

DIAGNOSTIC TEST

Directions: Work out each problem. Circle the letter that appears before your answer.

Answers are at the end of the chapter.

- Write as a fraction: 4.5%
 - $\frac{9}{2}$
 - $\frac{9}{20}$
 - $\frac{9}{200}$
 - $\frac{9}{2000}$
 - $\frac{4.5}{10}$
- Write $\frac{2}{5}\%$ as a decimal.
 - .40
 - .04
 - 40.0
 - .004
 - 4.00
- What is $62\frac{1}{2}\%$ of 80?
 - 5000
 - 500
 - 50
 - 5
 - .5
- Find 6% of b .
 - .6b
 - .06b
 - $\frac{b}{6}$
 - $\frac{b}{.06}$
 - $\frac{100b}{6}$
- 80 is 40% of what number?
 - 3200
 - 320
 - 32
 - 200
 - 20
- c is $83\frac{1}{3}\%$ of what number?
 - $\frac{5c}{6}$
 - $\frac{6c}{5}$
 - $\frac{7c}{8}$
 - $\frac{8c}{7}$
 - $\frac{2c}{3}$
- How many sixteenths are there in $87\frac{1}{2}\%$?
 - 7
 - 8
 - 10
 - 12
 - 14
- What percent of 40 is 16?
 - $2\frac{1}{2}$
 - 25
 - 30
 - 40
 - 45

9. Find 112% of 80.

- (A) 92
- (B) 89.6
- (C) 88
- (D) 70.5
- (E) 91

10. What percent of 60 is 72?

- (A) 105
- (B) 125
- (C) 120
- (D) $83\frac{1}{3}$
- (E) 110

1. FRACTIONAL AND DECIMAL EQUIVALENTS OF PERCENTS

Percent means “out of 100.” If you understand this concept, it then becomes very easy to change a percent to an equivalent decimal or fraction.

Example:

5% means 5 out of 100 or $\frac{5}{100}$, which is equal to .05

3.4% means 3.4 out of 100 or $\frac{3.4}{100}$, which is equivalent to $\frac{34}{1000}$ or .034

$c\%$ means c out of 100 or $\frac{c}{100}$, which is equivalent to $\frac{1}{100} \cdot c$ or $.01c$

$\frac{1}{4}\%$ means $\frac{1}{4}$ out of 100 or $\frac{\frac{1}{4}}{100}$, which is equivalent to $\frac{1}{100} \cdot .25$ or .0025

To change a percent to a decimal, therefore, we must move the decimal point two places to the *left*, as we are dividing by 100.

Example:

$$62\% = .62$$

$$.4\% = .004$$

$$3.2\% = .032$$

To change a decimal to a percent, we must reverse the above steps. We multiply by 100, which has the effect of moving the decimal point two places to the *right*, and insert the percent sign.

Example:

$$.27 = 27\%$$

$$.012 = 1.2\%$$

$$.003 = .3\%$$

To change a percent to a fraction, we remove the percent sign and divide by 100. This has the effect of putting the percent over 100 and then simplifying the resulting fraction.

Example:

$$25\% = \frac{25}{100} = \frac{1}{4}$$

$$70\% = \frac{70}{100} = \frac{7}{10}$$

$$.5\% = \frac{.5}{100} = \frac{5}{1000} = \frac{1}{200}$$

To change a fraction to a percent, we must reverse the above steps. We multiply by 100 and insert the percent sign.

Example:

$$\frac{4}{5} = \frac{4}{5} \cdot \frac{20}{20} \cdot 100\% = 80\%$$

$$\frac{3}{8} = \frac{3}{8} \cdot \frac{25}{25} \cdot 100\% = \frac{75}{2} \% = 37\frac{1}{2}\%$$

Some fractions do not convert easily, as the denominator does not divide into 100. Such fractions must be changed to decimals first by dividing the numerator by the denominator. Then convert the decimal to a percent as explained on the previous page. Divide to two places only, unless it clearly comes out even in one or two additional places.

Example:

$$\frac{8}{17} = 17 \overline{)8.00} = 47 \frac{1}{17} \%$$

$$\begin{array}{r} \underline{68} \\ 17 \overline{)8.00} \\ \underline{120} \\ 119 \\ \underline{1} \end{array}$$

$$\frac{4}{125} = 125 \overline{)4.000} = 3.2\%$$

$$\begin{array}{r} \underline{375} \\ 125 \overline{)4.000} \\ \underline{250} \\ 250 \\ \underline{250} \end{array}$$

Certain fractional and decimal equivalents of common percents occur frequently enough so that they should be memorized. Learning the values in the following table will make your work with percent problems much easier.

PERCENT	DECIMAL	FRACTION
50%	.5	$\frac{1}{2}$
25%	.25	$\frac{1}{4}$
75%	.75	$\frac{3}{4}$
10%	.1	$\frac{1}{10}$
30%	.3	$\frac{3}{10}$
70%	.7	$\frac{7}{10}$
90%	.9	$\frac{9}{10}$
$33\frac{1}{3}\%$. $\overline{33}$	$\frac{1}{3}$
$66\frac{2}{3}\%$. $\overline{66}$	$\frac{2}{3}$
$16\frac{2}{3}\%$. $\overline{16}$	$\frac{1}{6}$
$83\frac{1}{3}\%$. $\overline{83}$	$\frac{5}{6}$
20%	.2	$\frac{1}{5}$
40%	.4	$\frac{2}{5}$
60%	.6	$\frac{3}{5}$
80%	.8	$\frac{4}{5}$
$12\frac{1}{2}\%$.125	$\frac{1}{8}$
$37\frac{1}{2}\%$.375	$\frac{3}{8}$
$62\frac{1}{2}\%$.625	$\frac{5}{8}$
$87\frac{1}{2}\%$.875	$\frac{7}{8}$

Exercise 1

Work out each problem. Circle the letter that appears before your answer.

1. $3\frac{1}{2}\%$ may be written as a decimal as

(A) 3.5
(B) .35
 (C) .035
(D) .0035
(E) 3.05

2. Write as a fraction in simplest form: 85%.

(A) $\frac{13}{20}$
 (B) $\frac{17}{20}$
(C) $\frac{17}{10}$
(D) $\frac{19}{20}$
(E) $\frac{17}{2}$

3. Write 4.6 as a percent.

(A) 4.6%
(B) .46%
(C) .046%
(D) 46%
(E) 460%

4. Write $\frac{5}{12}$ as an equivalent percent.

(A) 41%
(B) 41.6%
 (C) $41\frac{2}{3}\%$
(D) 4.1%
(E) $.41\frac{2}{3}\%$

5. Write $\frac{1}{2}\%$ as a decimal.

(A) .5
 (B) .005
(C) 5.0
(D) 50.0
(E) .05

2. FINDING A PERCENT OF A NUMBER

Most percentage problems can be solved by using the proportion

$$\frac{\%}{100} = \frac{\text{part}}{\text{whole}}$$

Although this method will work, it often yields unnecessarily large numbers that make for difficult computation. As we look at different types of percent problems, we will compare methods of solution. In finding a percent of a number, it is usually easier to change the percent to an equivalent decimal or fraction and multiply by the given number.

Example:

Find 32% of 84.

Proportion Method

$$\begin{aligned} \frac{32}{100} &= \frac{x}{84} \\ 100x &= 2688 \\ x &= 26.88 \end{aligned}$$

Decimal Method

Change 32% to .32 and multiply.

$$\begin{array}{r} 84 \\ \times .32 \\ \hline 168 \\ 252 \\ \hline 26.88 \end{array}$$

Example:

Find $12\frac{1}{2}\%$ of 112.

Proportion Method

$$\begin{aligned} \frac{12\frac{1}{2}}{100} &= \frac{x}{112} \\ 100x &= 1400 \\ x &= 14 \end{aligned}$$

Decimal Method

$$\begin{array}{r} 112 \\ \times .125 \\ \hline 560 \\ 224 \\ \hline 14.000 \end{array}$$

Fraction Method

Change $12\frac{1}{2}\%$ to $\frac{1}{8}$

$$\frac{1}{8} \cdot 112 = 14$$

Which method do you think is the easiest? When the fractional equivalent of the required percent is among those given in the previous chart, the fraction method is by far the least time-consuming. It really pays to memorize those fractional equivalents.

Exercise 2

Work out each problem. Circle the letter that appears before your answer.

1. What is 40% of 40?

- (A) .16
- (B) 1.6
- (C) 16
- (D) 160
- (E) 1600

2. What is 42% of 67?

- (A) 2814
- (B) 281.4
- (C) 2.814
- (D) .2814
- (E) 28.14

3. Find $16\frac{2}{3}\%$ of 120.

- (A) 20
- (B) 2
- (C) 200
- (D) 16
- (E) 32

4. What is $\frac{1}{5}\%$ of 40?

- (A) 8
- (B) .8
- (C) .08
- (D) .008
- (E) .0008

5. Find $r\%$ of s .

- (A) $\frac{100s}{r}$
- (B) $\frac{rs}{100}$
- (C) $\frac{100r}{s}$
- (D) $\frac{r}{100s}$
- (E) $\frac{s}{100r}$

3. FINDING A NUMBER WHEN A PERCENT OF IT IS GIVEN

This type of problem may be solved using the proportion method, although this may again result in the unnecessary use of time. It is often easier to translate the words of such a problem into an algebraic statement, using decimal or fractional equivalents for the percents involved. Then it will become evident that we divide the given number by the given percent to solve.

Example:

7 is 5% of what number?

Proportion Method

$$\begin{aligned}\frac{5}{100} &= \frac{7}{x} \\ 5x &= 700 \\ x &= 140\end{aligned}$$

Equation Method

$$\begin{aligned}7 &= .05x \\ 700 &= 5x \\ 140 &= x\end{aligned}$$

Example:

40 is $66\frac{2}{3}\%$ of what number?

Proportion Method

$$\begin{aligned}\frac{66\frac{2}{3}}{100} &= \frac{40}{x} \\ 66\frac{2}{3}x &= 4000 \\ \frac{200}{3}x &= 4000 \\ 200x &= 12000 \\ 2x &= 120 \\ x &= 60\end{aligned}$$

Equation Method

$$\begin{aligned}40 &= \frac{2}{3}x \\ 120 &= 2x \\ 60 &= x\end{aligned}$$

Just think of the amount of time you will save and the extra problems you will get to do if you know that $66\frac{2}{3}\%$ is $\frac{2}{3}$ and use the equation method. Are you convinced that the common fraction equivalents in the previously given chart should be memorized?

Exercise 3

Work out each problem. Circle the letter that appears before your answer.

1. 72 is 12% of what number?

(A) 6
(B) 60
 (C) 600
(D) 86.4
(E) 8.64

2. 80 is $12\frac{1}{2}\%$ of what number?

(A) 10
(B) 100
(C) 64
 (D) 640
(E) 6400

3. $37\frac{1}{2}\%$ of what number is 27?

(A) 72
(B) $10\frac{1}{8}$
(C) 90
(D) 101.25
(E) 216

4. m is $p\%$ of what number?

(A) $\frac{mp}{100}$
(B) $\frac{100p}{m}$
(C) $\frac{m}{100p}$
(D) $\frac{p}{100m}$
 (E) $\frac{100m}{p}$

5. 50% of what number is r ?

(A) $\frac{1}{2}r$
(B) $5r$
(C) $10r$
 (D) $2r$
(E) $100r$

4. TO FIND WHAT PERCENT ONE NUMBER IS OF ANOTHER

This type of problem may also be solved using the proportion method. However, this may again result in the use of an unnecessary amount of time. It is often easier to put the part over the whole, simplify the resulting fraction, and multiply by 100.

Example:

30 is what percent of 1500?

Proportion Method

$$\begin{aligned}\frac{x}{100} &= \frac{30}{1500} \\ 1500x &= 3000 \\ x &= 2\%\end{aligned}$$

Fraction Method

$$\frac{30}{1500} = \frac{3}{150} = \frac{1}{50} \cdot 100 = 2\%$$

Example:

12 is what percent of 72?

Proportion Method

$$\begin{aligned}\frac{x}{100} &= \frac{12}{72} \\ 72x &= 1200\end{aligned}$$

Fraction Method

$$\frac{12}{72} = \frac{1}{6} = 16\frac{2}{3}\%$$

Time consuming long division is needed to find $x = 16\frac{2}{3}\%$. If you have memorized the fractional equivalents of common percents, this method requires only a few seconds.

Example:

What percent of 72 is 16?

Proportion Method

$$\begin{aligned}\frac{x}{100} &= \frac{16}{72} \\ 72x &= 1600 \\ x &= 22\frac{2}{9}\%\end{aligned}$$

Fraction Method

$$\frac{16}{72} = \frac{2}{9} \cdot 100 = \frac{200}{9} = 22\frac{2}{9}\%$$

Exercise 4

Work out each problem. Circle the letter that appears before your answer.

1. 4 is what percent of 80?

(A) 20
(B) 2
(C) 5
(D) .5
(E) 40

2. $\frac{1}{2}$ of 6 is what percent of $\frac{1}{4}$ of 60?

(A) 5
(B) 20
(C) 10
(D) 25
(E) 15

3. What percent of 96 is 12?

(A) $16\frac{2}{3}$
(B) $8\frac{1}{3}$
(C) $37\frac{1}{2}$
(D) 8
(E) $12\frac{1}{2}$

4. What percent of 48 is 48?

(A) 1
(B) 10
(C) 100
(D) 48
(E) 0

5. What percent of y is x ?

(A) $\frac{x}{y}$
(B) $\frac{x}{100y}$
(C) $\frac{xy}{100}$
(D) $\frac{100x}{y}$
(E) $\frac{100y}{x}$

5. PERCENTS GREATER THAN 100

When the percentage involved in a problem is greater than 100, the same methods apply. Remember that $100\% = 1$; $200\% = 2$; $300\% = 3$ and so forth. Therefore 150% will be equal to $100\% + 50\%$ or $1\frac{1}{2}$. Let us look at one example of each previously discussed problem, using percents greater than 100.

Example:

Find 175% of 60

Proportion Method

$$\begin{aligned}\frac{175}{100} &= \frac{x}{60} \\ 100x &= 10500 \\ x &= 105\end{aligned}$$

Decimal Method

$$\begin{array}{r} 60 \\ \times 1.75 \\ \hline 300 \\ 4200 \\ \hline 6000 \\ 105.00 \end{array}$$

Fraction Method

$$\begin{aligned}1\frac{3}{4} \cdot 60 \\ \frac{7}{4} \cdot \overset{15}{\cancel{60}} &= 105\end{aligned}$$

Example:

80 is 125% of what number?

Proportion Method

$$\begin{aligned}\frac{125}{100} &= \frac{80}{x} \\ 125x &= 8000 \\ x &= 64\end{aligned}$$

Decimal Method

$$\begin{aligned}80 &= 1.25x \\ 8000 &= 125x \\ x &= 64\end{aligned}$$

Fraction Method

$$\begin{aligned}80 &= 1\frac{1}{4}x \\ 80 &= \frac{5}{4}x \\ 320 &= 5x \\ x &= 64\end{aligned}$$

Example:

40 is what percent of 30?

Proportion Method

$$\begin{aligned}\frac{x}{100} &= \frac{40}{30} \\ 30x &= 4000 \\ x &= 133\frac{1}{3}\%\end{aligned}$$

Fraction Method

$$\frac{40}{30} = \frac{4}{3} = 1\frac{1}{3} = 133\frac{1}{3}\%$$

Exercise 5

Work out each problem. Circle the letter that appears before your answer.

1. 36 is 150% of what number?

(A) 24
(B) 54
(C) 26
(D) 12
(E) 48

2. What is 300% of 6?

(A) 2
(B) 3
(C) 12
(D) 18
(E) 24

3. What percent of 90 is 120?

(A) 75
(B) $133\frac{1}{3}$
(C) 125
(D) 120
(E) $1\frac{1}{3}$

4. 500 is 200% of what number?

(A) 250
(B) 1000
(C) 100
(D) 750
(E) 300

5. To multiply a number by $137\frac{1}{2}\%$, the number should be multiplied by

(A) 137.5
(B) 13750
(C) 1.375
(D) 13.75
(E) .1375

RETEST

Work out each problem. Circle the letter that appears before your answer.

1. Write as a fraction in lowest terms: .25%.

(A) $\frac{1}{4}$
 (B) $\frac{1}{40}$
 (C) $\frac{1}{400}$
 (D) $\frac{1}{4000}$
 (E) $\frac{1}{25}$

2. Write $\frac{3}{4}\%$ as a decimal.

(A) .75
 (B) 75.0
 (C) .075
 (D) .0075
 (E) 7.5

3. Find 12% of 80.

(A) 10
 (B) .96
 (C) .096
 (D) 960
 (E) 9.6

4. 18 is 20% of what number?

(A) 3.6
 (B) 90
 (C) 72
 (D) 21.6
 (E) 108

5. What is $b\%$ of 6?

(A) $\frac{3b}{50}$
 (B) $\frac{3}{50b}$
 (C) $\frac{50b}{3}$
 (D) $\frac{50}{3b}$
 (E) $\frac{b}{150}$

6. m is $62\frac{1}{2}\%$ of what number?

(A) $\frac{5m}{8}$
 (B) $\frac{8m}{5}$
 (C) $8m$
 (D) $\frac{5}{8m}$
 (E) $\frac{8}{5m}$

7. What percent of 12 is 2?

(A) 600
 (B) $12\frac{1}{2}$
 (C) $16\frac{2}{3}$
 (D) $6\frac{2}{3}$
 (E) 6

8. What is 140% of 70?

(A) 9800
 (B) 980
 (C) .98
 (D) 9.8
 (E) 98

9. How many fifths are there in 280%?

(A) 28
 (B) 1.4
 (C) 14
 (D) 56
 (E) 2.8

10. What percent of 12 is 16?

(A) $133\frac{1}{3}$
 (B) 125
 (C) 75
 (D) 80
 (E) $1\frac{1}{4}$

SOLUTIONS TO PRACTICE EXERCISES

Diagnostic Test

- (C) $4.5\% = \frac{4.5}{100} = \frac{45}{1000} = \frac{9}{200}$
- (D) $\frac{2}{5}\% = .4\% = .004$
- (C) $62\frac{1}{2}\% = \frac{5}{8} \cdot \frac{5}{8} \cdot \cancel{80} = 50$
- (B) $6\% = .06 \quad .06 \cdot b = .06b$
- (D) $80 = .40x \quad \text{Divide by } .40.$
 $200 = x$
- (B) $83\frac{1}{3}\% = \frac{5}{6} \quad \text{Multiply by 6. Divide by 5.}$
 $c = \frac{5}{6}x$
 $6c = 5x$
 $\frac{6c}{5} = x$
- (E) $87\frac{1}{2}\% = \frac{7}{8} = \frac{14}{16}$
- (D) $\frac{16}{40} = \frac{2}{5} = 40\%$
- (B) $112\% = 1.12$
 $1.12 \cdot 80 = 89.6$
- (C) $\frac{72}{60} = \frac{6}{5} = 120\%$

Exercise 1

- (C) $3\frac{1}{2}\% = 3.5\% = .035$ To change a percent to a decimal, move the decimal point two places to the *left*.
- (B) $85\% = \frac{85}{100} = \frac{17}{20}$
- (E) To change a decimal to a percent, move the decimal point two places to the *right*.
 $4.6 = 460\%$
- (C) $\frac{5}{12} \cdot \cancel{100}^{25} = \frac{125}{3} = 41\frac{2}{3}\%$
To change a fraction to a percent, multiply by 100.
- (B) $\frac{1}{2}\% = .5\% = .005$

Exercise 2

- (C) $40\% = \frac{2}{5}$
 $\frac{2}{5} \cdot \cancel{40}^8 = 16$
- (E) 67
 $\times \underline{42}$
 134
 $\underline{2680}$
 28.14
- (A) $16\frac{2}{3}\% = \frac{1}{6}$
 $\frac{1}{6} \cdot \cancel{120}^{20} = 20$
- (C) $\frac{1}{5}\% = .2\% = .002$
 40
 $\times \underline{.002}$
 $.0800$
- (B) $r\% = \frac{r}{100}$
 $\frac{r}{100} \cdot s = \frac{rs}{100}$

Exercise 3

$$1. \text{ (C)} \quad \begin{aligned} 72 &= .12x \\ 7200 &= 12x \\ x &= 600 \end{aligned}$$

$$2. \text{ (D)} \quad \begin{aligned} 80 &= \frac{1}{8}x \\ 640 &= x \end{aligned}$$

$$3. \text{ (A)} \quad \begin{aligned} \frac{3}{8}x &= 27 \\ 3x &= 216 \\ x &= 72 \end{aligned}$$

$$4. \text{ (E)} \quad \begin{aligned} m &= \frac{p}{100} \cdot x \\ 100m &= px \\ \frac{100m}{p} &= x \end{aligned}$$

$$5. \text{ (D)} \quad \begin{aligned} \frac{1}{2}x &= r \\ x &= 2r \end{aligned}$$

Exercise 4

$$1. \text{ (C)} \quad \frac{4}{80} = \frac{1}{20} \cdot 100 = 5\%$$

$$2. \text{ (B)} \quad \frac{1}{2} \text{ of } 6 = 3$$

$$\frac{1}{4} \text{ of } 60 = 15$$

$$\frac{3}{15} = \frac{1}{5} = 20\%$$

$$3. \text{ (E)} \quad \frac{12}{96} = \frac{1}{8} = 12\frac{1}{2}\%$$

$$4. \text{ (C)} \quad \frac{48}{48} = 1 = 100\%$$

$$5. \text{ (D)} \quad \frac{x}{y} \cdot 100 = \frac{100x}{y}$$

Exercise 5

$$1. \text{ (A)} \quad 36 = 1\frac{1}{2}x$$

$$36 = \frac{3}{2}x$$

$$72 = 3x$$

$$x = 24$$

$$2. \text{ (D)} \quad \begin{aligned} 300\% &= 3 \\ 6 \cdot 3 &= 18 \end{aligned}$$

$$3. \text{ (B)} \quad \frac{120}{90} = \frac{4}{3} = 133\frac{1}{3}\%$$

$$4. \text{ (A)} \quad \begin{aligned} 500 &= 2x \\ 250 &= x \end{aligned}$$

$$5. \text{ (C)} \quad 137.5\% = 1.375$$

Retest

$$1. \text{ (C)} \quad .25\% = \frac{.25}{100} = \frac{25}{10,000} = \frac{1}{400}$$

$$2. \text{ (D)} \quad \frac{3}{4}\% = .75\% = .0075$$

$$3. \text{ (E)} \quad \begin{aligned} 12\% &= .12 \\ .12 \cdot 80 &= 9.6 \end{aligned}$$

$$4. \text{ (B)} \quad \begin{aligned} 18 &= .20x && \text{Divide by } .20. \\ 90 &= x \end{aligned}$$

$$5. \text{ (A)} \quad b\% = \frac{b}{100} \quad \frac{b}{100} \cdot \frac{3}{5} = \frac{3b}{50}$$

$$6. \text{ (B)} \quad \begin{aligned} 62\frac{1}{2}\% &= \frac{5}{8} \\ m &= \frac{5}{8}x && \text{Multiply by } 8. \\ 8m &= 5x && \text{Divide by } 5. \\ \frac{8m}{5} &= x \end{aligned}$$

$$7. \text{ (C)} \quad \frac{2}{12} = \frac{1}{6} = 16\frac{2}{3}\%$$

$$8. \text{ (E)} \quad \begin{aligned} 140\% &= 1.40 \\ 1.40 \cdot 70 &= 98 \end{aligned}$$

$$9. \text{ (C)} \quad 280\% = \frac{280}{100} = \frac{28}{10} = \frac{14}{5}$$

$$10. \text{ (A)} \quad \frac{16}{12} = \frac{4}{3} = 133\frac{1}{3}\%$$