in question.

1. $\frac{a^2b}{b^2c} \div \frac{a^2c}{bc^2} =$

   (A) $\frac{1}{a}$

   (B) $\frac{1}{b}$

   (C) $\frac{b}{a}$

   (D) $\frac{c}{b}$

   (E) 1

2. $4^n + 4^n + 4^n + 4^n =$

   (A) 4^{4n}
   (B) 16^n
   (C) $4^{(n \cdot n \cdot n \cdot n)}$
   (D) $4^{(n+1)}$
   (E) 16^{4n}

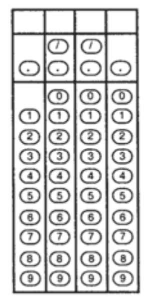
3. Which of the following expressions is a simplified form of $(-2x^2)^4$?

   (A) $16x^8$
   (B) $8x^6$
   (C) $-8x^8$
   (D) $-16x^6$
   (E) $-16x^8$

4. If $x = -1$, then $x^{-3} + x^{-2} + x^2 + x^3 =$

   (A) –2
   (B) –1
   (C) 0
   (D) 1
   (E) 2

5. What integer is equal to $4^{3/2} + 4^{3/2}$?



5. FUNCTION NOTATION

In a *function* (or *functional relationship*), the value of one variable depends upon the value of, or is "a function of," another variable. In mathematics, the relationship can be expressed in various forms. The new SAT uses the form $y = f(x)$—where y is a function of x. (Specific variables used may differ.) To find the value of the function for any value x, substitute the x-value for x wherever it appears in the function.

Example:

If $f(x) = 2x - 6x$, then what is the value of $f(7)$?

Solution:

The correct answer is –28. First, you can combine $2x - 6x$, which equals $-4x$. Then substitute (7) for x in the function: $-4(7) = -28$. Thus, $f(7) = -28$.

A problem on the new SAT may ask you to find the value of a function for either a number value (such as 7, in which case the correct answer will also be a number value) or for a variable expression (such as $7x$, in which case the correct answer will also contain the variable x). A more complex function problem might require you to apply two different functions or to apply the same function twice, as in the next example.

Example:

If $f(x) = \frac{2}{x^2}$, then $f\left(\frac{1}{2}\right) \times f\left(\frac{1}{x}\right) =$

(A) $4x$

(B) $\frac{1}{8x}$

(C) $16x$

(D) $\frac{1}{4x^2}$

(E) $16x^2$

Solution:

The correct answer is (E). Apply the function to each of the two x-values (in the first instance, you'll obtain a numerical value, while in the second instance you'll obtain an variable expression:

$$f\left(\frac{1}{2}\right) = \frac{2}{\left(\frac{1}{2}\right)^2} = \frac{2}{\frac{1}{4}} = 2 \times 4 = 8$$

$$f\left(\frac{1}{x}\right) = \frac{2}{\left(\frac{1}{x}\right)^2} = \frac{2}{\left(\frac{1}{x^2}\right)} = 2x^2$$

Then, combine the two results according to the operation specified in the question:

$$f\left(\frac{1}{2}\right) \times f\left(\frac{1}{x}\right) = 8 \times 2x^2 = 16x^2$$

Exercise 5

Work out each problem. Circle the letter that appears before your answer.

1. If $f(x) = 2x\sqrt{x}$, then for which of the following values of x does $f(x) = x$?
   (A) $\frac{1}{4}$
   (B) $\frac{1}{2}$
   (C) 2
   (D) 4
   (E) 8

2. If $f(a) = a^{-3} - a^{-2}$, then $f\left(\frac{1}{3}\right) =$
   (A) $-\frac{1}{6}$
   (B) $\frac{1}{6}$
   (C) 6
   (D) 9
   (E) 18

3. If $f(x) = x^2 + 3x - 4$, then $f(2 + a) =$
   (A) $a^2 + 7a + 6$
   (B) $2a^2 - 7a - 12$
   (C) $a^2 + 12a + 3$
   (D) $6a^2 + 3a + 7$
   (E) $a^2 - a + 6$

4. If $f(x) = x^2$ and $g(x) = x + 3$, then $g(f(x)) =$
   (A) $x + 3$
   (B) $x^2 + 6$
   (C) $x + 9$
   (D) $x^2 + 3$
   (E) $x^3 + 3x^2$

5. If $f(x) = \frac{x}{2}$, then $f(x^2) \div \left(f(x)\right)^2 =$
   (A) x^3
   (B) 1
   (C) $2x^2$
   (D) 2
   (E) $2x$

6. FUNCTIONS—DOMAIN AND RANGE

A function consists of a *rule* along with two sets—called the *domain* and the *range*. The domain of a function $f(x)$ is the set of all values of x on which the function $f(x)$ is defined, while the range of $f(x)$ is the set of all values that result by applying the rule to all values in the domain.

By definition, a function must assign *exactly one* member of the range to each member of the domain, and must assign at least one member of the domain to each member of the range. Depending on the function's rule and its domain, the domain and range might each consist of a finite number of values; or either the domain or range (or both) might consist of an infinite number of values.

Example:

In the function $f(x) = x + 1$, if the domain of x is the set {2,4,6}, then applying the rule that $f(x) = x + 1$ to all values in the domain yields the function's range: the set {3,5,7}. (All values other than 2, 4, and 6 are outside the domain of x, while all values other than 3, 5, and 7 are outside the function's range.)

Example:

In the function $f(x) = x^2$, if the domain of x is the set of all real numbers, then applying the rule that $f(x) = x^2$ to all values in the domain yields the function's range: the set of all non-negative real numbers. (Any negative number would be outside the function's range.)

Exercise 6

Work out each problem. Circle the letter that appears before your answer.

1. If $f(x) = \sqrt{x+1}$, and if the domain of x is the set {3,8,15}, then which of the following sets indicates the range of $f(x)$?

   (A) $\{-4, -3, -2, 2, 3, 4\}$
   (B) $\{2, 3, 4\}$
   (C) $\{4, 9, 16\}$
   (D) $\{3, 8, 15\}$
   (E) {all real numbers}

2. If $f(a) = 6a - 4$, and if the domain of a consists of all real numbers defined by the inequality $-6 < a < 4$, then the range of $f(a)$ contains all of the following members EXCEPT:

   (A) -24
   (B) $\sqrt{\frac{1}{6}}$
   (C) 0
   (D) 4
   (E) 20

3. If the range of the function $f(x) = x^2 - 2x - 3$ is the set $R = \{0\}$, then which of the following sets indicates the largest possible domain of x ?

   (A) $\{-3\}$
   (B) $\{3\}$
   (C) $\{-1\}$
   (D) $\{3, -1\}$
   (E) all real numbers

4. If $f(x) = \sqrt{x^2 - 5x + 6}$, which of the following indicates the set of all values of x at which the function is NOT defined?

   (A) $\{x \mid x < 3\}$
   (B) $\{x \mid 2 < x < 3\}$
   (C) $\{x \mid x < -2\}$
   (D) $\{x \mid -3 < x < 2\}$
   (E) $\{x \mid x < -3\}$

5. If $f(x) = \sqrt[3]{\frac{1}{x}}$, then the largest possible domain of x is the set that includes

   (A) all non-zero integers.
   (B) all non-negative real numbers.
   (C) all real numbers except 0.
   (D) all positive real numbers.
   (E) all real numbers.

7. LINEAR FUNCTIONS—EQUATIONS AND GRAPHS

A *linear function* is a function f given by the general form $f(x) = mx + b$, in which m and b are constants. In algebraic functions, especially where defining a line on the xy-plane is involved, the variable y is often used to represent $f(x)$, and so the general form becomes $y = mx + b$. In this form, each x-value (member of the domain set) can be paired with its corresponding y-value (member of the range set) by application of the function.

Example:

In the function $y = 3x + 2$, if the domain of x is the set of all positive integers less than 5, then applying the function over the entire domain of x results in the following set of (x,y) pairs: $S = \{(1,5), (2,8), (3,11), (4,14)\}$.

In addition to questions requiring you to solve a system of linear equations (by using either the substitution or addition-subtraction method), the new SAT includes questions requiring you to recognize any of the following:

- A linear function (equation) that defines two or more particular (x,y) pairs (members of the domain set and corresponding members of the range set). These questions sometimes involve real-life situations; you may be asked to construct a mathematical "model" that defines a relationship between, for example, the price of a product and the number of units of that product.
- The graph of a particular linear function on the xy-plane
- A linear function that defines a particular line on the xy-plane.

Variations on the latter two types of problems may involve determining the slope and/or y-intercept of a line defined by a function, or identifying a function that defines a given slope and y-intercept.

Example:

In the linear function f, if $f(-3) = 3$ and if the slope of the graph of f in the xy-plane is 3, what is the equation of the graph of f ?

(A) $y = 3x - 3$
(B) $y = 3x + 12$
(C) $y = x - 6$
(D) $y = -x$
(E) $y = 3x - 12$

Solution:

The correct answer is (B). In the general equation $y = mx + b$, slope (m) is given as 3. To determine b, substitute -3 for x and 3 for y, then solve for b: $3 = 3(-3) + b$; $12 = b$. Only in choice (B) does $m = 3$ and $b = 12$.

Exercise 7

Work out each problem. Circle the letter that appears before your answer.

1. XYZ Company pays its executives a starting salary of \$80,000 per year. After every two years of employment, an XYZ executive receives a salary raise of \$1,000. Which of the following equations best defines an XYZ executive's salary (S) as a function of the number of years of employment (N) at XYZ?

   (A) $S = \dfrac{1{,}000}{N} + 80{,}000$

   (B) $S = N + 80{,}000$

   (C) $S = \dfrac{80{,}000}{N} + 1{,}000$

   (D) $S = 1{,}000N + 80{,}000$

   (E) $S = 500N + 80{,}000$

2. In the linear function g, if $g(4) = -9$ and $g(-2) = 6$, what is the y-intercept of the graph of g in the xy-plane?

   (A) $-\dfrac{9}{2}$

   (B) $-\dfrac{5}{2}$

   (C) $\dfrac{2}{5}$

   (D) 1

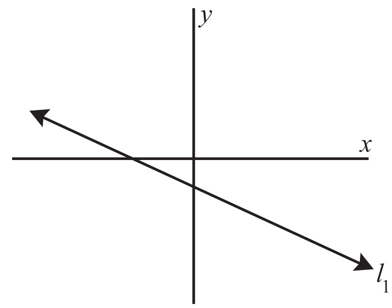
   (E) $\dfrac{3}{2}$

3. If two linear function f and g have identical domains and ranges, which of the following, each considered individually, could describe the graphs of f and g in the xy-plane?

   I. two parallel lines
   II. two perpendicular lines
   III. two vertical lines

   (A) I only
   (B) I and II only
   (C) II only
   (D) II and III only
   (E) I, II, and III

4. In the xy-plane below, if the scales on both axes are the same, which of the following could be the equation of a function whose graph is l_1 ?



   (A) $y = \dfrac{2}{3}x - 3$
   (B) $y = -2x + 1$
   (C) $y = x + 3$
   (D) $y = -3x - \dfrac{2}{3}$
   (E) $y = -\dfrac{2}{3}x - 3$

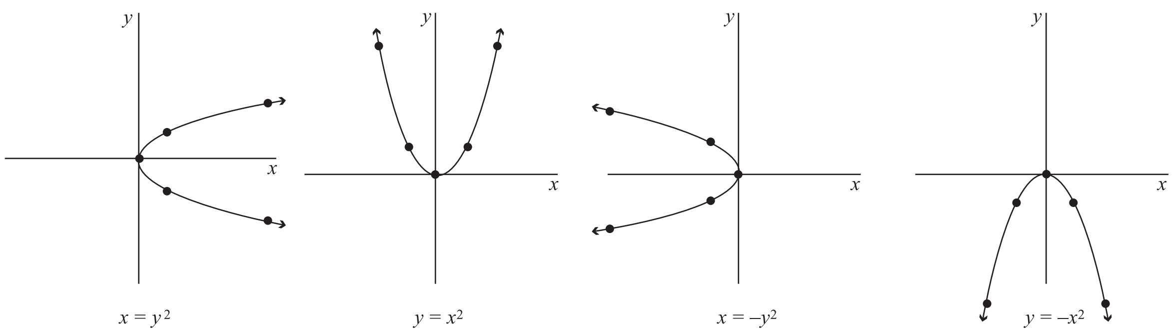
5. If h is a linear function, and if $h(2) = 3$ and $h(4) = 1$, then $h(-101) =$

   (A) –72
   (B) –58
   (C) 49
   (D) 92
   (E) 106

8. QUADRATIC FUNCTIONS—EQUATIONS AND GRAPHS

In Chapter 8, you learned to solve quadratic equations in the general form $ax^2 + bx + c = 0$ by factoring the expression on the left-hand side of this equation to find the equation's two roots— the values of x that satisfy the equation. (Remember that the two roots might be the same.) The new SAT may also include questions involving *quadratic functions* in the general form $f(x) = ax^2 + bx + c$. (Note that a, b, and c are constants and that a is the only essential constant.) In quadratic functions, especially where defining a graph on the xy-plane is involved, the variable y is often used to represent $f(x)$, and x is often used to represent $f(y)$.

The graph of a quadratic equation of the basic form $y = ax^2$ or $x = ay^2$ is a *parabola*, which is a U-shaped curve. The point at which the dependent variable is at its minimum (or maximum) value is the *vertex*. In each of the following four graphs, the parabola's vertex lies at the origin (0,0). Notice that the graphs are constructed by tabulating and plotting several (x,y) pairs, and then connecting the points with a smooth curve:



$x = y^2$

x	y
4	–2
1	–1
0	0
1	1
4	2

$y = x^2$

x	y
–2	4
–1	1
0	0
1	1
2	4

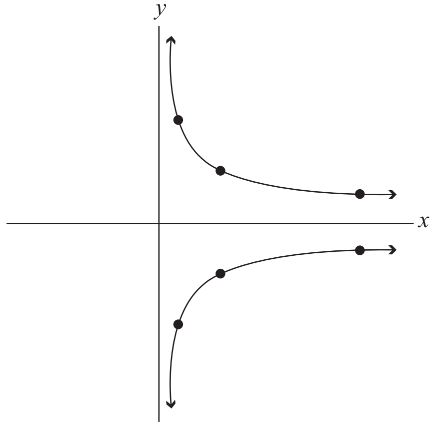
$x = -y^2$

x	y
–4	–2
–1	–1
0	0
–1	1
–4	2

$y = -x^2$

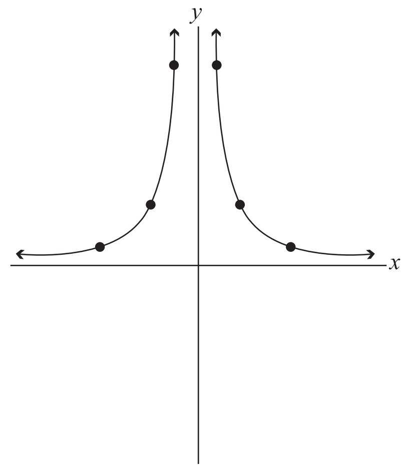
x	y
–2	–4
–1	–1
0	0
1	–1
2	–4

The graph of a quadratic equation of the basic form $x = \frac{1}{y^2}$ or $y = \frac{1}{x^2}$ is a *hyperbola*, which consists of two U-shaped curves that are symmetrical about a particular line, called the *axis of symmetry*. The axis of symmetry of the graph of $x = \frac{1}{y^2}$ is the x-axis, while the axis of symmetry in the graph of $y = \frac{1}{x^2}$ is the y-axis, as the next figure shows. Again, the graphs are constructed by tabulating and plotting some (x,y) pairs, then connecting the points:



$x = \frac{1}{y^2}$

x	y
$\frac{1}{4}$	-2
1	-1
4	$-\frac{1}{2}$
4	$\frac{1}{2}$
1	1
$\frac{1}{4}$	2

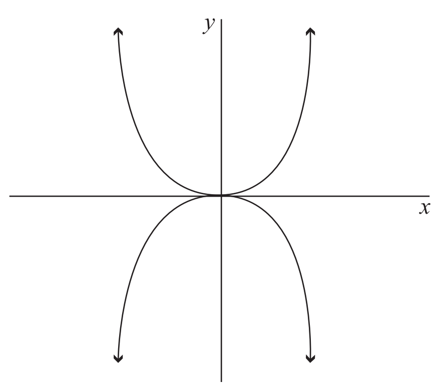


$y = \frac{1}{x^2}$

x	y
-2	$\frac{1}{4}$
-1	1
$-\frac{1}{2}$	4
$\frac{1}{2}$	4
1	1
2	$\frac{1}{4}$

The new SAT might include a variety of question types involving quadratic functions—for example, questions that ask you to recognize a quadratic equation that defines a particular graph in the *xy*-plane or to identify certain features of the graph of a quadratic equation, or compare two graphs

Example:



The graph shown in the *xy*-plane above could represent which of the following equations?

(A) $|x^2| = |y^2|$
(B) $x = |y^2|$
(C) $|y| = x^2$
(D) $y = |x^2|$
(E) $|x| = y^2$

Solution:

The correct answer is (C). The equation $|y| = x^2$ represents the union of the two equations $y = x^2$ and $-y = x^2$. The graph of $y = x^2$ is the parabola extending upward from the origin (0,0) in the figure, while the graph of $-y = x^2$ is the parabola extending downward from the origin.

Example:

In the *xy*-plane, the graph of $y+2=\frac{x^2}{2}$ shows a parabola that opens

(A) downward.
(B) upward.
(C) to the right.
(D) to the left.
(E) either upward or downward.

Solution:

The correct answer is (B). Plotting three or more points of the graph on the *xy*-plane should show the parabola's orientation. First, it is helpful to isolate *y* in the equation $y=\frac{x^2}{2}-2$. In this equation, substitute some simple values for *x* and solve for *y* in each case. For example, substituting 0, 2, and –2 for *x* gives us the three (*x*,*y*) pairs (0,–2), (2,0), and (–2,0). Plotting these three points on the *xy*-plane, then connecting them with a curved line, suffices to show a parabola that opens upward.

An SAT question might also ask you to identify a quadratic equation that defines two or more domain members and the corresponding members of the function's range (these questions sometimes involve "models" of real-life situations).

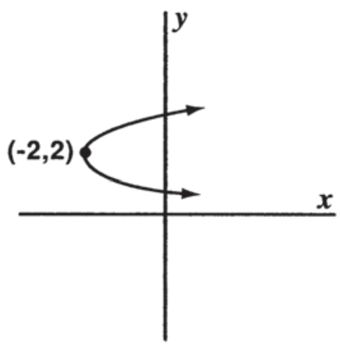
Exercise 8

Work out each problem. Circle the letter that appears before your answer.

1. Which of the following equations defines a function containing the (x,y) pairs $(1,-1)$, $(2,-4)$, $(3,-9)$, and $(4,-16)$?

(A) $y = -2x$
(B) $y = 2x$
(C) $y = x^2$
(D) $y = -x^2$
(E) $y = -2x^2$

2. The figure below shows a parabola in the xy-plane.



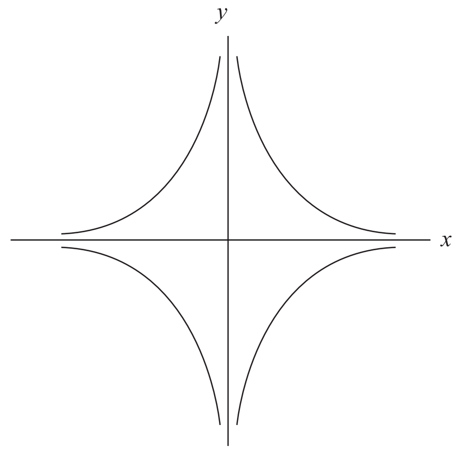
Which of the following equations does the graph best represent?

(A) $x = (y - 2)^2 - 2$
(B) $x = (y + 2)^2 - 2$
(C) $x = -(y - 2)^2 - 2$
(D) $y = (x - 2)^2 + 2$
(E) $y = (x - 2)^2 - 2$

3. In the xy-plane, which of the following is an equation whose graph is the graph of $y = \frac{x^2}{3}$ translated three units horizontally and to the left?

(A) $y = x^2$
(B) $y = \frac{x^2}{3} + 3$
(C) $y = \frac{x^2}{3} - 3$
(D) $y = \frac{(x-3)^2}{3}$
(E) $y = \frac{(x+3)^2}{3}$

4. Which of the following is the equations best defines the graph shown below in the xy-plane?



(A) $y = \frac{1}{x^2}$
(B) $x = \left|\frac{1}{y^2}\right|$
(C) $x = \frac{1}{y^2}$
(D) $|x| = \frac{1}{y^2}$
(E) $y = \left|\frac{1}{x^2}\right|$

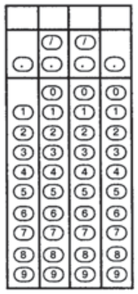
5. ABC Company projects that it will sell 48,000 units of product X per year at a unit price of \$1, 12,000 units per year at \$2 per unit, and 3,000 units per year at \$4 per unit. Which of the following equations could define the projected number of units sold per year (N), as a function of price per unit (P)?

(A) $N = \frac{48{,}000}{P^2 + 2}$
(B) $N = \frac{48{,}000}{P^2}$
(C) $N = \frac{48{,}000}{P + 14}$
(D) $N = \frac{48{,}000}{P + 4}$
(E) $N = \frac{48{,}000}{P^2 + 8}$

RETEST

Answer the following 10 questions, limiting your time to 15 minutes. Note that question 1 is a *grid-in* question, in which you provide the numerical solution. (All other questions are in multiple-choice format.)

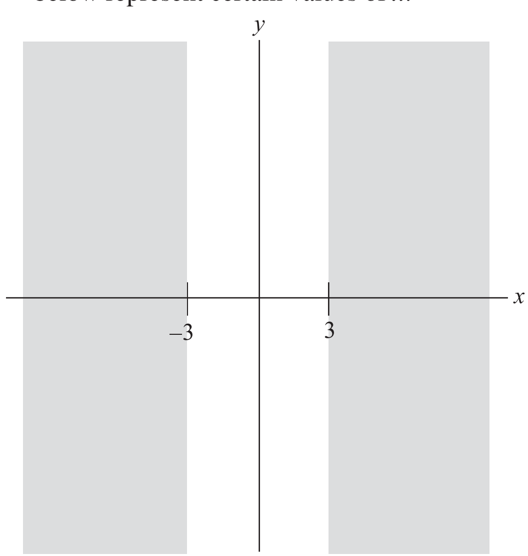
1. What is the fourth term in a geometric series with first term 2 and third term 72?



2. What is the intersection of the set of all positive integers divisible by 4 and the set of all positive integers divisible by 6?

   (A) All positive multiples of 4
   (B) All positive multiples of 6
   (C) All positive multiples of 8
   (D) All positive multiples of 12
   (E) All positive multiples of 24

3. The shaded regions of the *xy*-plane shown below represent certain values of *x*.



   Which of the following inequalities accounts for all such values of *x* ?

   (A) $|y| \geq 3$
   (B) $|x| \geq 3$
   (C) $|x| \leq 3$
   (D) $|y| \leq 3$
   (E) $|y| \leq -3$

4. If $-32 = \left(-\frac{1}{2}\right)^M$, then what is the value of M ?

   (A) –16
   (B) –6
   (C) –5
   (D) 5
   (E) 16

5. If $f(x) = \frac{1}{x+1}$, then $f\left(\frac{1}{x+1}\right) =$

   (A) x
   (B) $\frac{x+1}{2}$
   (C) 1
   (D) $x + 1$
   (E) $\frac{x+1}{x+2}$

6. If $f(x) = y = 1 - x^2$, and if the domain of x is all real numbers, which of the following sets indicates the range of the function?

   (A) $\{y \mid y \geq 1\}$
   (B) $\{y \mid y > 1\}$
   (C) $\{y \mid y \leq 1\}$
   (D) $\{y \mid y < 1\}$
   (E) $\{y \mid y \geq -2\}$

7. In the linear function *f*, if $f(-6) = -2$ and the slope of the graph of *f* in the *xy*-plane is –2, which of the following is true?

   (A) $f(-10) = -6$
   (B) $f(-6) = 0$
   (C) $f(-8) = 2$
   (D) $f(6) = 2$
   (E) $f(8) = 4$

8. Once a certain airplane attains its maximum speed of 300 miles per hour (mph), it begins decreasing speed as it approaches its destination. After every 50 miles, the plane decreases its airspeed by 20 mph. Which of the following equations best defines the number of miles the plane has traveled (m) after beginning to decrease speed as a function of the airplane's airspeed (s)?

   (A) $s=-\frac{5m}{2}+750$

   (B) $s=-\frac{2m}{5}+300$

   (C) $m=-\frac{5s}{2}+750$

   (D) $m=-\frac{5s}{2}+300$

   (E) $m=\frac{2s}{5}+300$

9. In the xy-plane, the graph of $3x = 2y^2$ shows a parabola with vertex at the point defined by the (x,y) pair:

   (A) (0,0)
   (B) (0,2)
   (C) (2,0)
   (D) (3,2)
   (E) (2,3)

10. A model rocket is shot straight up in the air from ground level. After 2 seconds and then again after 3 seconds, its height is 96 feet. Which of the following equations could define rocket's height, (h), as a function of the number of seconds after launch (t)?

    (A) $h = 10t^2 - 74t$
    (B) $h = 8t^2 - 64t$
    (C) $h = 64t - 8t^2$
    (D) $h = 80t - 16t^2$
    (E) $h = 96t - 10t^2$

SOLUTIONS TO PRACTICE EXERCISES

Diagnostic Test

1. The correct answer is 16,000. Solve for a in the general equation $a \cdot r^{(n-1)} = T$. Let $T = 256{,}000$, $r = 2$, and $n = 5$ (the number of terms in the sequence that includes the city's population 20, 15, 10, and 5 years ago, as well as its current population). Solving for a:

$$\begin{aligned} a\times 2^{(5-1)} &= 256{,}000 \\ a\times 2^{4} &= 256{,}000 \\ a\times 16 &= 256{,}000 \\ a &= 256{,}000 \div 16 \\ a &= 16{,}000 \end{aligned}$$

   Twenty years ago, Urbanville's population was 16,000.

2. **(E)** The union of the three sets of factors is a set that contains all factors of any one or more of the three sets. The factors of 30 include all factors of 15, as well as the integer 6 and 30 (but not the integer 25). Choice (B) desribes the union of the factors of 15 and the factors of 30, but not the factors of 75 (which include 25). The factors of 75 include all factors of 15, as well as the integer 25 (but not integers 6 and 30). Choice (D) desribes the union of the factors of 15 and the factors of 75, but not the factors of 30 (which include 6 and 30). Thus, among answer choices (A) through (D), none describes the intersection of all three sets.

3. **(C)** First, determine each of the three absolute values:

   $|-1-2| = |-3| = 3$

   $|5-6| = |-1| = 1$

   $|-3+4| = |1| = 1$

   The combine the three results: $3 - 1 - 1 = 1$.

4. **(D)** Multiply like base numbers by adding exponents, and divide like base numbers by subtracting the denominator exponent from the numerator exponent:

$$\frac{x^6y^3y}{y^6x^3x} = \frac{x^6y^4}{y^6x^4} = \frac{x^2}{y^2}$$

5. **(B)** Substitute the expression $(x + 1)$ for x in $\frac{1}{f(x)}\times f\left(\frac{1}{x}\right)$, then combine terms:

$$\frac{1}{f(x)}\times f\left(\frac{1}{x}\right) = \left(\frac{1}{x+1}\right)\left(\frac{1+x}{x}\right) = \frac{1}{x}$$

6. **(D)** The question asks which number among the five listed is outside the function's range. First, simplify the function. Note that $5^{-x} = \frac{1}{5^x}$, and therefore that $\frac{x}{5^{-x}} = \frac{x}{\frac{1}{5^x}} = (x)(5^x)$. To determine the function's range, apply the rule that $f(x) = (x)(5^x)$ to each member of the domain, in turn:

$$f(-2) = (-2)(5^{-2}) = \frac{-2}{5^2} = -\frac{2}{25}$$
$$f(-1) = (-1)(5^{-1}) = \frac{-1}{5^1} = -\frac{1}{5}$$

   $f(0) = (0)(5^0) = (0)(1) = 0$

   $f(2) = (2)(5^2) = (2)(25) = 50$

   The range of $f(x) = \frac{x}{5^{-x}}$ is the set $\{ -\frac{2}{25}, -\frac{1}{5}, 0, 50 \}$. Only answer choice (D) provides a number that is not in this range.

7. **(B)** To solve this problem, consider each answer choice in turn, substituting the (x,y) pairs provided in the question for x and y in the equation. Among the five equations, only the equation in choice (B) holds for *both* pairs:

$$\begin{aligned} y &= 2x+1 \\ (-1) &= 2(-1)+1 \\ (0) &= 2(-\tfrac{1}{2})+1 \end{aligned}$$

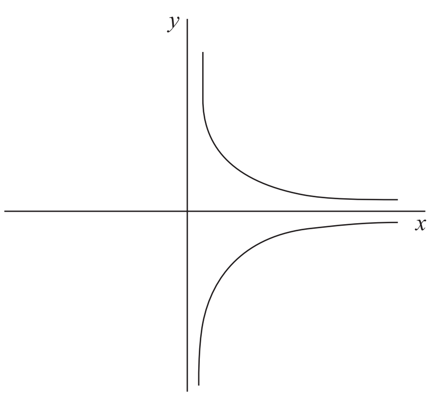
8. **(A)** One point on l is defined by $(-5,-6)$. A second point on l is defined by $(4,0)$, which is the point of x-intercept. With two points defined, you can find the line's slope (m) as follows:

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{0-(-6)}{4-(-5)} = \frac{6}{9} = \frac{2}{3}.$$

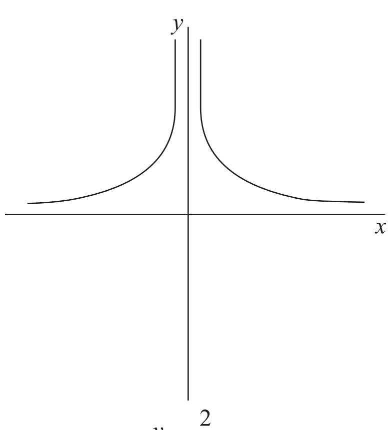
9. **(C)** The graph shows a parabola opening upward with vertex at (3,0). Of the five choices, only (A) and (C) provide equations that hold for the (x,y) pair (3,0). Eliminate choices (B), (D), and (E). In the equation given by choice (A), substituting any non-zero number for x yields a *negative* y-value. However, the graph shows no negative y-values. Thus, you can eliminate choice (A), and the correct answer must be (C).

   Also, when a parabola extends upward, the coefficient of x^2 in the equation must be positive.

10. **(E)** The following figure shows the graphs of the two equations:



$x = \frac{2}{y^2}$



$y = \frac{2}{x^2}$

   As you can see, the graphs are not mirror images of each other about any of the axes described in answer choices (A) through (D).

Exercise 1

1. **(E)**. Solve for T in the general equation $a \cdot r^{(n-1)} = T$. Let $a = 1{,}500$, $r = 2$, and $n = 6$ (the number of terms in the sequence that includes the value in 1950 and at every 12-year interval since then, up to and including the expected value in 2010). Solving for T:

$$\begin{aligned} 1{,}500 \times 2^{(6-1)} &= T \\ 1{,}500 \times 2^5 &= T \\ 1{,}500 \times 32 &= T \\ 48{,}000 &= T \end{aligned}$$

   Doubling every 12 years, the land's value will be \$48,000 in 2010.

2. **(A)** Solve for T in the general equation $a \cdot r^{(n-1)} = T$. Let $a = 4$, $r = 2$, and $n = 9$ (the number of terms in the sequence that includes the number of cells observable now as well as in 4, 8, 12, 16, 20, 24, 28, and 32 seconds). Solving for T:

$$\begin{aligned} 4 \times 2^{(9-1)} &= T \\ 4 \times 2^8 &= T \\ 4 \times 256 &= T \\ 1{,}024 &= T \end{aligned}$$

   32 seconds from now, the number of observable cancer cells is 1,024.

3. **(B)** In the standard equation, let $T = 448$, $r = 2$, and $n = 7$. Solve for a :

$$\begin{aligned} a \times 2^{(7-1)} &= 448 \\ a \times 2^6 &= 448 \\ a \times 64 &= 448 \\ a &= \frac{448}{64} \\ a &= 7 \end{aligned}$$

4. The correct answer is \$800. Solve for a in the general equation $a \cdot r^{(n-1)} = T$. Let $T = 2{,}700$. The value at the date of the purchase is the first term in the sequence, and so the value three years later is the fourth term; accordingly, $n = 4$. Given that painting's value increased by 50% (or $\frac{1}{2}$) per year on average, $r = 1.5 = \frac{3}{2}$. Solving for a:

$$a \times \left(\frac{3}{2}\right)^{(4-1)} = 2{,}700$$
$$a \times \left(\frac{3}{2}\right)^{3} = 2{,}700$$
$$a \times \frac{27}{8} = 2{,}700$$
$$a = 2{,}700 \times \frac{8}{27}$$
$$a = 800$$

At an increase of 50% per year, the collector must have paid \$800 for the painting three years ago.

5. The correct answer is 21. First, find r:

$$3 \times r^{(3-1)} = 147$$
$$3 \times r^{2} = 147$$
$$r^{2} = 49$$
$$r = 7$$

To find the second term in the sequence, multiply the first term (3) by r : $3 \cdot 7 = 21$.

Exercise 2

1. **(E)** The union of the two sets is the set that contains all integers — negative, positive, and zero (0).

2. The correct answer is 4. The positive factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24. The positive factors of 18 are 1, 2, 3, 6, 9, and 18.The two sets have in common four members: 1, 2, 3, and 6.

3. **(C)** 19 is a prime number, and therefore has only one prime factor: 19. There are two prime factors of 38: 2 and 19. The union of the sets described in choice (C) is the set that contains two members: 2 and 19.

4. **(A)** Through 10, the multiples of $\frac{5}{2}$, or $2\frac{1}{2}$, are $2\frac{1}{2}$, 5, $2\frac{1}{2}$, and 10. Through 10, the multiples of 2 are 2, 4, 6, 8, and 10. As you can see, the two sets desribed in choice (A) intersect at, but only at, every multiple of 10.

5. **(D)** You can express set R:{$|x| \leq 10$} as R:{$-10 \leq x \leq 10$}. The three sets have only one real number in common: the integer 10.

Exercise 3

1. **(D)** $|7 - 2| - |2 - 7| = |5| - |-5| = 5 - 5 = 0$

2. **(C)** If $b - a$ is a negative integer, then $a > b$, in which case $a - b$ must be a positive integer. (When you subtract one integer from another, the result is always an integer.) Choice (A), which incorporates the concept of absolute value, cannot be the correct answer, since the absolute value of any integer is by definition a positive integer.

3. **(E)** Either $x - 3 > 4$ or $x - 3 < -4$. Solve for x in both inequalities: $x > 7$; $x < -1$.

4. **(B)** If $x = 0$, $y = -1$. The point (0,–1) on the graph shows this functional pair. For all negative values of x, y is the absolute value of x, minus 1 (the graph is translated down one unit). The portion of the graph to the left of the y-axis could show these values. For all positive values of x, $y = x$, minus 1 (the graph is translated down one unit). The portion of the graph to the right of the y-axis could show these values.

5. **(D)** Substitute $\frac{1}{2}$ for x in the function:

$$\begin{aligned} f\left(\tfrac{1}{2}\right) &= \left|\frac{1}{\frac{1}{2}} - 3\right| - \tfrac{1}{2} \\ &= |2 - 3| - \tfrac{1}{2} \\ &= |-1| - \tfrac{1}{2} \\ &= 1 - \tfrac{1}{2} \\ &= \tfrac{1}{2} \end{aligned}$$

Exercise 4

1. **(E)** First, cancel common factors in each term. Then, multiply the first term by the reciprocal of the second term. You can now see that all terms cancel out:

$$\frac{a^2b}{b^2c} \div \frac{a^2c}{bc^2} = \frac{a^2}{bc} \div \frac{a^2}{bc} = \frac{a^2}{bc} \times \frac{bc}{a^2} = 1$$

2. **(D)** The expression given in the question is equivalent to $4 \cdot 4^n$. In this expression, base numbers are the same. Since the terms are multiplied together, you can combine exponents by adding them together: $4 \cdot 4^n = 4^{(n+1)}$.

3. **(A)** Raise both the coefficient –2 and variable x^2 to the power of 4. When raising an exponent to a power, multiply together the exponents:

$$(-2x^2)^4 = (-2)^4 x^{(2)(4)} = 16x^8$$

4. **(C)** Any term to a negative power is the same as "one over" the term, but raised to the *positive* power. Also, a negative number raised to a power is negative if the exponent is *odd*, yet positive if the exponent is *even*:

$$-1^{(-3)} + [-1^{(-2)}] + [-1^2] + [-1^3] = -\frac{1}{1} + \frac{1}{1} + 1 - 1 = 0$$

5. The correct answer is 16. Express fractional exponents as roots, calculate the value of each term, and then add:

$$4^{3/2} + 4^{3/2} = \sqrt{4^3} + \sqrt{4^3} = \sqrt{64} + \sqrt{64} = 8 + 8 = 16$$

Exercise 5

1. **(A)** One way to approach this problem is to substitute each answer choice for x in the function, then find $f(x)$. Only choice (A) provides a value for which $f(x) = x$:

$$f\left(\tfrac{1}{4}\right) = 2\left(\tfrac{1}{4}\right)\sqrt{\left(\tfrac{1}{4}\right)} = \left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right) = \tfrac{1}{4}$$

Another way to solve the problem is to let $x = 2x\sqrt{x}$, then solve for x by squaring both sides of the equation (look for a root that matches one of the answer choices):

$$x = 2x\sqrt{x}$$
$$1 = 2\sqrt{x}$$
$$\frac{1}{2} = \sqrt{x}$$
$$\frac{1}{4} = x$$

2. **(E)** First, note that any term raised to a negative power is equal to 1 divided by the term to the absolute value of the power. Hence:

$$a^{-3} - a^{-2} = \frac{1}{a^3} - \frac{1}{a^2}$$

Using this form of the function, substitute $\frac{1}{3}$ for a , then simplify and combine terms:

$$f\left(\tfrac{1}{3}\right) = \frac{1}{\left(\frac{1}{3}\right)^3} - \frac{1}{\left(\frac{1}{3}\right)^2} = \frac{1}{\frac{1}{27}} - \frac{1}{\frac{1}{9}} = 27 - 9 = 18$$

3. **(A)** In the function, substitute $(2 + a)$ for x. Since each of the answer choices indicates a quadratic expression, apply the distributive property of arithmetic, then combine terms:

$$\begin{aligned} f(2+a) &= (2+a)^2 + 3(2+a) - 4 \\ &= (2+a)(2+a) + 6 + 3a - 4 \\ &= 4 + 4a + a^2 + 6 + 3a - 4 \\ &= a^2 + 7a + 6 \end{aligned}$$

4. **(D)** Substitute $f(x)$ for x in the function $g(x) = x + 3$:

$$g(f(x)) = f(x) + 3$$

Then substitute x^2 for $f(x)$:

$$g(f(x)) = x^2 + 3$$

5. **(D)** $f(x^2) = \frac{x^2}{2}$, and $\left(f(x)\right)^2 = \left(\frac{x}{2}\right)^2$.

Accordingly, $f(x^2) \div \left(f(x)\right)^2 = \frac{x^2}{2} \div \left(\frac{x}{2}\right)^2 = \frac{x^2}{2} \cdot \frac{4}{x^2} = 2$.

Exercise 6

1. **(B)** To determine the function's range, apply the rule $\sqrt{x+1}$ to 3, 8, and 15:

$$\sqrt{(3)+1} = \sqrt{4} = +2$$
$$\sqrt{(8)+1} = \sqrt{9} = +3$$
$$\sqrt{(15)+1} = \sqrt{16} = +4$$

Choice (B) provides the members of the range. Remember that $\sqrt{x}$ means the *positive* square root of x.

2. **(E)** To determine the function's range, apply the rule $(6a - 4)$ to -6 and to 4. The range consists of all real numbers between the two results:

$$6(-6) - 4 = -40$$
$$6(4) - 4 = 20$$

The range of the function can be expressed as the set $R = \{b \mid -40 < b < 20\}$. Of the five answer choices, only (E) does not fall within the range.

3. **(D)** The function's range contains only one member: the number 0 (zero). Accordingly, to find the domain of x, let $f(x) = 0$, and solve for all possible roots of x:

$$x^2 - 2x - 3 = 0$$
$$(x-3)(x+1) = 0$$
$$x - 3 = 0,\ x + 1 = 0$$
$$x = 3,\ x = -1$$

Given that $f(x) = 0$, the largest possible domain of x is the set $\{3, -1\}$.

4. **(B)** The question asks you to recognize the set of values outside the domain of x. To do so, first factor the trinomial within the radical into two binomials:

$$f(x) = \sqrt{x^2 - 5x + 6} = \sqrt{(x-3)(x-2)}$$

The function is undefined for all values of x such that $(x - 3)(x - 2) < 0$ because the value of the function would be the square root of a negative number (not a real number). If $(x - 3)(x - 2) < 0$, then one binomial value must be negative while the other is positive. You obtain this result with any value of x greater than 2 but less than 3—that is, when $2 < x < 3$.

5. **(C)** If $x = 0$, then the value of the fraction is undefined; thus, 0 is outside the domain of x. However, the function can be defined for any other real-number value of x. (If $x > 0$, then applying the function yields a positive number; if $x < 0$, then applying the function yields a negative number.)

Exercise 7

1. **(E)** After the first 2 years, an executive's salary is raised from $80,000 to $81,000. After a total of 4 years, that salary is raised to $82,000. Hence, two of the function's (N,S) pairs are (2, $81,000) and (4, $82,000). Plugging both of these (N,S) pairs into each of the five equations, you see that only the equation in choice (E) holds (try plugging in additional pairs to confirm this result):

$$(81{,}000) = (500)(2) + 80{,}000$$
$$(82{,}000) = (500)(4) + 80{,}000$$
$$(83{,}000) = (500)(6) + 80{,}000$$

2. **(D)** The points (4,–9) and (–2,6) both lie on the graph of g, which is a straight line. The question asks for the line's y-intercept (the value of b in the general equation $y = mx + b$). First, determine the line's slope:

$$\text{slope } (m) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-(-9)}{-2-4} = \frac{15}{-6} = -\frac{5}{2}$$

In the general equation ($y = mx + b$), $m = -\frac{5}{2}$. To find the value of b, substitute either (x,y) value pair for x and y, then solve for b. Substituting the (x,y) pair (–2,6):

$$y = -\tfrac{5}{2}x + b$$
$$6 = -\tfrac{5}{2}(-2) + b$$
$$6 = 5 + b$$
$$1 = b$$

3. **(B)** In the xy-plane, the domain and range of any line other than a vertical or horizontal line is the set of all real numbers. Thus, option III (two vertical lines) is the only one of the three options that *cannot* describe the graphs of the two functions.

4. **(E)** The line shows a negative y-intercept (the point where the line crosses the vertical axis) and a negative slope less than –1 (that is, slightly more horizontal than a 45º angle). In equation (E), $-\frac{2}{3}$ is the slope and –3 is the y-intercept. Thus, equation (E) matches the graph of the function.

5. **(E)** The function h includes the two functional pairs (2,3) and (4,1). Since h is a linear function, its graph on the xy-plane is a straight line. You can determine the equation of the graph by first finding its slope (m):

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-3}{4-2} = \frac{-2}{2} = -1.$$

Plug either (x,y) pair into the standard equation $y = mx + b$ to define the equation of the line. Using the pair (2,3):

$$y = -x + b$$
$$3 = -2 + b$$
$$5 = b$$

The line's equation is $y = -x + 5$. To determine which of the five answer choices provides a point that also lies on this line, plug in the value –101 (as provided in the question) for x:

$y = -(-101) + 5 = 101 + 5 = 106$.

Exercise 8

1. **(D)** To solve this problem, consider each answer choice in turn, substituting the (x,y) pairs provided in the question for x and y in the equation. Among the five equations, only the equation in choice (D) holds for all four pairs.

2. **(A)** The graph shows a parabola opening to the right with vertex at (–2,2). If the vertex were at the origin, the equation defining the parabola might be $x = y^2$. Choices (D) and (E) define vertically oriented parabolas (in the general form $y = x^2$) and thus can be eliminated. Considering the three remaining equations, (A) and (C) both hold for the (x,y) pair (–2,2), but (B) does not. Eliminate (B). Try substituting 0 for y in equations (A) and (C), and you'll see that only in equation (A) is the corresponding x-value greater than 0, as the graph suggests.

3. **(E)** The equation $y = \frac{x^2}{3}$ is a parabola with vertex at the origin and opening upward. To see that this is the case, substitute some simple values for x and solve for y in each case. For example, substituting 0, 3, and –3 for x gives us the three (x,y) pairs (0,0), (3,3), and (–3,3). Plotting these three points on the xy-plane, then connecting them with a curved line, suffices to show a parabola with vertex (0,0) — opening upward. Choice (E) provides an equation whose graph is identical to the graph of $y = \frac{x^2}{3}$, except translated three units to the left. To confirm this, again, substitute simple values for x and solve for y in each case. For example, substituting 0, –3, and –6 for x gives us the three (x,y) pairs (0,3), (–3,0), and (–6,–3). Plotting these three points on the xy-plane, then connecting them with a curved line, suffices to show the same parabola as the previous one, except with vertex (–3,0) instead of (0,0).

4. **(D)** The equation $|x| = \frac{1}{y^2}$ represents the union of the two equations $x = \frac{1}{y^2}$ and $-x = \frac{1}{y^2}$. The graph of the former equation is the hyperbola shown to the right of the y-axis in the figure, while the graph of the latter equation is the hyperbola shown to the left of the y-axis in the figure.

5. **(B)** In this problem, S is a function of P. The problem provides three (P,S) number pairs that satisfy the function: (1, 48,000), (2, 12,000) and (4, 3,000). For each of the answer choices, plug each of these three (P,S) pairs in the equation given. Only the equation given in choice (B) holds for all three (P,S) pairs:

$$48{,}000 = \frac{48{,}000}{(1)^2} = 48{,}000$$
$$12{,}000 = \frac{48{,}000}{(2)^2} = \frac{48{,}000}{4} = 12{,}000$$
$$3{,}000 = \frac{48{,}000}{(4)^2} = \frac{48{,}000}{16} = 3{,}000$$

Retest

1. The correct answer is 432. First, find r:

$$2 \times r^{(3-1)} = 72$$
$$2 \times r^2 = 72$$
$$r^2 = 36$$
$$r = 6$$

To find the fourth term in the sequence, solve for T in the standard equation (let $r = 6$ and $n = 4$):

$$2 \times 6^{(4-1)} = T$$
$$2 \times 6^3 = T$$
$$2 \times 216 = T$$
$$432 = T$$

2. **(D)** The set of positive integers divisible by 4 includes all multiples of 4: 4, 8, 12, 16, ... The set of positive integers divisible by 6 includes all multiples of 6: 6, 12, 18, 24, ... The least common multiple of 4 and 6 is 12. Thus, common to the two sets are all multiples of 12, but no other elements.

3. **(B)** The shaded region to the left of the y-axis accounts for all values of x that are less than or equal to –3 . In other words, this region is the graph of $x \le -3$. The shaded region to the right of the y-axis accounts for all values of x that are greater than or equal to 3 . In other words, this region is the graph of $x \ge 3$.

4. **(C)** Note that $(-2)^5 = -32$. So, the answer to the problem must involve the number 5. However, the 2 in the number $\frac{1}{2}$ is in the denominator, and you must move it to the numerator. Since a negative number reciprocates its base, $\left(-\frac{1}{2}\right)^{-5} = -32$.

5. **(E)** Substitute $\frac{1}{x+1}$ for x, then simplify:

$$f\left(\frac{1}{x+1}\right) = \frac{1}{\frac{1}{x+1}+1} = \frac{1}{\frac{1}{x+1}+\frac{x+1}{x+1}} = \frac{1}{\frac{1+(x+1)}{x+1}}$$
$$= \frac{x+1}{1+(x+1)} = \frac{x+1}{x+2}$$

6. **(C)** According to the function, if $x = 0$, then $y = 1$. (The function's range includes the number 1.) If you square any real number x other than 0, the result is a number greater than 0. Accordingly, for any non-zero value of x, $1 - x^2 < 1$. The range of the function includes 1 and all numbers less than 1.

7. **(C)** The graph of f is a straight line, one point on which is (–6,–2). In the general equation $y = mx + b$, $m = -2$. To find the value of b, substitute the (x,y) value pair (–6,–2) for x and y, then solve for b:

$$\begin{aligned} y &= -2x + b \\ (-2) &= -2(-6) + b \\ -2 &= 12 + b \\ -14 &= b \end{aligned}$$

The equation of the function's graph is $y = -2x - 14$. Plugging in each of the five (x,y) pairs given, you can see that this equation holds only for choice (C).

8. **(C)** You can easily eliminate choices (A) and (B) because each one expresses speed (s) as a function of miles (m), just the reverse of what the question asks for. After the first 50 miles, the plane's speed decreases from 300 mph to 280 mph. After a total of 100 miles, the speed has decreased to 260. Hence, two of the function's (s,m) pairs are (280,50) and (260,100). Plugging both of these (s,m) pairs into each of the five equations, you see that only the equation in choice (C) holds (try plugging in additional pairs to confirm this result):

$$\begin{aligned} (50) &= -\frac{5(280)}{2} + 750 \\ 50 &= -\frac{1400}{2} + 750 \\ 50 &= -700 + 750 \\ 50 &= 50 \end{aligned}$$

$$\begin{aligned} (100) &= -\frac{5(260)}{2} + 750 \\ 100 &= --\frac{1300}{2} + 750 \\ 100 &= -650 + 750 \\ 100 &= 100 \end{aligned}$$

9. **(A)** The graph of any quadratic equation of the incomplete form $x = ay^2$ (or $y = ax^2$) is a parabola with vertex at the origin (0,0). Isolating x in the equation $3x = 2y^2$ shows that the equation is of that form:

$$x = \frac{2y^2}{3}$$

To confirm that the vertex of the graph of $x = \frac{2y^2}{3}$ lies at (0,0), substitute some simple values for y and solve for x in each case. For example, substituting 0, 1, and –1 for y gives us the three (x,y) pairs (0,0), ($\frac{2}{3}$,1), and ($\frac{2}{3}$,–1). Plotting these three points on the xy-plane, then connecting them with a curved line, suffices to show a parabola with vertex (0,0) — opening to the right.

10. **(D)** The question provides two (t,h) number pairs that satisfy the function: (2,96) and (3,96). For each of the answer choices, plug each of these two (t,h) pairs in the equation given. Only the equation given in choice (D) holds for both (t,h) pairs:

$(96) = 80(2) - 16(2)^2 = 160 - 64 = 96$

$(96) = 80(3) - 16(3)^2 = 240 - 144 = 96$

Note that the equation in choice (C) holds for $f(2) = 96$ but *not* for $f(3) = 96$.