# Averages

7

## DIAGNOSTIC TEST

**Directions: Work out each problem. Circle the letter that appears before your answer.**

**Answers are at the end of the chapter.**

1. Find the average of the first ten positive even integers.
   (A) 9
   (B) 10
   (C) 11
   (D) 12
   (E) $5\frac{1}{2}$

2. What is the average of $x - 4$, $x$, and $x + 4$?
   (A) $3x$
   (B) $x$
   (C) $x - 1$
   (D) $x + 1$
   (E) $\frac{3x-8}{3}$

3. Find the average of $\sqrt{.09}$, .4, and $\frac{1}{2}$.
   (A) .31
   (B) .35
   (C) .04
   (D) .4
   (E) .45

4. Valerie received test grades of 93 and 88 on her first two French tests. What grade must she get on the third test to have an average of 92?
   (A) 95
   (B) 100
   (C) 94
   (D) 96
   (E) 92

5. The average of W and another number is A. Find the other number.
   (A) $A - W$
   (B) $A + W$
   (C) $\frac{1}{2}(A - W)$
   (D) $\frac{1}{2}(A + W)$
   (E) $2A - W$

6. The weight of three packages are 4 lb. 10 oz., 6 lb. 13 oz, and 3 lb. 6 oz. Find the average weight of these packages.
   (A) 4 lb. 43 oz.
   (B) 4 lb. $7\frac{1}{2}$ oz.
   (C) 4 lb. 15 oz.
   (D) 4 lb. 6 oz.
   (E) 4 lb. 12 oz.

7. If Barbara drove for 4 hours at 50 miles per hour and then for 2 more hours at 60 miles per hour, what was her average rate, in miles per hour, for the entire trip?
   (A) 55
   (B) $53\frac{1}{3}$
   (C) $56\frac{2}{3}$
   (D) 53
   (E) $54\frac{1}{2}$

8. Mr. Maron employs three secretaries at a salary of $140 per week and five salespeople at a salary of $300 per week. What is the average weekly salary paid to an employee?
   (A) $55
   (B) $190
   (C) $240
   (D) $200
   (E) $185

9. Which of the following statements are always true?
   I. The average of any three consecutive even integers is the middle integer.
   II. The average of any three consecutive odd integers is the middle integer.
   III. The average of any three consecutive multiples of 5 is the middle number.
   (A) I only
   (B) II only
   (C) I and II only
   (D) I and III only
   (E) I, II, and III

10. Mark has an average of 88 on his first four math tests. What grade must he earn on his fifth test in order to raise his average to 90?
   (A) 92
   (B) 94
   (C) 96
   (D) 98
   (E) 100

# 1. SIMPLE AVERAGE

Most students are familiar with the method for finding an average and use this procedure frequently during the school year. To find the average of *n* numbers, find the sum of all the numbers and divide this sum by *n*.

***Example:***

Find the average of 12, 17, and 61.

***Solution:***

$$\begin{array}{r} 12 \\ 17 \\ +\quad 61 \\ \hline 3\overline{)90} \\ 30 \end{array}$$

When the numbers to be averaged form an evenly spaced series, the average is simply the middle number. If we are finding the average of an even number of terms, there will be no middle number. In this case, the average is halfway between the two middle numbers.

***Example:***

Find the average of the first 40 positive even integers.

***Solution:***

Since these 40 addends are evenly spaced, the average will be half way between the 20th and 21st even integers. The 20th even integer is 40 (use your fingers to count if needed) and the 21st is 42, so the average of the first 40 positive even integers that range from 2 to 80 is 41.

The above concept must be clearly understood as it would use up much too much time to add the 40 numbers and divide by 40. Using the method described, it is no harder to find the average of 100 evenly spaced terms than it is of 40 terms.

In finding averages, be sure the numbers being added are all of the same form or in terms of the same units. To average fractions and decimals, they must all be written as fractions or all as decimals.

***Example:***

Find the average of $87\frac{1}{2}\%$, $\frac{1}{4}$, and .6

***Solution:***

Rewrite each number as a decimal before adding.

$$\begin{array}{r} .875 \\ .25 \\ +\ \ .6 \\ \hline 3\overline{)1.725} \\ .575 \end{array}$$

## Exercise 1

Work out each problem. Circle the letter that appears before your answer.

1. Find the average of $\sqrt{.49}$, $\frac{3}{4}$, and 80%.
   (A) .72
   (B) .75
   (C) .78
   (D) .075
   (E) .073

2. Find the average of the first 5 positive integers that end in 3.
   (A) 3
   (B) 13
   (C) 18
   (D) 23
   (E) 28

3. The five men on a basketball team weigh 160, 185, 210, 200, and 195 pounds. Find the average weight of these players.
   (A) 190
   (B) 192
   (C) 195
   (D) 198
   (E) 180

4. Find the average of $a$, $2a$, $3a$, $4a$, and $5a$.
   (A) $3a^5$
   (B) $3a$
   (C) $2.8a$
   (D) $2.8a^5$
   (E) 3

5. Find the average of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.
   (A) $\frac{1}{9}$
   (B) $\frac{13}{36}$
   (C) $\frac{1}{27}$
   (D) $\frac{13}{12}$
   (E) $\frac{1}{3}$

# 2. TO FIND A MISSING NUMBER WHEN AN AVERAGE IS GIVEN

In solving this type of problem, it is easiest to use an algebraic equation that applies the definition of average. That is,

$$\text{average} = \frac{\text{sum of terms}}{\text{number of terms}}$$

***Example:***

The average of four numbers is 26. If three of the numbers are 50, 12, and 28, find the fourth number.

***Solution:***

$$\frac{50+12+28+x}{4} = 26$$
$$50+12+28+x = 104$$
$$90+x = 104$$
$$x = 14$$

An alternative method of solution is to realize that the number of units below 26 must balance the number of units above 26. 50 is 24 units *above* 26. 12 is 14 units *below* 26. 28 is 2 units *above* 26. Therefore, we presently have 26 units (24 + 2) *above* 26 and only 14 units *below* 26. Therefore the missing number must be 12 units *below* 26, making it 14. When the numbers are easy to work with, this method is usually the fastest. Just watch your arithmetic.

## *Exercise 2*

Work out each problem. Circle the letter that appears before your answer.

1. Dick's average for his freshman year was 88, his sophomore year was 94, and his junior year was 91. What average must he have in his senior year to leave high school with an average of 92?
   (A) 92
   (B) 93
   (C) 94
   (D) 95
   (E) 96

2. The average of $X$, $Y$, and another number is $M$. Find the missing number.
   (A) $3M - X + Y$
   (B) $3M - X - Y$
   (C) $\frac{M+X+Y}{3}$
   (D) $M - X - Y$
   (E) $M - X + Y$

3. The average of two numbers is $2x$. If one of the numbers is $x + 3$, find the other number.
   (A) $x - 3$
   (B) $2x - 3$
   (C) $3x - 3$
   (D) $-3$
   (E) $3x + 3$

4. On consecutive days, the high temperature in Great Neck was 86°, 82°, 90°, 92°, 80°, and 81°. What was the high temperature on the seventh day if the average high for the week was 84°?
   (A) 79°
   (B) 85°
   (C) 81°
   (D) 77°
   (E) 76°

5. If the average of five consecutive integers is 17, find the largest of these integers.
   (A) 17
   (B) 18
   (C) 19
   (D) 20
   (E) 21

# 3. WEIGHTED AVERAGE

When some numbers among terms to be averaged occur more than once, they must be given the appropriate weight. For example, if a student received four grades of 80 and one of 90, his average would not be the average of 80 and 90, but rather the average of 80, 80, 80, 80, and 90.

***Example:***

Mr. Martin drove for 6 hours at an average rate of 50 miles per hour and for 2 hours at an average rate of 60 miles per hour. Find his average rate for the entire trip.

***Solution:***

$$\frac{6(50)+2(60)}{8}=\frac{300+120}{8}=\frac{420}{8}=52\frac{1}{2}$$

Since he drove many more hours at 50 miles per hour than at 60 miles per hour, his average rate should be closer to 50 than to 60, which it is. In general, average rate can always be found by dividing the total distance covered by the total time spent traveling.

## *Exercise 3*

Work out each problem. Circle the letter that appears before your answer.

1. In a certain gym class, 6 girls weigh 120 pounds each, 8 girls weigh 125 pounds each, and 10 girls weigh 116 pounds each. What is the average weight of these girls?
   (A) 120
   (B) 118
   (C) 121
   (D) 122
   (E) 119

2. In driving from San Francisco to Los Angeles, Arthur drove for three hours at 60 miles per hour and for 4 hours at 55 miles per hour. What was his average rate, in miles per hour, for the entire trip?
   (A) 57.5
   (B) 56.9
   (C) 57.1
   (D) 58.2
   (E) 57.8

3. In the Linwood School, five teachers earn \$15,000 per year, three teachers earn \$17,000 per year, and one teacher earns \$18,000 per year. Find the average yearly salary of these teachers.
   (A) \$16,667
   (B) \$16,000
   (C) \$17,000
   (D) \$16,448
   (E) \$16,025

4. During the first four weeks of summer vacation, Danny worked at a camp earning \$50 per week. During the remaining six weeks of vacation, he worked as a stock boy earning \$100 per week. What was his average weekly wage for the summer?
   (A) \$80
   (B) \$75
   (C) \$87.50
   (D) \$83.33
   (E) \$82

5. If $M$ students each received a grade of $P$ on a physics test and $N$ students each received a grade of $Q$, what was the average grade for this group of students?
   (A) $\frac{P+Q}{M+N}$
   (B) $\frac{PQ}{M+N}$
   (C) $\frac{MP+NQ}{M+N}$
   (D) $\frac{MP+NQ}{P+Q}$
   (E) $\frac{M+N}{P+Q}$

# RETEST

Work out each problem. Circle the letter that appears before your answer.

1. Find the average of the first 14 positive odd integers.
   (A) 7.5
   (B) 13
   (C) 14
   (D) 15
   (E) 14.5

2. What is the average of $2x - 3$, $x + 1$, and $3x + 8$?
   (A) $6x + 6$
   (B) $2x - 2$
   (C) $2x + 4$
   (D) $2x + 2$
   (E) $2x - 4$

3. Find the average of $\frac{1}{5}$, 25%, and .09.
   (A) $\frac{2}{3}$
   (B) .18
   (C) .32
   (D) 20%
   (E) $\frac{1}{4}$

4. Andy received test grades of 75, 82, and 70 on three French tests. What grade must he earn on the fourth test to have an average of 80 on these four tests?
   (A) 90
   (B) 93
   (C) 94
   (D) 89
   (E) 96

5. The average of $2P$, $3Q$, and another number is $S$. Represent the third number in terms of $P$, $Q$, and $S$.
   (A) $S - 2P - 3Q$
   (B) $S - 2P + 3Q$
   (C) $3S - 2P + 3Q$
   (D) $3S - 2P - 3Q$
   (E) $S + 2P - 3Q$

6. The students of South High spent a day on the street collecting money to help cure birth defects. In counting up the collections, they found that 10 cans contained \$5.00 each, 14 cans contained \$6.50 each, and 6 cans contained \$7.80 each. Find the average amount contained in each of these cans.
   (A) \$6.14
   (B) \$7.20
   (C) \$6.26
   (D) \$6.43
   (E) \$5.82

7. The heights of the five starters on the Princeton basketball team are 6′ 6″, 6′ 7″, 6′ 9″, 6′ 11″, and 7′. Find the average height of these men.
   (A) $6' \, 8\frac{1}{5}''$
   (B) $6' \, 9''$
   (C) $6' \, 9\frac{3}{5}''$
   (D) $6' \, 9\frac{1}{5}''$
   (E) $6' \, 9\frac{1}{2}''$

8. Which of the following statements is always true?
   I. The average of the first twenty odd integers is 10.5
   II. The average of the first ten positive integers is 5.
   III. The average of the first 4 positive integers that end in 2 is 17.
   (A) I only
   (B) II only
   (C) III only
   (D) I and III only
   (E) I, II, and III

9. Karen drove 40 miles into the country at 40 miles per hour and returned home by bus at 20 miles per hour. What was her average rate in miles per hour for the round trip?

(A) 30

(B) $25\frac{1}{2}$

(C) $26\frac{2}{3}$

(D) 20

(E) $27\frac{1}{3}$

10. Mindy's average monthly salary for the first four months she worked was $300. What must be her average monthly salary for each of the next 8 months so that her average monthly salary for the year is $350?

(A) $400
(B) $380
(C) $390
(D) $375
(E) $370

# SOLUTIONS TO PRACTICE EXERCISES

## Diagnostic Test

1. (C) The integers are 2, 4, 6, 8, 10, 12, 14, 16, 18, 20. Since these are evenly spaced, the average is the average of the two middle numbers, 10 and 12, or 11.

2. (B) These numbers are evenly spaced, so the average is the middle number $x$.

3. (D) $\sqrt{.09} = .3$

$$\frac{1}{2} = .5$$
$$.4 = .4$$
$$3\overline{)\,1.2}$$
$$.4$$

4. (A) 93 is 1 above 92; 88 is 4 below 92. So far, she has 1 point above 92 and 4 points below 92. Therefore, she needs another 3 points above 92, making a required grade of 95.

5. (E) $\frac{W+x}{2} = A$

$W + x = 2A$

$x = 2A - W$

6. (C)

$$\begin{array}{r} 4 \text{ lb. } 10 \text{ oz.} \\ 6 \text{ lb. } 13 \text{ oz.} \\ +\ \ 3 \text{ lb. } \ \ 6 \text{ oz.} \\ \hline 13 \text{ lb. } 29 \text{ oz.} \end{array}$$

$\frac{13\text{lb. } 29\text{oz.}}{3} = \frac{12\text{lb. } 45\text{oz.}}{3} = 4$ lb. 15 oz.

7. (B)

$$4(50) = 200$$
$$2(60) = \underline{120}$$
$$6\overline{)320}$$
$$53\frac{1}{3}$$

8. (C)

$$3(140) = 420$$
$$5(300) = \underline{1500}$$
$$8\overline{)1920}$$
$$240$$

9. (E) The average of any three numbers that are evenly spaced is the middle number.

10. (D) Since 88 is 2 below 90, Mark is 8 points below 90 after the first four tests. Thus, he needs a 98 to make the required average of 90.

## Exercise 1

1. (B) $\sqrt{.49} = .7$

$$\frac{3}{4} = .75$$
$$80\% = \underline{.80}$$
$$3\overline{)2.25}$$
$$.75$$

2. (D) The integers are 3, 13, 23, 33, 43. Since these are evenly spaced, the average is the middle integer, 23.

3. (A) 160 + 185 + 210 + 200 + 195 = 950

$\frac{950}{5} = 190$

4. (B) These numbers are evenly spaced, so the average is the middle number, $3a$.

5. (B) $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{6}{12} + \frac{4}{12} + \frac{3}{12} = \frac{13}{12}$

To divide this sum by 3, multiply by $\frac{1}{3}$

$\frac{13}{12} \cdot \frac{1}{3} = \frac{13}{36}$

## Exercise 2

1. (D) 88 is 4 below 92; 94 is 2 above 92; 91 is 1 below 92. So far, he has 5 points below 92 and only 2 above. Therefore, he needs another 3 points above 92, making the required grade 95.

2. (B) $\frac{X+Y+x}{3} = M$

   $X + Y + x = 3M$

   $x = 3M - X - Y$

3. (C) $\frac{(x+3)+n}{2} = 2x$

   $x+3+n = 4x$

   $n = 3x - 3$

4. (D) 86° is 2 above the average of 84; 82° is 2 below; 90° is 6 above; 92° is 8 above; 80° is 4 below; and 81° is 3 below. So far, there are 16° above and 9° below. Therefore, the missing term is 7° below the average, or 77°.

5. (C) 17 must be the middle integer, since the five integers are consecutive and the average is, therefore, the middle number. The numbers are 15, 16, 17, 18, and 19.

## Exercise 3

1. (A)
$$\begin{array}{r} 6(120) = \phantom{0}720 \\ 8(125) = 1000 \\ 10(116) = \underline{1160} \\ 24\overline{)2880} \\ 120 \end{array}$$

2. (C)
$$\begin{array}{r} 3(60) = 180 \\ 4(55) = \underline{220} \\ 7\overline{)400} \\ 57\frac{1}{7}, \end{array}$$

   which is 57.1 to the nearest tenth.

3. (B)
$$\begin{array}{r} 5(15{,}000) = 75{,}000 \\ 3(17{,}000) = 51{,}000 \\ 1(18{,}000) = \underline{18{,}000} \\ 9\overline{)144{,}000} \\ 16{,}000 \end{array}$$

4. (A)
$$\begin{array}{r} 4(50) = 200 \\ 6(100) = \underline{600} \\ 10\overline{)800} \\ 80 \end{array}$$

5. (C)
$$\begin{array}{r} M(P) = MP \\ N(Q) = NQ \\ \hline MP + NQ \end{array}$$

   Divide by the number of students, $M + N$.

## Retest

1. (C) The integers are 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27. Since these are evenly spaced, the average is the average of the two middle numbers 13 and 15, or 14.

2. (D)
$$\begin{array}{r} 2x - 3 \\ x + 1 \\ \underline{+3x + 8} \\ 6x + 6 \end{array}$$
$$\frac{6x+6}{3} = 2x + 2$$

3. (B)
$$\frac{1}{5} = .20$$
$$25\% = .25$$
$$\begin{array}{r} .09 = \underline{.09} \\ 3\overline{)\underline{.54}} \\ .18 \end{array}$$

4. (B) 75 is 5 below 80; 82 is 2 above 80; 70 is 10 below 80. So far, he is 15 points below and 2 points above 80. Therefore, he needs another 13 points above 80, or 93.

5. (D)
$$\frac{2P + 3Q + x}{3} = S$$
$$2P + 3Q + x = 3S$$
$$x = 3S - 2P - 3Q$$

6. (C)
$$10(\$5.00) = \$50$$
$$14(\$6.50) = \$91$$
$$6(\$7.80) = \$46.80$$
$$30\overline{)\$187.80}$$
$$\$6.26$$

7. (B) 6′6″ + 6′7″ + 6′11″ + 6′9″ + 7′ = 31′33″ = 33′9″
$$\frac{33'9''}{5} = 6'9''$$

8. (C) I. The average of the first twenty *positive* integers is 10.5.

   II. The average of the first ten positive integers is 5.5.

   III. The first four positive integers that end in 2 are 2, 12, 22, and 32. Their average is 17.

9. (C) Karen drove for 1 hour into the country and returned home by bus in 2 hours. Since the total distance traveled was 80 miles, her average rate for the round trip was $\frac{80}{3}$ or $26\frac{2}{3}$ miles per hour.

10. (D) Since \$300 is \$50 below \$350, Mindy's salary for the first four months is \$200 below \$350. Therefore, her salary for each of the next 8 months must be $\frac{\$200}{8}$ or \$25 above the average of \$350, thus making the required salary \$375.