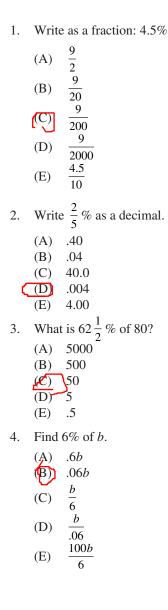
Percent



DIAGNOSTIC TEST

Directions: Work out each problem. Circle the letter that appears before your answer.

Answers are at the end of the chapter.



- 5. 80 is 40% of what number?
 - (A) 3200
 - (B) 320
 - (C) 32
 - (D) 200 (E) 20
- 6. $c ext{ is } 83 \frac{1}{3}\%$ of what number?
 - $\frac{5c}{6}$ $\frac{6c}{5}$ $\frac{7c}{8}$ $\frac{8c}{7}$ $\frac{2c}{3}$ (A) (B)

 - (C)
 - (D)
 - (E)
- 7. How many sixteenths are there in 87 $\frac{1}{2}$ %?
 - (A) 7
 - (B) 8
 - (C) 10
 - (D) 12 (E) 14
- 8. What percent of 40 is 16?
 - (A) $2\frac{1}{2}$ (B) 25
 - (**C**) 30
 - 40 (D)
 - (E) 45

70 Chapter 5

9. Find 112% of 80.	10. What percent of 60 is 72?
(A) 92 (B) 89.6	(A) 105 (B) 125
(C) 88	(B) 125 120
(D) 70.5 (E) 91	(D) $83\frac{1}{3}$
(L) 71	(E) 110 ³

1. FRACTIONAL AND DECIMAL EQUIVALENTS OF PERCENTS

Percent means "out of 100." If you understand this concept, it then becomes very easy to change a percent to an equivalent decimal or fraction.

Example:

5% means 5 out of 100 or $\frac{5}{100}$, which is equal to .05 3.4% means 3.4 out of 100 or $\frac{3.4}{100}$, which is equivalent to $\frac{34}{1000}$ or .034 c% means c out of 100 or $\frac{c}{100}$, which is equivalent to $\frac{1}{100} \cdot c$ or .01c $\frac{1}{4}$ % means $\frac{1}{4}$ out of 100 or $\frac{4}{100}$, which is equivalent to $\frac{1}{100} \cdot .25$ or .0025

To change a percent to a decimal, therefore, we must move the decimal point two places to the *left*, as we are dividing by 100.

Example:

62% = .62.4% = .004 3.2% = .032

To change a decimal to a percent, we must reverse the above steps. We multiply by 100, which has the effect of moving the decimal point two places to the *right*, and insert the percent sign.

Example:

$$.27 = 27\%$$

 $.012 = 1.2\%$
 $.003 = .3\%$

To change a percent to a fraction, we remove the percent sign and divide by 100. This has the effect of putting the percent over 100 and then simplifying the resulting fraction.

Example:

$$25\% = \frac{25}{100} = \frac{1}{4}$$
$$70\% = \frac{70}{100} = \frac{7}{10}$$
$$.5\% = \frac{.5}{100} = \frac{5}{1000} = \frac{1}{200}$$

To change a fraction to a percent, we must reverse the above steps. We multiply by 100 and insert the percent sign.

Example:

$$\frac{4}{5} = \frac{4}{5} \cdot \frac{20}{100} \% = 80\%$$
$$\frac{3}{8} = \frac{3}{8} \cdot \frac{25}{100} \% = \frac{75}{2} \% = 37\frac{1}{2}\%$$



Some fractions do not convert easily, as the denominator does not divide into 100. Such fractions must be changed to decimals first by dividing the numerator by the denominator. Then convert the decimal to a percent as explained on the previous page. Divide to two places only, unless it clearly comes out even in one or two additional places.

Example:

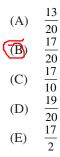
$\frac{8}{17} = 17 \overline{\big)8.00} = 47 \frac{1}{17} \%$	$\frac{4}{125} = 125 \overline{\big)} \frac{.032}{4.000} = 3.2\%$
<u>68</u>	<u>3 75</u>
1 20	250
1 19	250
1	

Certain fractional and decimal equivalents of common percents occur frequently enough so that they should be memorized. Learning the values in the following table will make your work with percent problems much easier.

DECIMAL	FRACTION
.5	$\frac{1}{2}$
.25	$\frac{1}{4}$
.75	$\frac{3}{4}$
.1	$\frac{1}{10}$
.3	$\frac{3}{10}$
.7	$\frac{7}{10}$
.9	$\frac{9}{10}$
.33	$\frac{1}{3}$
.66	$\frac{2}{3}$
.16	$\frac{1}{6}$
.83	$\frac{5}{6}$
.2	$\frac{1}{5}$
.4	$\frac{2}{5}$
.6	$\frac{3}{5}$
.8	$\frac{4}{5}$
.125	$\frac{1}{8}$
.375	$ \frac{1}{2} \\ \frac{1}{4} \\ \frac{3}{3} \\ \frac{1}{4} \\ \frac{1}{10} \\ \frac{3}{10} \\ \frac{7}{7} \\ \frac{10}{9} \\ \frac{9}{10} \\ \frac{1}{3} \\ \frac{2}{3} \\ \frac{1}{6} \\ \frac{5}{6} \\ \frac{1}{5} \\ \frac{2}{5} \\ \frac{3}{5} \\ \frac{4}{5} \\ \frac{1}{8} \\ \frac{3}{8} \\ \frac{5}{8} \\ \frac{7}{8} \\ \frac{7}{8} \\ \frac{1}{8} \\ \frac{1}{8} \\ \frac{3}{8} \\ \frac{5}{8} \\ \frac{7}{8} \\ \frac{1}{8} \\ \frac{1}{8} \\ \frac{3}{8} \\ \frac{5}{8} \\ \frac{7}{8} \\ \frac{1}{8} \\ \frac{1}{8} \\ \frac{5}{8} \\ \frac{7}{8} \\ \frac{1}{8} \\ \frac{1}{8} \\ \frac{5}{8} \\ \frac{7}{8} \\ \frac{1}{8} \\$
.625	$\frac{5}{8}$
.875	$\frac{7}{8}$
	.5 .25 .75 .1 .3 .7 .9 $.3\overline{3}$ $.6\overline{6}$ $.1\overline{6}$ $.8\overline{3}$.2 .4 .6 .8 .125 .375 .625

Work out each problem. Circle the letter that appears before your answer.

- 1. $3\frac{1}{2}$ % may be written as a decimal as
 - (A) 3.5
 - (B) .35
 - .035
 - (D) .0035
 - (E) 3.05
- 2. Write as a fraction in simplest form: 85%.



(A) 41% (B) 41.6% (D) $41\frac{2}{3}\%$ (D) 4.1%

4. Write $\frac{5}{12}$ as an equivalent percent.

- (E) $.41\frac{2}{3}\%$
- 5. Write $\frac{1}{2}$ % as a decimal.
 - (A) .5
 - **B** .005
 - (C) 5.0
 - (D) 50.0
 - (E) .05

- Write 4.6 as a percent. 3.
 - (A) 4.6%
 - (B) .46%
 - (C) .046%
 - (D) 46%
 - (E) 460%

2. FINDING A PERCENT OF A NUMBER

Most percentage problems can be solved by using the proportion

 $\frac{\%}{100} = \frac{\text{part}}{\text{whole}}$.

Although this method will work, it often yields unnecessarily large numbers that make for difficult computation. As we look at different types of percent problems, we will compare methods of solution. In finding a percent of a number, it is usually easier to change the percent to an equivalent decimal or fraction and multiply by the given number.

Example:

Proportion Method	<i>Decimal Method</i> Change 32% to .32 and multiply.
$\frac{32}{100} = \frac{x}{84}$ $100x = 2688$ $x = 26.88$	$84 \times \underline{.32} \\ 168 \\ \underline{252} \\ 26.88$

Example:

Find $12\frac{1}{2}$ % of 112.		
Proportion Method	Decimal Method	Fraction Method
$\frac{12\frac{1}{2}}{100} = \frac{x}{112}$ $100x = 1400$ $x = 14$	$ \begin{array}{r} 112 \\ \times \underline{.125} \\ 560 \\ 2 24 \\ \underline{11 2} \\ 14.000 \end{array} $	Change $12\frac{1}{2}\%$ to $\frac{1}{8}$ $\frac{1}{8}\cdot\cancel{14}{2}=14$

Which method do you think is the easiest? When the fractional equivalent of the required percent is among those given in the previous chart, the fraction method is by far the least time-consuming. It really pays to memorize those fractional equivalents.

Work out each problem. Circle the letter that appears before your answer.

 What is 40% of 40? .16 (B) 1.6 (C) . 16 (D) 160 (E) 1600 	4. What is $\frac{1}{5}\%$ of 40? (A) 8 (B) .8 (C) .08 (D) .008 (E) .0008
 2. What is 42% of 67? (A) 2814 (B) 281.4 (C) 2.814 (D) .2814 (D) 28.14 	5. Find $r\%$ of s . (A) $\frac{100s}{r}$ (B) $\frac{rs}{100}$ (C) $\frac{100r}{s}$
3. Find $16\frac{2}{3}\%$ of 120. (A) 20 (B) 2 (C) 200 (D) 16 (E) 32	(D) $\frac{r}{100s}$ (E) $\frac{s}{100r}$

3. FINDING A NUMBER WHEN A PERCENT OF IT IS GIVEN

This type of problem may be solved using the proportion method, although this may again result in the unnecessary use of time. It is often easier to translate the words of such a problem into an algebraic statement, using decimal or fractional equivalents for the percents involved. Then it will become evident that we divide the given number by the given percent to solve.

Example:

7 is 5% of what number?Proportion MethodEquation Method $\frac{5}{100} = \frac{7}{x}$ 7 = .05x5x = 700700 = 5xx = 140140 = x

Example:

40 is $66\frac{2}{3}$ % of what number? Proportion Method Equation Method $\frac{66\frac{2}{3}}{100} = \frac{40}{x}$ $66\frac{2}{3}x = 4000$ $\frac{40}{3} = \frac{2}{3}x$ 120 = 2x 60 = x 200x = 12000 2x = 120 x = 60

Just think of the amount of time you will save and the extra problems you will get to do if you know that $66\frac{2}{3}\%$ is $\frac{2}{3}$ and use the equation method. Are you convinced that the common fraction equivalents in the previously given chart should be memorized?

Work out each problem. Circle the letter that appears before your answer.

- 1. 72 is 12% of what number?
 - (A) 6
 - (B) 60
 - **600**
 - (D) 86.4
 - (E) 8.64
- 2. 80 is $12\frac{1}{2}$ % of what number?
 - (A) 10
 - (B) 100
 - (C) 64
 - (1) 640 (E) 6400
- 3. $37\frac{1}{2}$ % of what number is 27?
 - A 72
 - (B) $10\frac{1}{8}$
 - (C) 90
 - (D) 101.25
 - (E) 216

(A) 100 100p(B) т т (C) 100*p* $\frac{p}{100m}$ (D) 100m(E) р 5. 50% of what number is r? $\frac{1}{2}r$ (A)

4. m is p% of what number?

тp

- (B) 5*r*
- (C) 10*r*
- (D) 2r
- (E) 100r

4. TO FIND WHAT PERCENT ONE NUMBER IS OF ANOTHER

This type of problem may also be solved using the proportion method. However, this may again result in the use of an unnecessary amount of time. It is often easier to put the part over the whole, simplify the resulting fraction, and multiply by 100.

Example:

30 is what percent of 1500?	
Proportion Method	Fraction Method
$\frac{x}{100} = \frac{30}{1500}$ $1500x = 3000$ $x = 2\%$	$\frac{30}{1500} = \frac{3}{150} = \frac{1}{50} \cdot 100 = 2\%$

Example:

12 is what percent of 72?

Proportion Method	Fraction Method
$\frac{x}{100} = \frac{12}{72}$ $72x = 1200$	$\frac{12}{72} = \frac{1}{6} = 16\frac{2}{3}\%$

Time consuming long division is needed to find $x = 16\frac{2}{3}$ %. If you have memorized the fractional equivalents of common percents, this method requires only a few seconds.

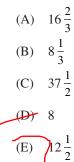
Example:

What percent of 72 is 16?

Proportion Method	Fraction Method
$\frac{x}{100} = \frac{16}{72}$ $72x = 1600$ $x = 22\frac{2}{9}\%$	$\frac{16}{72} = \frac{2}{9} \cdot 100 = \frac{200}{9} = 22\frac{2}{9}\%$

Work out each problem. Circle the letter that appears before your answer.

- 1. 4 is what percent of 80?
 - (A) 20
 - (B) 2
 - 5
 - (D) .5
 - (E) 40
- 2. $\frac{1}{2}$ of 6 is what percent of $\frac{1}{4}$ of 60? (A) 5
 - (A) 5 -(B) 20
 - (C) 10
 - (D) 25
 - (E) 15
- 3. What percent of 96 is 12?



- 4. What percent of 48 is 48?
 - (A) 1
 - (B) 10
 - (D) 100 (D) 48
 - (D) +c(E) 0
- 5. What percent of y is x?

(A)
$$\frac{x}{y}$$

- (B) $\frac{x}{100y}$
- (C) xy
- (C) $\frac{xy}{100}$ (D) $\frac{100x}{y}$ (E) $\frac{100y}{y}$

x

5. PERCENTS GREATER THAN 100

When the percentage involved in a problem is greater than 100, the same methods apply. Remember that 100% = 1; 200% = 2; 300% = 3 and so forth. Therefore 150% will be equal to 100% + 50% or $1\frac{1}{2}$. Let us look at one example of each previously discussed problem, using percents greater than 100.

Example:

Find 175% of 60

Proportion Method	Decimal Method	Fraction Method
	60	
$\frac{175}{100} = \frac{x}{60}$ $100x = 10500$ $x = 105$	$\times \underline{1.75}$ 300 4200 $\underline{6000}$	$1\frac{3}{4} \cdot 60$ $\frac{7}{\cancel{4}} \cdot \cancel{60}^{15} = 105$
	105.00	

Example:

80 is 125% of what number?

Proportion Method	Decimal Method	Fraction Method
$\frac{125}{100} = \frac{80}{x}$ $125x = 8000$ $x = 64$	80 = 1.25x 8000 = 125x	$80 = 1\frac{1}{4}x$ $80 = \frac{5}{4}x$
	x = 64	320 = 5x $x = 64$

Example:

40 is what percent of 30?

Proportion Method	Fraction Method
$x_{-}40$	
100 30	
30x = 4000	40 4 1 122 1 π
$x = 133\frac{1}{3}\%$	$\frac{40}{30} = \frac{4}{3} = 1\frac{1}{3} = 133\frac{1}{3}\%$

Work out each problem. Circle the letter that appears before your answer.

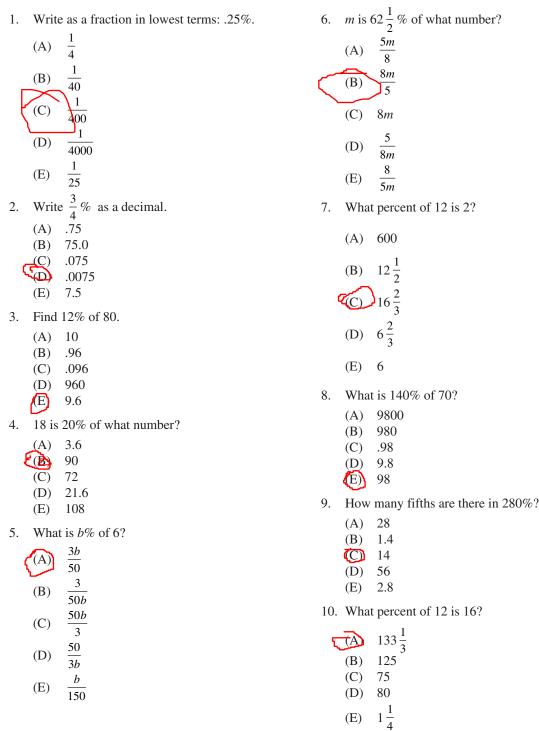
- 1. 36 is 150% of what number?
 - (B) 24 (B) 54

 - (C) 26
 - (D) 12
 - (E) 48
- 2. What is 300% of 6?
 - (A) 2
 - (B) 3
 - (C) 12
 - (D) 18
 - (E) 24
- 3. What percent of 90 is 120?
 - (A) 75 (B) $133\frac{1}{3}$ (C) 125 (D) 120 (E) $1\frac{1}{3}$

- 500 is 200% of what number? 4.
 - (A) 250
 - (B) 1000
 - (C) 100
 - (D) 750
 - (E) 300
- 5. To multiply a number by $137 \frac{1}{2}$ %, the number should be multiplied by
 - (A) 137.5
 - (B) 13750
 - **(C)** 1.375
 - (D) 13.75 (E) .1375

RETEST

Work out each problem. Circle the letter that appears before your answer.



SOLUTIONS TO PRACTICE EXERCISES

Diagnostic Test

1. (C)
$$4.5\% = \frac{4.5}{100} = \frac{45}{1000} = \frac{9}{200}$$

2. (D)
$$\frac{2}{5}\% = .4\% = .004$$

3. (C)
$$62\frac{1}{2}\% = \frac{5}{8} \cdot \frac{5}{8} \cdot \frac{5}{80} = 50$$

- 4. (B) 6% = .06 $.06 \cdot b = .06b$
- 5. (D) 80 = .40x Divide by .40. 200 = x

6. (B)
$$83\frac{1}{3}\% = \frac{5}{6}$$
 Multiply by 6. Divide by 5.
 $c = \frac{5}{6}x$
 $6c = 5x$
 $\frac{6c}{5} = x$

7. (E)
$$87\frac{1}{2}\% = \frac{7}{8} = \frac{14}{16}$$

8. (D)
$$\frac{10}{40} = \frac{2}{5} = 40\%$$

9. (B)
$$112\% = 1.12$$

 $1.12 \cdot 80 = 89.6$

10. (C)
$$\frac{72}{60} = \frac{6}{5} = 120\%$$

Exercise 1

1. (C) $3\frac{1}{2}\% = 3.5\% = .035$ To change a percent to a decimal, move the decimal point two places to the *left*.

2. (B)
$$85\% = \frac{85}{100} = \frac{17}{20}$$

3. (E) To change a decimal to a percent, move the decimal point two places to the *right*.
4.6 = 460%

4. (C)
$$\frac{5}{\cancel{12}} \cdot \cancel{100}_{3}^{25} = \frac{125}{3} = 41\frac{2}{3}\%$$

To change a fraction to a percent, multiply by 100.

5. (B)
$$\frac{1}{2}$$
 % = .5% = .005

Exercise 2

1. (C)
$$40\% = \frac{2}{5}$$

 $\frac{2}{5} \cdot \frac{8}{40} = 16$
2. (E) 67
 $\times \frac{42}{134}$
 $\frac{26\ 80}{28.14}$
3. (A) $16\frac{2}{3}\% = \frac{1}{6}$
 $\frac{1}{6} \cdot \frac{120}{20} = 20$
4. (C) $\frac{1}{5}\% = .2\% = .002$
 40
 $\times \frac{.002}{.0800}$
5. (B) $r\% = \frac{r}{100}$
 $\frac{r}{100} \cdot s = \frac{rs}{100}$

11		-
Exer	cise	-5
		_

1. (C)
$$72 = .12x$$

 $7200 = 12x$
 $x = 600$
2. (D) $80 = \frac{1}{8}x$
 $640 = x$
3. (A) $\frac{3}{8}x = 27$
 $3x = 216$
 $x = 72$
4. (E) $m = \frac{p}{100} \cdot x$
 $100m = px$
 $\frac{100m}{p} = x$
5. (D) $\frac{1}{2}x = r$
 $x = 2r$

Exercise 4 4 1 ⁵

1. (C)
$$\frac{4}{80} = \frac{1}{20} \cdot 100 = 5\%$$

2. (B) $\frac{1}{2}$ of 6 = 3
 $\frac{1}{4}$ of 60 = 15
 $\frac{3}{15} = \frac{1}{5} = 20\%$

3. (E) $\frac{12}{96} = \frac{1}{8} = 12\frac{1}{2}\%$ 4. (C) $\frac{48}{48} = 1 = 100\%$

5. (D)
$$\frac{x}{y} \cdot 100 = \frac{100x}{y}$$

Exercise 5

1. (A)
$$36 = 1\frac{1}{2}x$$

 $36 = \frac{3}{2}x$
 $72 = 3x$
 $x = 24$
2. (D) $300\% = 3$
 $6 \cdot 3 = 18$
3. (B) $\frac{120}{90} = \frac{4}{3} = 133\frac{1}{3}\%$
4. (A) $500 = 2x$
 $250 = x$

5. (C) 137.5% = 1.375

Retest

1. (C)
$$.25\% = \frac{.25}{100} = \frac{25}{10,000} = \frac{1}{400}$$

2. (D) $\frac{3}{4}\% = .75\% = .0075$
3. (E) $12\% = .12$

. (E)
$$12\% = .12$$

 $.12 \cdot 80 = 9.6$

4. (B)
$$18 = .20x$$
 Divide by .20.
 $90 = x$

5. (A)
$$b\% = \frac{b}{100}$$
 $\frac{b}{100} \cdot \frac{3}{6} = \frac{3b}{50}$
6. (B) $62\frac{1}{2}\% = \frac{5}{8}$
 $m = \frac{5}{8}x$ Multiply by 8.
 $8m = 5x$ Divide by 5.
 $\frac{8m}{5} = x$
7. (C) $\frac{2}{12} = \frac{1}{6} = 16\frac{2}{3}\%$
8. (E) $140\% = 1.40$
 $1.40 \cdot 70 = 98$
9. (C) $280\% = \frac{280}{100} = \frac{28}{10} = \frac{14}{5}$

10. (A)
$$\frac{16}{12} = \frac{4}{3} = 133\frac{1}{3}\%$$