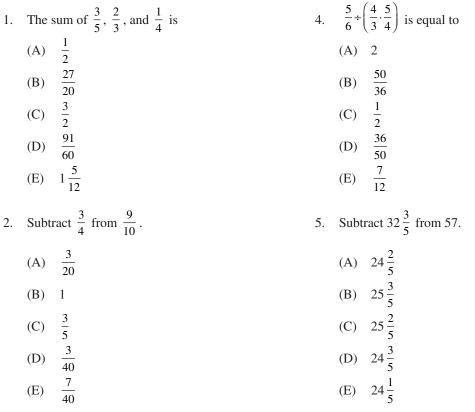# Operations with Fractions

2

DIAGNOSTIC TEST

Directions: Work out each problem. Circle the letter that appears before your answer.

Answers are at the end of the chapter.

1. The sum of $\frac{3}{5}$, $\frac{2}{3}$, and $\frac{1}{4}$ is

   (A) $\frac{1}{2}$

   (B) $\frac{27}{20}$

   (C) $\frac{3}{2}$

   (D) $\frac{91}{60}$

   (E) $1\frac{5}{12}$

2. Subtract $\frac{3}{4}$ from $\frac{9}{10}$.

   (A) $\frac{3}{20}$

   (B) 1

   (C) $\frac{3}{5}$

   (D) $\frac{3}{40}$

   (E) $\frac{7}{40}$

3. The number 582,354 is divisible by

   (A) 4
   (B) 5
   (C) 8
   (D) 9
   (E) 10

4. $\frac{5}{6} \div \left(\frac{4}{3} \cdot \frac{5}{4} \right)$ is equal to

   (A) 2

   (B) $\frac{50}{36}$

   (C) $\frac{1}{2}$

   (D) $\frac{36}{50}$

   (E) $\frac{7}{12}$

5. Subtract $32\frac{3}{5}$ from 57.

   (A) $24\frac{2}{5}$

   (B) $25\frac{3}{5}$

   (C) $25\frac{2}{5}$

   (D) $24\frac{3}{5}$

   (E) $24\frac{1}{5}$

6. Divide $4\frac{1}{2}$ by $1\frac{1}{8}$.

    (A) $\frac{1}{4}$

    (B) 4

    (C) $\frac{8}{9}$

    (D) $\frac{9}{8}$

    (E) $3\frac{1}{2}$

7. Which of the following fractions is the largest?

    (A) $\frac{1}{2}$

    (B) $\frac{11}{16}$

    (C) $\frac{5}{8}$

    (D) $\frac{21}{32}$

    (E) $\frac{3}{4}$

8. Which of the following fractions is closest

    to $\frac{2}{3}$?

    (A) $\frac{11}{15}$

    (B) $\frac{7}{10}$

    (C) $\frac{4}{5}$

    (D) $\frac{1}{2}$

    (E) $\frac{5}{6}$

9. Simplify $\dfrac{4-\dfrac{9}{10}}{\dfrac{2}{3}+\dfrac{1}{2}}$.

    (A) $\frac{93}{5}$

    (B) $\frac{93}{35}$

    (C) $\frac{147}{35}$

    (D) $\frac{147}{5}$

    (E) $\frac{97}{35}$

10. Find the value of $\dfrac{\dfrac{1}{a}+\dfrac{1}{b}}{\dfrac{1}{a}-\dfrac{1}{b}}$ when $a = 3$, $b = 4$.

    (A) 7

    (B) 2

    (C) 1

    (D) $\frac{1}{7}$

    (E) $\frac{2}{7}$

1. ADDITION AND SUBTRACTION

To add or subtract fractions, they must have the same *denominator*. To add several fractions, this common denominator will be the least number into which each given denominator will divide evenly.

Example:

Add $\dfrac{1}{2}+\dfrac{1}{3}+\dfrac{1}{4}+\dfrac{1}{5}$

Solution:

The common denominator must contain two factors of 2 to accommodate the 4, and also a factor of 3 and one of 5. That makes the least common denominator 60. Rename each fraction to have 60 as the denominator by dividing the given denominator into 60 and multiplying the quotient by the given numerator.

$$\frac{30}{60}+\frac{20}{60}+\frac{15}{60}+\frac{12}{60}=\frac{77}{60}=1\frac{17}{60}$$

When only two fractions are being added, a shortcut method can be used: $\dfrac{a}{b}+\dfrac{c}{d}=\dfrac{ad+bc}{bd}$. That is, in order to add two fractions, add the two cross products and place this sum over the product of the given denominators.

Example:

$$\frac{4}{5}+\frac{7}{12}$$

Solution:

$$\frac{4(12)+5(7)}{5(12)}=\frac{48+35}{60}=\frac{83}{60}=1\frac{23}{60}$$

A similar shortcut applies to the subtraction of two fractions:

$$\frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$$

Example:

$$\frac{4}{5}-\frac{7}{12}=\frac{4(12)-5(7)}{5(12)}=\frac{48-35}{60}=\frac{13}{60}$$

Exercise 1

Work out each problem. Circle the letter that appears before your answer.

1.   The sum of $\dfrac{1}{2}+\dfrac{2}{3}+\dfrac{3}{4}$ is

(A) $\dfrac{6}{9}$

(B) $\dfrac{23}{12}$

(C) $\dfrac{23}{36}$

(D) $\dfrac{6}{24}$

(E) $2\dfrac{1}{3}$

2.   The sum of $\dfrac{5}{17}$ and $\dfrac{3}{15}$ is

(A) $\dfrac{126}{255}$

(B) $\dfrac{40}{255}$

(C) $\dfrac{8}{32}$

(D) $\dfrac{40}{32}$

(E) $\dfrac{126}{265}$

3.   From the sum of $\dfrac{3}{4}$ and $\dfrac{5}{6}$ subtract the sum of $\dfrac{1}{4}$ and $\dfrac{2}{3}$.

(A)   2

(B) $\dfrac{1}{2}$

(C) $\dfrac{36}{70}$

(D) $\dfrac{2}{3}$

(E) $\dfrac{5}{24}$

4.   Subtract $\dfrac{3}{5}$ from $\dfrac{9}{11}$.

(A) $-\dfrac{12}{55}$

(B) $\dfrac{12}{55}$

(C)   1

(D) $\dfrac{3}{8}$

(E) $\dfrac{3}{4}$

5.   Subtract $\dfrac{5}{8}$ from the sum of $\dfrac{1}{4}$ and $\dfrac{2}{3}$.

(A)   2

(B) $\dfrac{3}{2}$

(C) $\dfrac{11}{24}$

(D) $\dfrac{8}{15}$

(E) $\dfrac{7}{24}$

2. MULTIPLICATION AND DIVISION

In multiplying fractions, always try to divide out any common factor of any denominator with any numerator to keep your numbers as small as possible. Remember that if all numbers divide out in the numerator, you are left with a numerator of 1. The same goes for the denominator. If all numbers in both numerator and denominator divide out, you are left with $\frac{1}{1}$ or 1.

Example:

$$\text{Multiply } \frac{3}{5} \cdot \frac{15}{33} \cdot \frac{11}{45}$$

Solution:

$$\frac{\cancel{3}}{\cancel{5}} \cdot \frac{\cancel{15}}{\cancel{33}} \cdot \frac{\cancel{11}}{\cancel{45}} = \frac{1}{15}$$

In dividing fractions, we multiply by the multiplicative inverse.

Example:

$$\text{Divide } \frac{5}{18} \text{ by } \frac{5}{9}$$

Solution:

$$\frac{\cancel{5}}{\cancel{18}} \cdot \frac{\cancel{9}}{\cancel{5}} = \frac{1}{2}$$

Exercise 2

Work out each problem. Circle the letter that appears before your answer.

1. Find the product of $\frac{3}{2}$, 6, $\frac{4}{9}$, and $\frac{1}{12}$.

    (A)   3

    (B) $\frac{1}{3}$

    (C) $\frac{14}{23}$

    (D) $\frac{1}{36}$

    (E) $\frac{5}{12}$

2. Find $\frac{7}{8} \cdot \frac{2}{3} \div \frac{1}{8}$.

    (A) $\frac{3}{14}$

    (B) $\frac{7}{96}$

    (C) $\frac{21}{128}$

    (D) $\frac{14}{3}$

    (E) $\frac{8}{3}$

3. $\frac{3}{5} \div \left(\frac{1}{2} \cdot \frac{3}{10} \right)$ is equal to

    (A)   4

    (B) $\frac{1}{4}$

    (C) $\frac{12}{5}$

    (D) $\frac{5}{12}$

    (E) $\frac{12}{15}$

4. Find $\frac{2}{3}$ of $\frac{7}{12}$.

    (A) $\frac{7}{8}$

    (B) $\frac{7}{9}$

    (C) $\frac{8}{7}$

    (D) $\frac{8}{9}$

    (E) $\frac{7}{18}$

5. Divide 5 by $\frac{5}{12}$.

    (A) $\frac{25}{12}$

    (B) $\frac{1}{12}$

    (C) $\frac{5}{12}$

    (D)   12

    (E) $\frac{12}{5}$

3. SIMPLIFYING FRACTIONS

All fractional answers should be left in simplest form. There should be no factor that can still be divided into numerator and denominator. In simplifying fractions involving very large numbers, it is helpful to tell at a glance whether or not a given number will divide evenly into both numerator and denominator. Certain tests for divisibility assist with this.

If a number is divisible by	Then
2	its last digit is 0, 2, 4, 6, or 8
3	the sum of the digits is divisible by 3
4	the number formed by the last 2 digits is divisible by 4
5	the last digit is 5 or 0
6	the number meets the tests for divisibility by 2 and 3
8	the number formed by the last 3 digits is divisible by 8
9	the sum of the digits is divisible by 9

Example:

By what single digit number should we simplify $\frac{135,492}{428,376}$?

Solution:

Since both numbers are even, they are at least divisible by 2. The sum of the digits in the numerator is 24. The sum of the digits in the denominator is 30. Since these sums are both divisible by 3, each number is divisible by 3. Since these numbers meet the divisibility tests for 2 and 3, they are each divisible by 6.

Example:

Simplify to simplest form: $\frac{43,672}{52,832}$

Solution:

Since both numbers are even, they are at least divisible by 2. However, to save time, we would like to divide by a larger number. The sum of the digits in the numerator is 22, so it is not divisible by 3. The number formed by the last two digits of each number is divisible by 4, making the entire number divisible by 4. The numbers formed by the last three digits of each number is divisible by 8. Therefore, each number is divisible by 8. Dividing by 8, we have $\frac{5459}{6604}$. Since these numbers are no longer even and divisibility by 3 was ruled out earlier, there is no longer a single digit factor common to numerator and denominator. It is unlikely, at the level of this examination, that you will be called on to divide by a two-digit number.

Exercise 3

Work out each problem. Circle the letter that appears before your answer.

1. Which of the following numbers is divisible by 5 and 9?

    (A)   42,235
    (B)   34,325
    (C)   46,505
    (D)   37,845
    (E)   53,290

2. Given the number 83,21p, in order for this number to be divisible by 3, 6, and 9, p must be

    (A)   4
    (B)   5
    (C)   6
    (D)   0
    (E)   9

3. If $n!$ means $n(n - 1)(n - 2) ... (4)(3)(2)(1)$, so that $4! = (4)(3)(2)(1) = 24$, then 19! is divisible by

    I.   17
    II.   54
    III.   100
    IV.   39
    (A)   I and II only
    (B)   I only
    (C)   I and IV only
    (D)   I, II, III, and IV
    (E)   none of the above

4. The fraction $\dfrac{432}{801}$ can be simplified by dividing numerator and denominator by

    (A)   2
    (B)   4
    (C)   6
    (D)   8
    (E)   9

5. The number 6,862,140 is divisible by

    I.   3
    II.   4
    III.   5
    (A)   I only
    (B)   I and III only
    (C)   II and III only
    (D)   I, II, and III
    (E)   III only

4. OPERATIONS WITH MIXED NUMBERS

To add or subtract mixed numbers, it is again important to find common denominators. If it is necessary to borrow in subtraction, you must borrow in terms of the common denominator.

Example:

$$23\frac{1}{3} - 6\frac{2}{5}$$

Solution:

$$23\frac{1}{3} = 23\frac{5}{15}$$

$$-6\frac{2}{5} = -6\frac{6}{15}$$

Since we cannot subtract $\frac{6}{15}$ from $\frac{5}{15}$, we borrow $\frac{15}{15}$ from 23 and rewrite our problem as

$$22\frac{20}{15}$$
$$-6\frac{6}{15}$$

In this form, subtraction is possible, giving us an answer of $16\frac{14}{15}$.

Example:

Add $17\frac{3}{4}$ to $43\frac{3}{5}$

Solution:

Again we first rename the fractions to have a common denominator. This time it will be 20.

$$17\frac{3}{4} = \quad 17\frac{15}{20}$$

$$+43\frac{3}{5} = +43\frac{12}{20}$$

When adding, we get a sum of $60\frac{27}{20}$, which we change to $61\frac{7}{20}$.

To multiply or divide mixed numbers, always rename them as improper fractions first.

Example:

Multiply $3\frac{3}{5} \cdot 1\frac{1}{9} \cdot 2\frac{3}{4}$

Solution:

$$\frac{\cancel{18}^{2}}{\cancel{5}} \cdot \frac{\cancel{10}^{2}}{\cancel{9}} \cdot \frac{11}{\cancel{4}_{2}} = 11$$

Example:

Divide $3\frac{3}{4}$ by $5\frac{5}{8}$

Solution:

$$\frac{15}{4} \div \frac{45}{8} = \frac{\cancel{15}}{\cancel{4}} \cdot \frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{45}}} = \frac{2}{3}$$

Exercise 4

Work out each problem. Circle the letter that appears before your answer.

1. Find the sum of $1\frac{1}{6}$, $2\frac{2}{3}$, and $3\frac{3}{4}$.

    (A) $7\frac{5}{12}$

    (B) $6\frac{6}{13}$

    (C) $7\frac{7}{12}$

    (D) $6\frac{1}{3}$

    (E) $7\frac{1}{12}$

2. Subtract $45\frac{5}{12}$ from 61.

    (A) $15\frac{7}{12}$

    (B) $15\frac{5}{12}$

    (C) $16\frac{7}{12}$

    (D) $16\frac{5}{12}$

    (E) $17\frac{5}{12}$

3. Find the product of $32\frac{1}{2}$ and $5\frac{1}{5}$.

    (A) 26

    (B) 13

    (C) 169

    (D) $160\frac{1}{10}$

    (E) $160\frac{2}{7}$

4. Divide $17\frac{1}{2}$ by 70.

    (A) $\frac{1}{4}$

    (B) 4

    (C) $\frac{1}{2}$

    (D) $4\frac{1}{2}$

    (E) $\frac{4}{9}$

5. Find $1\frac{3}{4} \cdot 12 \div 8\frac{2}{5}$.

    (A) $\frac{2}{5}$

    (B) $\frac{5}{288}$

    (C) $2\frac{1}{5}$

    (D) $\frac{1}{2}$

    (E) $2\frac{1}{2}$

5. COMPARING FRACTIONS

There are two methods by which fractions may be compared to see which is larger (or smaller).

Method I—Rename the fractions to have the same denominator. When this is done, the fraction with the larger numerator is the larger fraction.

Example:

Which is larger, $\dfrac{5}{6}$ or $\dfrac{8}{11}$?

Solution:

The least common denominator is 66.

$\dfrac{5}{6} = \dfrac{55}{66} \qquad \dfrac{8}{11} = \dfrac{48}{66}$

Therefore, $\dfrac{5}{6}$ is the larger fraction.

Method II—To compare $\dfrac{a}{b}$ with $\dfrac{c}{d}$, compare the cross products as follows:

If $ad > bc$, then $\dfrac{a}{b} > \dfrac{c}{d}$

If $ad < bc$, then $\dfrac{a}{b} < \dfrac{c}{d}$

If $ad = bc$, then $\dfrac{a}{b} = \dfrac{c}{d}$

Using the example above, to compare $\dfrac{5}{6}$ with $\dfrac{8}{11}$, compare $5 \cdot 11$ with $6 \cdot 8$. Since $5 \cdot 11$ is greater, $\dfrac{5}{6}$ is the larger fraction.

Sometimes, a combination of these methods must be used in comparing a series of fractions. When a common denominator can be found easily for a series of fractions, Method I is easier. When a common denominator would result in a very large number, Method II is easier.

Example:

Which of the following fractions is the largest?

(A) $\dfrac{3}{5}$

(B) $\dfrac{21}{32}$

(C) $\dfrac{11}{16}$

(D) $\dfrac{55}{64}$

(E) $\dfrac{7}{8}$

Solution:

To compare the last four, we can easily use a common denominator of 64.

$\dfrac{21}{32} = \dfrac{42}{64} \qquad \dfrac{11}{16} = \dfrac{44}{64} \qquad \dfrac{55}{64} \qquad \dfrac{7}{8} = \dfrac{56}{64}$

The largest of these is $\dfrac{7}{8}$. Now we compare $\dfrac{7}{8}$ with $\dfrac{3}{5}$ using Method II. $7 \cdot 5 > 8 \cdot 3$; therefore, $\dfrac{7}{8}$ is the greatest fraction.

Exercise 5

Work out each problem. Circle the letter that appears before your answer.

1. Arrange these fractions in order of size, from largest to smallest: $\frac{4}{15}, \frac{2}{5}, \frac{1}{3}$.

   (A) $\frac{4}{15}, \frac{2}{5}, \frac{1}{3}$

   (B) $\frac{4}{15}, \frac{1}{3}, \frac{2}{5}$

   (C) $\frac{2}{5}, \frac{1}{3}, \frac{4}{15}$

   (D) $\frac{1}{3}, \frac{4}{15}, \frac{2}{5}$

   (E) $\frac{1}{3}, \frac{2}{5}, \frac{4}{15}$

2. Which of the following fractions is the smallest?

   (A) $\frac{3}{4}$

   (B) $\frac{5}{6}$

   (C) $\frac{7}{8}$

   (D) $\frac{19}{24}$

   (E) $\frac{13}{15}$

3. Which of the following fractions is the largest?

   (A) $\frac{3}{5}$

   (B) $\frac{7}{10}$

   (C) $\frac{5}{8}$

   (D) $\frac{3}{4}$

   (E) $\frac{13}{20}$

4. Which of the following fractions is closest to $\frac{3}{4}$?

   (A) $\frac{1}{2}$

   (B) $\frac{7}{12}$

   (C) $\frac{5}{6}$

   (D) $\frac{11}{12}$

   (E) $\frac{19}{24}$

5. Which of the following fractions is closest to $\frac{1}{2}$?

   (A) $\frac{5}{12}$

   (B) $\frac{8}{15}$

   (C) $\frac{11}{20}$

   (D) $\frac{31}{60}$

   (E) $\frac{7}{15}$

6. COMPLEX FRACTIONS

To simplify complex fractions, fractions that contain fractions within them, multiply every term by the smallest number needed to clear all fractions in the given numerator and denominator.

Example:

$$\dfrac{\dfrac{1}{6}+\dfrac{1}{4}}{\dfrac{1}{2}+\dfrac{1}{3}}$$

Solution:

The smallest number into which 6, 4, 2, and 3 will divide is 12. Therefore, multiply every term of the fraction by 12 to simplify the fraction.

$$\dfrac{2+3}{6+4}=\dfrac{5}{10}=\dfrac{1}{2}$$

Example:

$$\dfrac{\dfrac{3}{4}-\dfrac{2}{3}}{1+\dfrac{1}{2}}$$

Solution:

Again, we multiply every term by 12. Be sure to multiply the 1 by 12 also.

$$\dfrac{9-8}{12+6}=\dfrac{1}{18}$$

Exercise 6

Work out each problem. Circle the letter that appears before your answer.

1. Write as a fraction in simplest form: $\dfrac{\dfrac{2}{3}+\dfrac{1}{6}+\dfrac{1}{4}}{\dfrac{2}{3}-\dfrac{1}{2}}$

   (A) $\dfrac{13}{2}$

   (B) $\dfrac{7}{2}$

   (C) $\dfrac{13}{4}$

   (D) $\dfrac{4}{13}$

   (E) $\dfrac{49}{12}$

2. Simplify: $\dfrac{\dfrac{5}{6}-\dfrac{2}{3}}{\dfrac{5}{12}-\dfrac{1}{6}}$

   (A) $\dfrac{5}{12}$

   (B) $\dfrac{5}{6}$

   (C) $\dfrac{2}{3}$

   (D) $\dfrac{1}{6}$

   (E) $\dfrac{7}{12}$

3. Find the value of $\dfrac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{ab}}$ when $a = 2$ and $b = 3$.

(A) $\dfrac{5}{6}$

(B) 5

(C) $4\dfrac{1}{6}$

(D) $1\dfrac{1}{5}$

(E) $2\dfrac{2}{5}$

4. Find the value of $\dfrac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{ab}}$ when $a = \dfrac{1}{2}$ and $b = \dfrac{1}{3}$.

(A) $\dfrac{5}{6}$

(B) 5

(C) $4\dfrac{1}{6}$

(D) $1\dfrac{1}{5}$

(E) $2\dfrac{2}{5}$

5. Find the value of $\dfrac{2\frac{1}{3}}{5\frac{1}{2}+3\frac{1}{3}}$.

(A) $\dfrac{4}{17}$

(B) $\dfrac{21}{25}$

(C) $\dfrac{7}{6}$

(D) $\dfrac{12}{51}$

(E) $\dfrac{14}{53}$

RETEST

Work out each problem. Circle the letter that appears before your answer.

1. The sum of $\frac{4}{5}$, $\frac{3}{4}$, and $\frac{1}{3}$ is

   (A) $\frac{8}{12}$

   (B) $\frac{113}{60}$

   (C) $\frac{1}{5}$

   (D) $\frac{10}{9}$

   (E) $\frac{11}{6}$

2. Subtract $\frac{2}{3}$ from $\frac{11}{15}$.

   (A) $\frac{3}{4}$

   (B) $\frac{7}{5}$

   (C) $\frac{5}{7}$

   (D) $\frac{1}{15}$

   (E) $\frac{1}{3}$

3. If 52,34p is divisible by 9, the digit represented by p must be
   (A) 1
   (B) 2
   (C) 3
   (D) 4
   (E) 5

4. $\left(\frac{3}{5}+\frac{1}{4}\right)\div\frac{34}{15}$ is equal to

   (A) $\frac{5}{3}$

   (B) $\frac{5}{8}$

   (C) $\frac{8}{3}$

   (D) $\frac{8}{5}$

   (E) $\frac{3}{8}$

5. Subtract $62\frac{2}{3}$ from 100.

   (A) $37\frac{1}{3}$

   (B) $38\frac{1}{3}$

   (C) $37\frac{2}{3}$

   (D) $38\frac{2}{3}$

   (E) $28\frac{2}{3}$

6. Divide $2\frac{2}{5}$ by $4\frac{8}{10}$.

   (A) 2

   (B) $\frac{1}{2}$

   (C) $\frac{288}{25}$

   (D) $\frac{25}{288}$

   (E) $2\frac{1}{4}$

7. Which of the following fractions is the smallest?

   (A) $\frac{7}{12}$

   (B) $\frac{8}{15}$

   (C) $\frac{11}{20}$

   (D) $\frac{5}{6}$

   (E) $\frac{2}{3}$

8. Which of the following fractions is closest to $\frac{1}{4}$?

   (A) $\frac{4}{15}$

   (B) $\frac{3}{10}$

   (C) $\frac{3}{20}$

   (D) $\frac{1}{5}$

   (E) $\frac{1}{10}$

9. Simplify: $\dfrac{\dfrac{5}{2}+\dfrac{2}{3}}{\dfrac{3}{4}+\dfrac{5}{6}}$

    (A) 2

    (B) $\dfrac{1}{2}$

    (C) 12

    (D) $\dfrac{1}{4}$

    (E) 4

10. Find the value of $\dfrac{\dfrac{1}{ab}}{\dfrac{1}{a}+\dfrac{1}{b}}$ when $a = 4$, $b = 5$.

    (A) 9

    (B) 20

    (C) $\dfrac{1}{9}$

    (D) $\dfrac{1}{20}$

    (E) $\dfrac{9}{40}$

SOLUTIONS TO PRACTICE EXERCISES

Diagnostic Test

1. (D) Change all fractions to sixtieths.

$$\frac{36}{60}+\frac{40}{60}+\frac{15}{60}=\frac{91}{60}$$

2. (A) $\dfrac{9}{10}-\dfrac{3}{4}=\dfrac{36-30}{40}=\dfrac{6}{40}=\dfrac{3}{20}$

3. (D) The sum of the digits is 27, which is divisible by 9.

4. (C) $\dfrac{5}{6}\div\left(\dfrac{4}{3}\cdot\dfrac{5}{4}\right)=\dfrac{5}{6}\div\dfrac{5}{3}=\dfrac{\cancel{5}}{\cancel{6}_2}\cdot\dfrac{\cancel{3}}{\cancel{5}}=\dfrac{1}{2}$

5. (A) $57\;=56\dfrac{5}{5}$

$$32\dfrac{3}{5}=32\dfrac{3}{5}$$

$$24\dfrac{2}{5}$$

6. (B) $\dfrac{9}{2}\div\dfrac{9}{8}=\dfrac{\cancel{9}}{\cancel{2}}\cdot\dfrac{\cancel{8}^{\,4}}{\cancel{9}}=4$

7. (E) Use a common denominator of 32.

$$\frac{1}{2}=\frac{16}{32}\qquad\frac{11}{16}=\frac{22}{32}\qquad\frac{5}{8}=\frac{20}{32}\qquad\frac{21}{32}$$

$$\frac{3}{4}=\frac{24}{32}$$

Of these, $\dfrac{3}{4}$ is the largest.

8. (B) Use a common denominator of 30.

$$\frac{11}{15}=\frac{22}{30}\qquad\frac{7}{10}=\frac{21}{30}\qquad\frac{4}{5}=\frac{24}{30}$$

$$\frac{1}{2}=\frac{15}{30}\qquad\frac{5}{6}=\frac{25}{30}$$

Since $\dfrac{2}{3}=\dfrac{20}{30}$, the answer closest to $\dfrac{2}{3}$ is $\dfrac{7}{10}$.

9. (B) Multiply every term of the fraction by 30.

$$\frac{120-27}{20+15}=\frac{93}{35}$$

10. (A) $\dfrac{\dfrac{1}{3}+\dfrac{1}{4}}{\dfrac{1}{3}-\dfrac{1}{4}}$

Multiply every term by 12.

$$\frac{4+3}{4-3}=7$$

Exercise 1

1. (B) Change all fractions to twelfths.

$$\frac{6}{12}+\frac{8}{12}+\frac{9}{12}=\frac{23}{12}$$

2. (A) Use the cross product method.

$$\frac{5(15)+17(3)}{17(15)}=\frac{75+51}{255}=\frac{126}{255}$$

3. (D) $\dfrac{3}{4}+\dfrac{5}{6}=\dfrac{18+20}{24}=\dfrac{38}{24}=\dfrac{19}{12}$

$$\frac{1}{4}+\frac{2}{3}=\frac{3+8}{12}=\frac{11}{12}$$

$$\frac{19}{12}-\frac{11}{12}=\frac{8}{12}=\frac{2}{3}$$

4. (B) $\dfrac{9}{11}-\dfrac{3}{5}=\dfrac{45-33}{55}=\dfrac{12}{55}$

5. (E) $\dfrac{1}{4}+\dfrac{2}{3}=\dfrac{3+8}{12}=\dfrac{11}{12}$

$$\frac{11}{12}-\frac{5}{8}=\frac{88-60}{96}=\frac{28}{96}=\frac{7}{24}$$

Exercise 2

1. (B) $\dfrac{\cancel{3}}{\cancel{2}}\cdot\dfrac{\cancel{6}}{1}\cdot\dfrac{\cancel{4}^{\,2}}{\cancel{9}_3}\cdot\dfrac{1}{\cancel{12}_2}=\dfrac{1}{3}$

2. (D) $\dfrac{7}{\cancel{8}}\cdot\dfrac{2}{3}\cdot\dfrac{\cancel{8}}{1}=\dfrac{14}{3}$

3. (A) $\dfrac{3}{5}\div\dfrac{3}{20}$

$$\frac{\cancel{3}}{\cancel{5}}\cdot\frac{\cancel{20}^{\,4}}{\cancel{3}}=4$$

4. (E) $\dfrac{\cancel{2}}{3}\cdot\dfrac{7}{\cancel{12}_6}=\dfrac{7}{18}$

5. (D) $\dfrac{\cancel{5}}{1}\cdot\dfrac{12}{\cancel{5}}=12$

Exercise 3

1. (D) The digits must add to a number divisible by 9. All answers are divisible by 5. $3 + 7 + 8 + 4 + 5 = 27$, which is divisible by 9.

2. (A) The sum of the digits must be divisible by 9, and the digit must be even. $8 + 3 + 2 + 1 = 14$. Therefore, we choose (A) because $14 + 4 = 18$, which is divisible by 9.

3. (D) $19! = 19 \cdot 18 \cdot 17 \cdot 16 \ldots 3 \cdot 2 \cdot 1$. This is divisible by 17, since it contains a factor of 17. It is divisible by 54, since it contains factors of 9 and 6. It is divisible by 100, since it contains factors of 10, 5, and 2. It is divisible by 39, since it contains factors of 13 and 3.

4. (E) The sum of the digits in both the numerator and denominator are divisible by 9.

5. (D) The sum of the digits is 27, which is divisible by 3. The number formed by the last two digits is 40, which is divisible by 4. The number ends in 0 and is therefore divisible by 5.

Exercise 4

1. (C) $1\frac{1}{6} = 1\frac{2}{12}$

   $2\frac{2}{3} = 2\frac{8}{12}$

   $3\frac{3}{4} = 3\frac{9}{12}$

   $6\frac{19}{12} = 7\frac{7}{12}$

2. (A) $61 \quad = 60\frac{12}{12}$

   $\underline{45\frac{5}{12}} = \underline{45\frac{5}{12}}$

   $15\frac{7}{12}$

3. (C) $\dfrac{\overset{13}{\cancel{65}}}{\cancel{2}} \cdot \dfrac{\overset{13}{\cancel{26}}}{\cancel{5}} = 169$

4. (A) $17\frac{1}{2} \div 70 = \frac{35}{2} \div 70 = \frac{\cancel{35}}{2} \cdot \frac{1}{\cancel{70}} = \frac{1}{4}$

5. (E) $\dfrac{\cancel{7}}{\cancel{4}} \cdot \dfrac{\overset{3}{\cancel{12}}}{1} \cdot \dfrac{5}{\cancel{42}} = \frac{5}{2} = 2\frac{1}{2}$

Exercise 5

1. (C) $\frac{2}{5} = \frac{6}{15} \qquad \frac{1}{3} = \frac{5}{15}$

2. (A) To compare (A), (B), (C), and (D), use a common denominator of 24.

   $\frac{3}{4} = \frac{18}{24} \qquad \frac{5}{6} = \frac{20}{24} \qquad \frac{7}{8} = \frac{21}{24} \qquad \frac{19}{24}$

   Of these, $\frac{3}{4}$ is the smallest. To compare $\frac{3}{4}$ with $\frac{13}{15}$, use cross products. Since $(3)(15) < (4)(14)$, $\frac{3}{4} < \frac{13}{15}$. Therefore, (A) is the smallest.

3. (D) To compare (A), (B), (D), and (E), use a common denominator of 20.

   $\frac{3}{5} = \frac{12}{20} \qquad \frac{7}{10} = \frac{14}{20} \qquad \frac{3}{4} = \frac{15}{20} \qquad \frac{13}{20}$

   Of these, $\frac{3}{4}$ is the largest. To compare $\frac{3}{4}$ with $\frac{5}{8}$, use cross products. Since $(3)(8) > (4)(5)$, $\frac{3}{4}$ is the larger fraction.

4. (E) Use a common denominator of 24.

   $\frac{1}{2} = \frac{12}{24} \qquad \frac{7}{12} = \frac{14}{24} \qquad \frac{5}{6} = \frac{20}{24} \qquad \frac{11}{12} = \frac{22}{24}$

   $\frac{19}{24}$

   Since $\frac{3}{4} = \frac{18}{24}$, the answer closest to $\frac{3}{4}$ is (E), $\frac{19}{24}$.

5. (D) Use a common denominator of 60.

   $\frac{5}{12} = \frac{25}{60} \qquad \frac{8}{15} = \frac{32}{60} \qquad \frac{11}{20} = \frac{33}{60} \qquad \frac{31}{60}$

   $\frac{7}{15} = \frac{28}{60}$

   Since $\frac{1}{2} = \frac{30}{60}$, the answer closest to $\frac{1}{2}$ is (D), $\frac{31}{60}$.

Exercise 6

1. (A) Multiply every term of the fraction by 12.

$$\frac{8+2+3}{8-6}=\frac{13}{2}$$

2. (C) Multiply every term of the fraction by 12.

$$\frac{10-8}{5-2}=\frac{2}{3}$$

3. (B) $\dfrac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{6}}$ Multiply every term by 6.

$$\frac{3+2}{1}=5$$

4. (A) $\dfrac{1}{\frac{1}{2}}=2$ $\qquad \dfrac{1}{\frac{1}{3}}=3$ $\qquad \dfrac{1}{\frac{1}{6}}=6$

$$\frac{2+3}{6}=\frac{5}{6}$$

5. (E) $\dfrac{\frac{7}{3}}{\frac{11}{2}+\frac{10}{3}}$ Multiply every term by 6.

$$\frac{14}{33+20}=\frac{14}{53}$$

Retest

1. (B) Rename all fractions as sixtieths.

$$\frac{48}{60}+\frac{45}{60}+\frac{20}{60}=\frac{113}{60}$$

2. (D) $\dfrac{11}{15}-\dfrac{2}{3}=\dfrac{11}{15}-\dfrac{10}{15}=\dfrac{1}{15}$

3. (D) The sum of the digits must be divisible by 9.
   $5 + 2 + 3 + 4 + 4 = 18$, which is divisible by 9.

4. (E) $\dfrac{17}{20}\div\dfrac{34}{15}$

$$\frac{\overset{1}{\cancel{17}}}{\underset{4}{\cancel{20}}}\cdot\frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{34}}}=\frac{3}{8}$$

5. (A) $\begin{array}{r}100 \;=99\frac{3}{3}\\ -62\frac{2}{3}=62\frac{2}{3}\\ \hline 37\frac{1}{3}\end{array}$

6. (B) $\dfrac{12}{5}\div\dfrac{48}{10}=\dfrac{\overset{1}{\cancel{12}}}{\underset{1}{\cancel{5}}}\cdot\dfrac{\overset{2}{\cancel{10}}}{\underset{4}{\cancel{48}}}=\dfrac{2}{4}=\dfrac{1}{2}$

7. (B) Use a common denominator of 60.

$$\frac{7}{12}=\frac{35}{60} \qquad \frac{8}{15}=\frac{32}{60} \qquad \frac{11}{20}=\frac{33}{60} \qquad \frac{5}{6}=\frac{50}{60}$$
$$\frac{2}{3}=\frac{40}{60}$$

Of these, $\dfrac{8}{15}$ is the smallest.

8. (A) Use a common denominator of 60.

$$\frac{4}{15}=\frac{16}{60} \qquad \frac{3}{10}=\frac{18}{60} \qquad \frac{3}{20}=\frac{9}{60} \qquad \frac{1}{5}=\frac{12}{60}$$
$$\frac{1}{10}=\frac{6}{60}$$

Since $\dfrac{1}{4}=\dfrac{15}{60}$, the answer closest to $\dfrac{1}{4}$ is $\dfrac{4}{15}$.

9. (A) Multiply every term of the fraction by 12.

$$\frac{30+8}{9+10}=\frac{38}{19}=2$$

10. (C) $\dfrac{\frac{1}{20}}{\frac{1}{4}+\frac{1}{5}}$ Multiply every term by 20.

$$\frac{1}{5+4}=\frac{1}{9}$$